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Section : A

Subject : Differential Equation

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Date : 20 August 20

Question -1 ①

$$\frac{dy}{dt} = e^{y-t} \; Sec(y) \; (1+t^2) \quad y(0) = 0$$

Sol:-

$$\frac{dy}{dt} = e^{y-t} \; Sec(y) \; (1+t^2) \quad y(0) = 0$$

So $x = 0 \quad y = 0$

$$dy = e^{y}.e^{-t} \; Sec(y) \; (1+t^2) \; dt$$

$$\frac{1}{e^{y}.Sec(y)} dy = (1+t^2) \, e^{-t} \, dt$$

As $\cos(y) = \frac{1}{Sec(y)}$

$$\Rightarrow \int e^{-y} \cos y \, dy = \int (1+t^2) \, e^{-t} \, dt$$

using integration by parts

$$\Rightarrow e^{-y} \int \cos dx - \int \left( \int \cos y . \frac{d}{dy} e^{-y} \right) = (1+t^2)$$

$$\int e^{-t} - \int \left( \int e^{-t} \frac{d}{dt} (1+t^2) \right. \quad eq(1)$$

L.H.S

$$e^{-y} \int \cos(y) \, dx - \int \left( \int \cos y . \frac{d}{dx} e^{-y} \right.$$

$$e^{-y} \sin(y) - \int (\sin y . e^{-y} (-1)$$

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$e^{-y}\sin y + \int (\sin y \cdot e^{-y})$

$e^{-y}\sin y + \int (e^{-y}\sin y)$

Again using integration by parts

$e^{-y}\sin y + e^{-y}(\cos y) - \int (\sin y \frac{d}{dy} e^{-y}$

$e^{-y}\sin y + e^{-y}(-\cos y) - \int -\cos y \frac{e^{-y}}{-1}$

$e^{-y}\sin y - e^{-y}\cos y - \int (\cos y\, e^{-y})$

Since $\int (\cos y e^{-y}) = L.H.S$

Since it is again L.H.S to the first one so L.H.S will become

$L.H.S = e^{-y}(\sin y - \cos y) - L.H.S$

$2L.H.S = e^{-y}(\sin y - \cos y)$

$L.H.S = \frac{e^{-y}(\sin y - \cos y)}{2}$

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Now taking R.H.S

$\Rightarrow \int (1+t^2)\, e^{-t}\, dt$

$\Rightarrow (1+t^2) \int e^{-t} \int \left( \int e^{-t} \frac{d}{dt} (1+t^2) \right)$

$\Rightarrow (1+t^2)\, e^{-t} - \int \left( -e^{-t} (2t) \right)$

$\Rightarrow -(1+t^2)\, e^{-t} + \int (2t)\, \dot{e} )$

Again using integration by parts

$\Rightarrow -(1+t^2)\, e^{-t} + \left( 2t \int e^{-t} - \int \left( \int e^{-t} \frac{d}{dt} 2t \right) \right)$

$\Rightarrow -(1+t^2)\, e^{-t} + \left( -2t\, e^{-t} \int (-e^{-t}\, 2) \right)$

$\Rightarrow -(1+t^2) e^{-t} + \left( -2t\, e^{-t} + \int (2e^{-t}) \right)$

$\Rightarrow -(1+t^2)\, e^{-t} + (-2te^{-t} - 2e^{-t}) + C$

$\Rightarrow -(1+t^2)\, e^{-t} - 2te^{-t} - 2e^{-t} + C$

$\Rightarrow -e^{-t} - e^{-t}\, t^2 - 2te^{-t} - 2e^{-t} + C$

$\Rightarrow -(t^2 + 2t + 3)\, e^{-t} + C = R.H.S$

Now take L.H.S = R.H.S

$$e^{-y} \frac{(\sin y - \cos y)}{2} = -(t^2 + 2t + 3)\, e^{-t} + C$$

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We know that

$t = 0 \quad y = 0$

$$\Rightarrow \frac{1}{t} = (0-1) = -3 + C$$

$$C = 5/2$$

Put value of C

$$\frac{e^{-y}}{2}(\sin y - \cos y) = -(x^2 + 2t + 3)e^{-t} + 5/2$$

Answer

Question 2

$(\sqrt{x+y} + \sqrt{x-y}\, dx - \sqrt{x+y} - \sqrt{x-y}\, dy = 0$

Sol:-

$(\sqrt{x+y} + \sqrt{x-y}\, dx - (\sqrt{x+y} - \sqrt{x-y}\, dy = 0$

$$\frac{dy}{dx} = \frac{\sqrt{x+y}}{\sqrt{x+y}} \quad \frac{+\sqrt{x-y}}{\sqrt{x-y}}$$

This is Homogeneous Differential eq in $x$ and $y$ to solve this $y = vx$

$\Rightarrow dy/dx = v + x\, dv/dx$

Thus eq(1) becomes

$= v + x\, dv/dx = \sqrt{x+vx} / \sqrt{x+vx} + \sqrt{x-vx} / x - vx$

$$= v + x\frac{dv}{dx} = \frac{\sqrt{1+v} \quad + \sqrt{1-v}}{\sqrt{1+v} \quad - \sqrt{1-v}}$$

$$= v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}} \times \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} + \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{1 + \cancel{v} + 1 - \cancel{v} + 2\sqrt{1-v^2}}{2v}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{2}(1 + \sqrt{1-v^2})}{\cancel{2}v}$$

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$$x\frac{dv}{dx} = \frac{\sqrt{1-v^2}\,(1+\sqrt{1+v^2})}{v}$$

$$\frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \frac{dx}{x}$$

$$\int\frac{v\,dx}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \frac{dx}{x}$$

Put $1+\sqrt{1-v^2} = t$

$$\Rightarrow 1/2\,(1-v^2)^{-1/2}\,(-2v)\,dv = dt$$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\int\frac{-dt}{t} = \frac{dx}{x}$$

$$-\ln t = \ln x + \ln c$$

$$-\ln(1+\sqrt{1+v^2}) = \ln cx$$

$$\ln(1+\sqrt{1-v^2}) = -\ln cx$$

$$\ln(1+\sqrt{1-v^2} = \ln(cx)^{-1}$$

$$1+\sqrt{1-v^2} = \frac{1}{cx}$$

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$$= \quad 1+\sqrt{\frac{x^2-y^2}{x^2}} = \frac{1}{cx}$$

$$= \quad 1+\frac{\sqrt{x^2-y^2}}{x^2} = \frac{1}{cx}$$

$$= \quad x+\sqrt{x^2-y^2} = \frac{1}{c}$$

$x+\sqrt{x^2-y^2} = c \quad \because 1/c = c.$

which is Required Solution.

Question 3

$(D^4 - D^2)y = 3x^2 + 4\sin x - 2\cos x$

Sol:-

$(D^4 - D^2)\, y = 3x^2 + 4e^{3ix} - 2\cos ix$

$F(D)y = f(x)$

Complementry solve ~~by~~ yc

$m^4 + m^2 = 0$

$m^2(m^2+1) = 0$

$m^2 = 0 \quad , \quad (m^2+1)(m^2-1)$

$m^2 = -1 \quad , m^2 = 1$

$m = \pm i \quad , m = \pm 1$

$m_1 = 0 \;, m_2 = +1 \;, m_3 = -1 \;, m_4 = 0 + i$

So the roots are real distinct & complex

$= c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x} + e^{\alpha t}(c_4 \cos \beta x + \sin \beta x)$

$= c_1 e^{0x} + c_2 e^{x} + c_3 e^{-x} + e^{0x}(c_4 \cos x + \sin x$

Particular y.p

$yp = \frac{1}{F(D)} = f(x)$

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$$= \frac{1}{D^4 - D^2} \cdot 3x^2 + 4e^{ix} - 2\cos ix$$

$$= \frac{1}{D^4 + D^2} 3x^2 + \frac{1}{D^4 + D^2} \cdot 4e^{ix} - \frac{1}{D^4 + D^2} 2e^{ix}$$

$$= \frac{1}{(2)^4 + (2)^2} 3x^2 + \frac{1}{2^4 + 2^2} 4e^{ix} - \frac{1}{(i) + (i)} 2e^{ix}$$

$$= \frac{1}{16+4} 3x^2 + \frac{1}{1-1} 4e^{ix} - \frac{1}{1-1} 2e^{ix}$$

$$= \frac{1}{16+4} 3x^2 + \frac{1}{0} 4e^{ix} - \frac{1}{0} 2e^{ix}$$

$$= \frac{1}{20} ix^2 + \frac{1}{4D^2 + 2D} x 4e^{ix} - \frac{1}{4(i) + 2(i)}$$

$$+ \frac{1}{-4i + 2i} x 4e^{ix} - \frac{1}{-4i + 2i} x 2e^{ix}$$

$$= \frac{1}{20} 3x^3 + \cancel{\frac{1}{-2i}} - \cancel{\frac{1}{-2i}}$$

$$= \frac{1}{20} 3x^2 - \frac{1}{2i} x 4\cos x + i\sin x + \frac{1}{2i\cos x + i\sin x} x 2$$

Answer.