

NAME=INAM UL HASSAN

ID=7693

PAPER=ADVANCE FLUID MECHANIC

EXAME=FINAL TERM

Question no 1

Drag:

A body which is wholly immersed in a homogeneous fluid may be subjected to two kind of force arising from relative motion between body and fluid these forces are termed as drag and lift. If the force parallel to the motion then it is termed as drag force

There are two Components

1) Pressure Drag (FP)

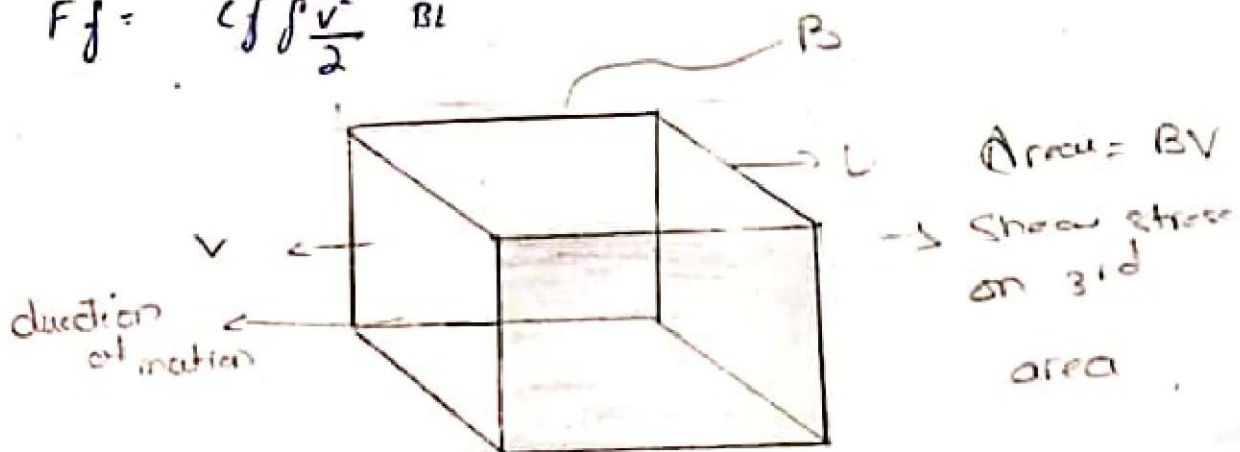
It is equal to integration of Components in direction of motion of all pressure forces exerted on surface of body.

$$F_P = C_P \int \frac{\rho v^2}{2} A$$

2) Friction Drag: (Ff)

It is equal to integration of Components of shear stress along surface of body in direction of motion.

$$F_f = C_f \int \frac{\rho v^2}{2} BL$$



→ Friction Drag of Boundary Layer

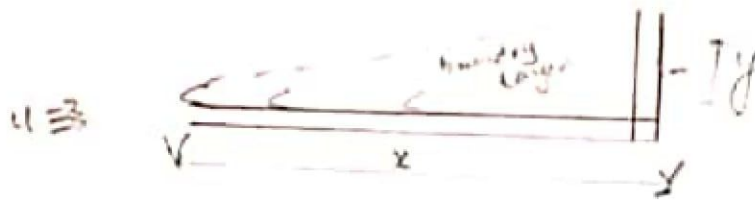
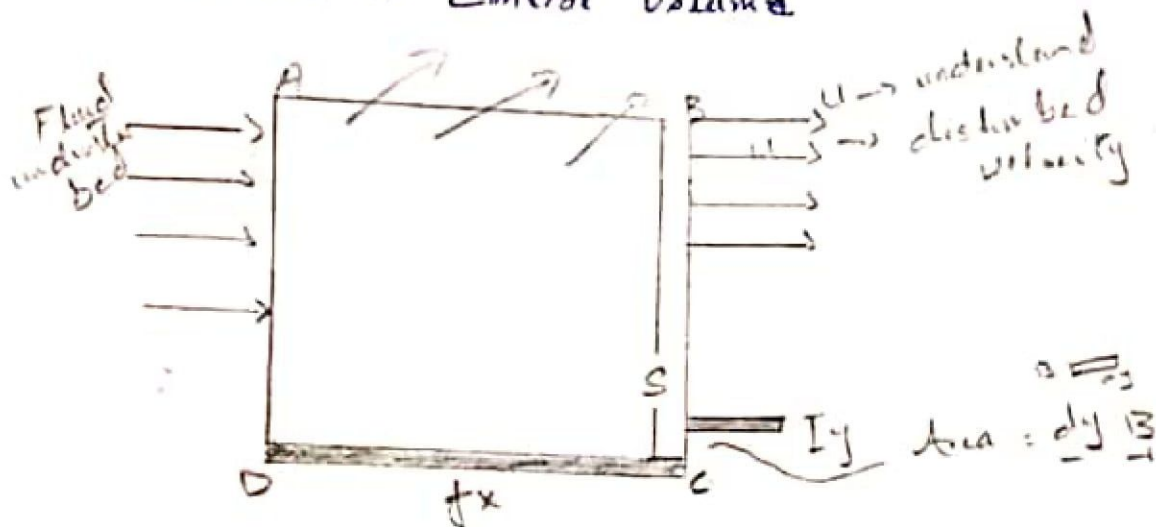


Fig shows growth of boundary layer along one side of smooth plate inside the fluid.

Now Consider a Control volume



Where δ is thickness of boundary layer and U is undisturbed velocity

Thus

$$-F_x = \text{drag} = (\text{rate in momentum in } x \text{ direction})$$

(leaving through $\{BC + \text{rate of momentum through } AB) - \text{rate of momentum entering through } DA)$

$$\Delta P = P_{out} - P_{in}$$

Thus according to momentum

$$\Sigma F = \frac{d(P)}{dt} = \frac{d(mv)}{dt}$$

(3)

where

$$\frac{dm}{dt} = \int \rho \, dms$$

$$F = \int \rho \, dV$$

$$F = \int A \cdot v \cdot v$$

$$F = \int A v^2$$

$$DA \rightarrow \int v \, (UBS)$$

$$BC \rightarrow \int_B \int u^2 \, dy$$

$$AB \rightarrow \int v \, (UBS - B \cdot \int u \, dy)$$

Putting value

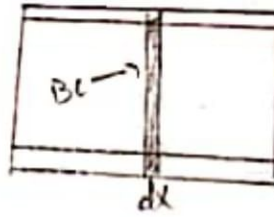
$$F_x = \int_B \int u(u-u) \, dy$$

$$F_x = \int_B u^2 s \, dy$$

Now to find local small shear stress

$$\tau_0 = \frac{dF_x}{B \cdot dx - \text{area}}$$

$$F_x = \int_B u^2 s \, dy$$



$$\tau_0 = s u^2 \alpha \frac{ds}{dx} \text{ in general equation of shear stress}$$

(4)

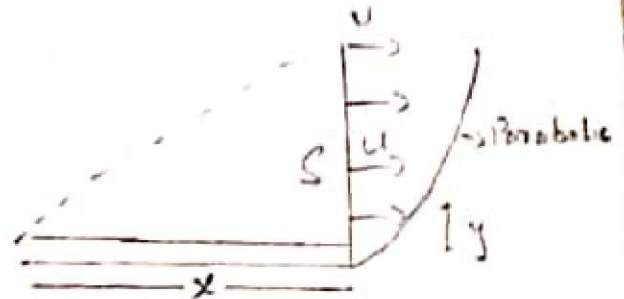
→ Laminar boundary layer:

$$\frac{u}{U} = \eta = \left(\frac{y}{\delta}\right)$$

Assume

$$\eta = \frac{y}{\delta} \text{ or } y = \eta \delta$$

thus $\frac{u}{U} = f(\eta)$ or $u = u(\eta)$



In case of laminar flow

$$\begin{aligned} \tau_0 &= \mu \left(\frac{du}{dy} \right) \\ &= \frac{\mu U}{\delta} \left(\frac{df}{d\eta}(\eta) \right) \end{aligned}$$

Solving the equation

$$\tau_0 = \frac{\mu U B}{\delta} \rightarrow \textcircled{1}$$

As general equation is $\tau_0 = \int_0^\delta \rho u^2 \alpha \frac{ds}{dx}$

Equating both equation

$$\frac{\mu U B}{\delta} = \int_0^\delta \rho u^2 \alpha \frac{ds}{dx}$$

or

$$\delta dx = \frac{\mu B}{\rho \alpha U} dx$$

(5)

Integration the equation

$$\frac{S^2}{2} = \frac{\mu B}{\int \mu x} x + C$$

Now at $x=0$ $S=0$ thus $C=0$

$$\frac{S^2}{2} = \frac{\mu B}{\int \mu x}$$

or

$$S = \frac{\sqrt{2\mu B}}{\sqrt{\mu \alpha}} x \quad \text{or} \quad \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\mu}}$$

and dividing by " x "

$$S = \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\mu}} \cdot \frac{x}{\sqrt{x} \cdot \sqrt{x}}$$

where $\alpha = 0.135$

$$B = 1.63$$

$$Rx = \frac{\mu x}{\mu}$$

$$S = \frac{4.91}{\sqrt{Rx}} x \quad \text{or} \quad \frac{S}{x} = \frac{4.91}{\sqrt{Rx}}$$

Now

$$\tau_0 = \frac{\mu \dot{U} B}{S}$$

thus putting value

$$\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{Rx}$$

where Rx is local Reynolds number

$$\text{Now } F_D = B \int_0^x \frac{\tau_0 dx}{\text{stress}}$$

④

Putting values

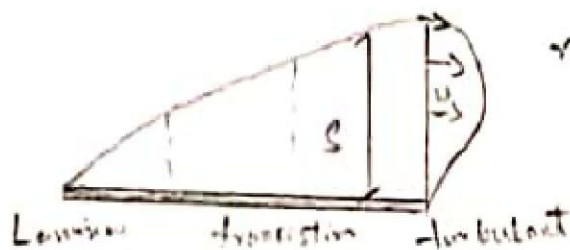
$$F_f = 0.664 B \sqrt{\mu L U^3}$$

As general equation is

$$F_f = c_f \int \frac{v^2}{2} RL \rightarrow \text{equating both equations}$$

$$c_f = 1.328 \frac{\mu}{\rho L U} = \frac{1.328}{\sqrt{R}}$$

Turbulent boundary layer



resistance is less
so curve become
slight

fig show that velocity distribution in turbulent boundary layer shows a much steeper gradient near wall and flatter through out remaining layer

The shear stress greater than turbulent than in laminar layer.

As we have

$$\tau_0 = f \frac{\rho v^2}{8}$$

Now we have obtained an approximate relation between v and U by using pipe factor equation of

$$\frac{v}{U_{max}} = \frac{1}{1 + 1.33rf}$$

Using friction factor of 0.028 from chart which is middle critical value

$$\text{So } \boxed{U = 1.235v}$$

Now we have

$$\tau_0 = f \int \frac{v^2}{\delta}$$

As we know

$$f = \frac{0.316}{R^{0.25}}$$

$$\text{thus } \tau_0 = \frac{0.316}{\left(\frac{Dv}{U}\right)^{1/4}} \int \frac{v^2}{\delta}$$

where $v = \frac{U}{1.235}$ Thus

$$\tau_0 = \frac{0.316}{\left(\frac{D}{v} \left(\frac{v}{1.235}\right)\right)^{1/4}} \int \left(\frac{v}{1.235}\right)^2$$

$$D = 25$$

$$\text{thus } \tau_0 = \frac{0.023 \rho v^2}{\left(\frac{54}{v}\right)^{1/4}}$$

As we have

$$\tau_0 = f v^2 \alpha \frac{ds}{dx}$$

(8)

Equating both and integration for boundary condition of

$$x=0, S=0$$

thus
$$S = \left(\frac{0.0257}{x}\right)^{4/5} \left(\frac{v}{vx}\right)^{1/5} x$$

For $\alpha = 0.0972$

$$\boxed{\frac{S}{x} = \frac{0.377}{(Rx)^{1/5}}$$

Putting values in equation

$$\tau_0 = 0.0587 \int \frac{v^2}{2} \left(\frac{v}{vx}\right)^{1/5}$$

Now
$$Ff = B \int \tau_0 dx$$

$$Ff = 0.0785 \int \frac{v^2}{2} \left(\frac{v}{vL}\right) BL$$

As
$$Ff = cf \int \frac{v^2}{2} BL$$

equating both

$$\boxed{cf = \frac{0.0735}{R^{1/5}}$$

For $R > 10^7$

$$\boxed{cf = \frac{0.455}{(\log R)^{2.58}} \quad A_{01}$$

Question 1
Part b

(7)

Let Specific energy

$$E = y + \frac{v^2}{2g}$$

The flow Q per unit width b can be expressed as

$$q = \frac{Q}{b}$$

Now average velocity will be

$$v = \frac{Q}{A} = \frac{q/b}{by} = \frac{q}{y}$$

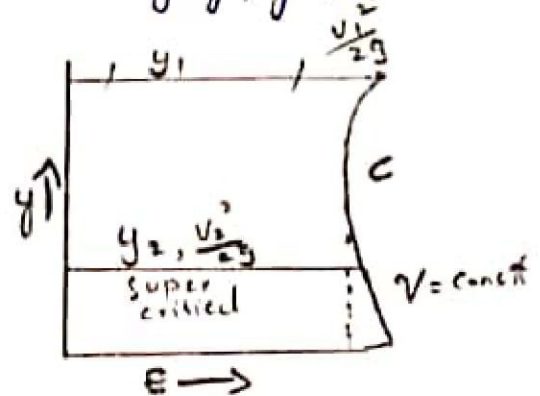
thus

$$E = y + \frac{v^2}{2g} \Rightarrow y + \frac{1}{2g} \left(\frac{q^2}{y^2} \right)$$

$$(E-y) = \frac{1}{2g} \left(\frac{q^2}{y^2} \right) \text{ or } (E-y)y^2 = \frac{q^2}{2g}$$

Thus plot of E vs y will be parabolic for particular q , there will be two kind of possible values of y , for a given E .

The Equation is cubic with three roots, with third root being negative point c represents dividing point b/w two regime of flow



Thus for given q , E value of E is minimum & Flow at that point is critical depth y_c & velocity at that point is critical velocity v_c

Thus

$$E = y + \frac{1}{2g} \left(\frac{q^2}{y^2} \right)$$

(10)

For minimum Specific energy $\frac{dE}{dy} = 0$

Thus

$$\frac{dE}{dy} = 1 - \frac{2}{2g} \left(\frac{q^2}{y^3} \right) = 0$$

$$\Rightarrow \frac{q^2}{gy^3} = 1 \Rightarrow q^2 = gy^3$$

$$\frac{q^2}{g} = y^3 \Rightarrow \left(\frac{q^2}{g} \right)^{1/3} = y_{cr}$$

Now

$$q^2 = gy^3$$

or

$$q = vy \Rightarrow v^2 y^2 = gy^3$$

or

$$v^2 = gy_{cr}$$

or

$$v_{cr} = \sqrt{gy_{cr}}$$

Question no 2

(11)

Given:

Water flows at rate $Q = 35 \text{ m}^3/\text{s}$

Bed slope, $S_0 = 0.0005$

$n = 0.0219$

Width of bed in student FD = 7693 mm

Required:

Depth of rectangular channel = ?

Critical Depth, $y_c = ?$

Critical velocity, $v_c = ?$

Flow is critical or subcritical = ?

Sol:

Manning Equation

$$Q = \left(\frac{1}{n} R_n^{2/3} S_0^{1/2} \right) A \quad \text{--- (1)}$$

$$\text{Area} = 7693 \times d$$

$$\text{Perimeter} = d + 7693 + d$$

$$\begin{aligned} \text{Hydraulic Radius } R_n &= \frac{\text{Area}}{\text{Perimeter}} \\ &= \frac{7693}{2d + 7693} \end{aligned}$$

Put values in eq 1

$$Q = \left(\frac{1}{n} R_n^{2/3} S_0^{1/2} \right) A$$

(12)

$$3.5 \left(\frac{1}{0.0219} \cdot \left(\frac{7693 d}{2d + 7693} \right)^{2/3} \right)^{1/2} \cdot 7693 d$$

$$\frac{3.5 \times 0.0219}{(0.0008)^{1/2}} = \left(\frac{7693 d}{2d + 7693} \right)^{2/3} \times 7693 d$$

$$2.709 = \sqrt[2]{\left(\frac{7693 d}{2d + 7693} \right)^{2/3}} \times 7693 d$$

$$2.709 = \frac{7693 d}{2d + 7693} \times 7693 d$$

$$5.418d + 21046 = 60.35d$$

$$21.046 = 60.35d - 5.418d$$

$$21.046 = 54.932 d$$

$$\frac{21.046}{54.932} = d$$

$$d = 0.38 \text{ m}$$

So the depth of channel is 0.38 m

Now
As $q =$ discharge per unit width

$$q = \frac{Q}{b}$$

$$= \frac{3.5}{7693}$$

$$q = 0.45$$

→ Critical Depth y_c
using equation

(13)

$$y_c = \left(\frac{q^2}{g}\right)^{1/3}$$

$$= \left(\frac{(0.450)^2}{9.81}\right)^{1/3}$$

$$y_{cr} = 0.274 \text{ m}$$

→ Critical Velocity, V_{cr}

Using equation

$$V_{cr} = \sqrt{gy_{cr}}$$

$$V_{cr} = \sqrt{(9.81)(0.274)}$$

$$V_{cr} = 1.63 \text{ m/s}$$

$$\rightarrow y = 0.38 \text{ m} \quad y_{cr} = 0.27 \text{ m}$$

$$V_{cr} = 1.63 \text{ m/s}$$

As $y > y_{cr}$
and

$$V < V_{cr}$$

So flow is subcritical flow.

$$V = \frac{Q}{A} = \frac{35}{\frac{7693}{1000000}} = 117 \text{ m/s}$$

Question no 3

(4)

Given Data

Width of smooth plate, $B = 200\text{mm}$
 $= 0.2\text{m}$

Length of smooth plate, $L = 800\text{mm}$
 $= 0.8\text{m}$

Oil with specific gravity, $S = 0.89$

Undisturbed velocity, $U = 5\text{m/s}$

Kinematic viscosity, $\nu = 0.93 \times 10^{-4}\text{m}^2/\text{s}$

Required Data

Friction drag on one side of a smooth plate, $f_d = ?$

Sol

Check the flow

$$\text{Use } \nu = 0.93 \times 10^{-4}\text{m}^2/\text{s}$$

$$R = \frac{LU}{\nu} = \frac{(0.8)(5)}{0.93 \times 10^{-4}}$$

$$R = 43010.75 < 500,000$$

thus flow is laminar

Now

$$C_f = \frac{1.328}{\sqrt{R}} \Rightarrow \frac{1.328}{\sqrt{43010.75}}$$

$$C_f = 6.403 \times 10^{-3}$$

$$C_f = 0.0064$$

(15)

$$\Rightarrow F_f = C_f f \frac{v^2}{2} BL$$

$$= (0.0064) (\rho_{\text{soil}} \times \gamma_{\text{water}}) \times \left(\frac{5}{2}\right)^2 \times (0.2) (0.8)$$

$$= (0.0064) (0.89 \times 1000) \times \frac{5^2}{2} \times (0.2) (0.8)$$

$$\boxed{11.392 \text{ N}}$$

