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Program : BE (Electrical)

Subject : LCS (Theory)

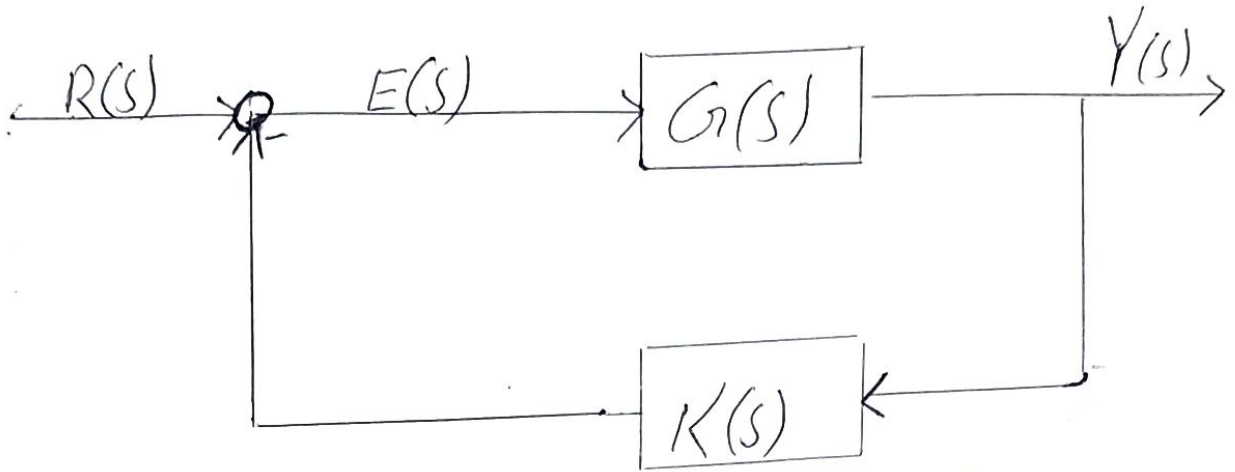
Exam : Summer (2020) Final

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Submitted to: Dr. Rafiq Mansoor

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Q#(1) Find Transfer function by using feedback loop (1)

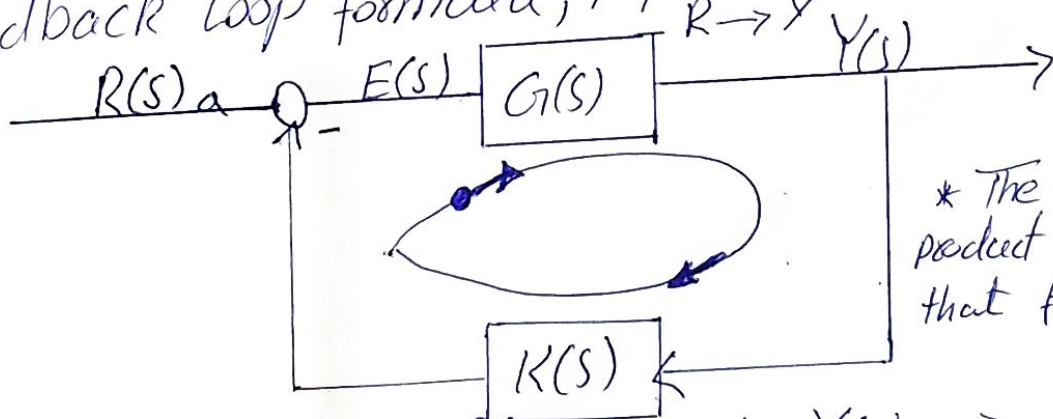


\* When we computing Transfer functions from outside to inside the feedback:

$$E(s) = R(s) - K(s)G(s)E(s) \Rightarrow \frac{E(s)}{R(s)} = \frac{1}{1 + G(s)K(s)}$$

$$Y(s) = G(s)E(s) \Rightarrow \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)K(s)}$$

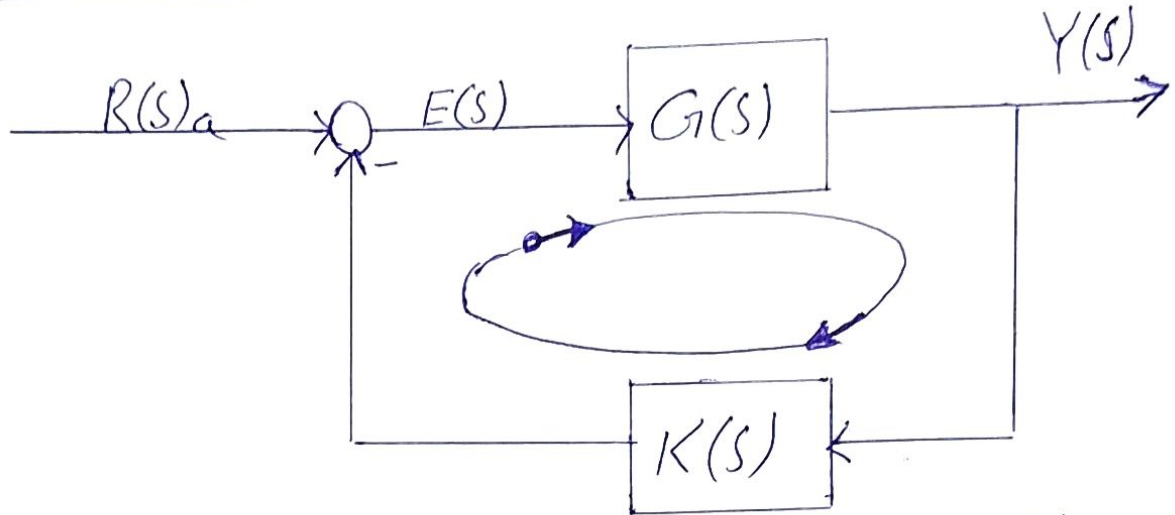
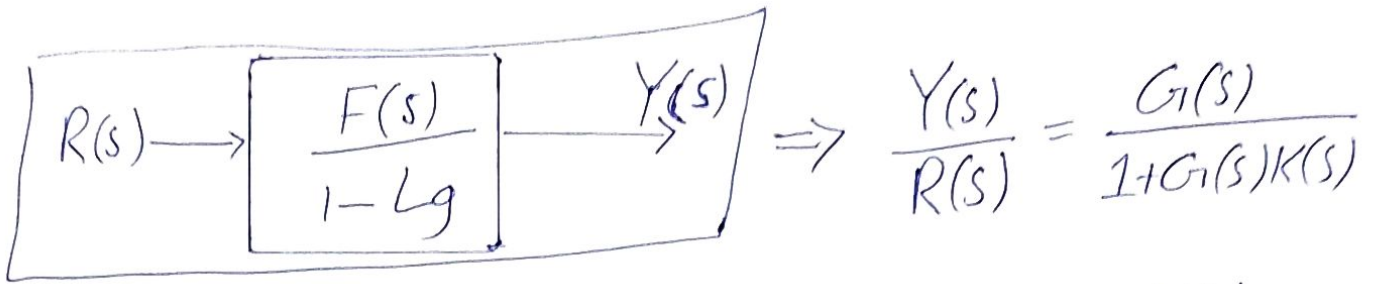
Feedback loop formula, TF  $R \rightarrow Y$



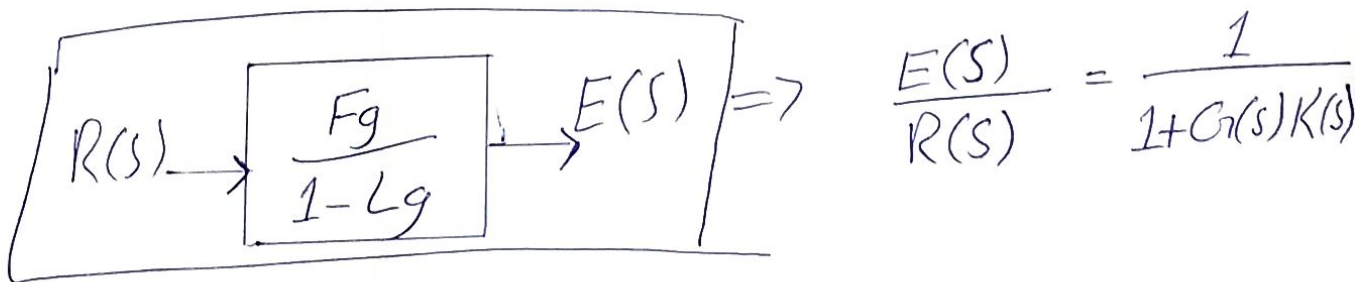
\* The loop gain is the product of all TF that form the loop

$F_g$ : Forward gain from  $R(s)$  to  $Y(s) \Rightarrow G(s)$

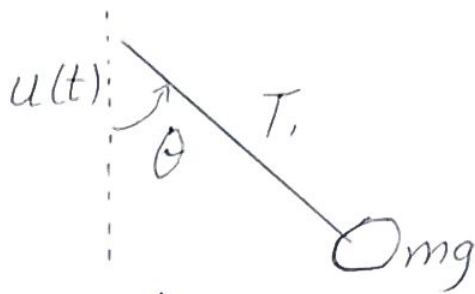
$L_g$ : Loop gain:  $G(s) \Rightarrow K(s)(-1)$



$F_g$ : Forward gain from  $R(s)$  to  $E(s) \Rightarrow 1$   
 $L_g$ : Loop gain:  $G(s) \Rightarrow K(s)(-1)$



Q#2 (a): Find linearization of following diagram:



Motion of Pendulum:

$$mL^2 \ddot{\theta}(t) + mgL \sin \theta(t) = u(t)$$

$$\ddot{\theta}(t) + \underbrace{\frac{g \sin \theta(t)}{L} - \frac{u(t)}{mL^2}}_{f(\theta, u)} = 0$$

Linearize it at  $\theta_0 = \pi$

Find  $u_0$   $\pi + \frac{g \sin \pi}{L} - \frac{u_0}{mL^2} = 0 \rightarrow u_0 = 0$

New coordinates:

$$\theta = \theta_0 + \delta\theta = \pi + \delta\theta$$

$$u = u_0 + \delta u = 0 + \delta u$$

Taylor series expansion of

$$f(\theta, u) \text{ at } \theta = \pi, u = 0$$

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$$\left. \frac{\partial f(\theta, u)}{\partial \theta} \right|_{\substack{\theta=\pi \\ u=0}} = \frac{g \cos \theta}{L} \Big|_{\theta=\pi} = \frac{g}{L}$$

$$\left. \frac{\partial f(\theta, u)}{\partial u} \right|_{\substack{\theta=\pi \\ u=0}} = \frac{-1}{mL^2}$$

$$\delta \theta + \left. \frac{\partial f(\theta, u)}{\partial \theta} \right|_{\substack{\theta=\pi \\ u=0}} \delta \theta + \left. \frac{\partial f(\theta, u)}{\partial u} \right|_{\substack{\theta=\pi \\ u=0}} \delta u = 0$$

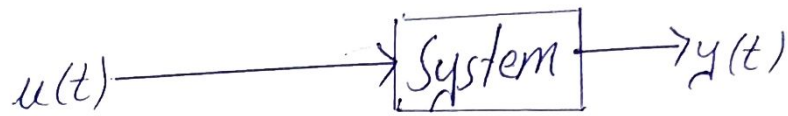
$$\delta \ddot{\theta} - \frac{g}{L} \delta \theta - \frac{1}{mL^2} \delta u = 0$$



Q# (2) (b)

Linear System:-

A system having principle of superposition



$$\left. \begin{array}{l} u_1(t) \rightarrow y_1(t) \\ u_2(t) \rightarrow y_2(t) \end{array} \right\} \Rightarrow a_1 u_1(t) + a_2 u_2(t) \rightarrow a_1 y_1(t) + a_2 y_2(t)$$

Why do we linearization:-

- \* Real system are inherently nonlinear. (Linear systems do not exist!) Ex.  $f(t) = Kx(t)$ ,  $V(t) = Ri(t)$
- \* Transfer Function models are only for Linear time-invariant (LTI) systems.
- \* Many control analysis/design techniques are available for linear systems.

- \* Nonlinear systems are difficult to deal with mathematically
- \* often we linearize nonlinear system before analysis and design

Q #3 (a)

|       |  |              |   |   |
|-------|--|--------------|---|---|
| $s^4$ |  | 1            | 2 | 3 |
| $s^3$ |  | 1            | 2 |   |
| $s^2$ |  | 1            | 2 |   |
| $s^1$ |  | <del>0</del> | 2 |   |
| $s^0$ |  | 2            |   |   |

- No sign changes in the first column
- No Roots in RHP But some roots are on imag axis
- If zero row appears in Routh array, Q has roots either on the imaginary axis or in RHP



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- Take derivation of an auxiliary polynomial  
(which is a factor of  $Q(s)$ )  $s^2+2$