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SECTION: A

SUBJECT: ADVANCED FLUID MECHANICS

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Q # 01 (a)

→ Force on immersed bodies :-

A body which is wholy immersed in a homogenous fluid may be subjected to two kind of forces arraising from relative motion b/w body and fluid. These forces are termed as drag and lift. Depending on forces either parallel or right angle to motion.

Drag force on immersed body can have 2 components :-

① Pressure drag :- (FP) equal to the integration of component in direction of motion of all pressure forces exerted on surface of body.

$$F_p = C_p \rho \frac{v^2}{2} \cdot A \quad \left(C_p \text{ depends upon shape} \right)$$

② Friction drag :- Equal to the integration of component at all shear stress along the surface in direction of motion.

$$F_p = C_f \rho \frac{v^2}{2} (BL) \quad \left(C_f \text{ depends upon velocity} \right)$$

→ As we have $\Sigma F_x = 0$ $\therefore m = \rho v$
where $F_x = \frac{\Delta P}{\Delta t} = \frac{\Delta mv}{\Delta t}$

$F_x = \frac{\Delta \rho \cdot vol \cdot v}{\Delta t} = \Delta \rho Q v$

$F_x =$ rate of change of $BC + AB - AD$
$AD = \rho U (U \delta B)$ $BC = \rho B (u^2 dy)$

$AB = \rho u (U B \delta) - B \int_0^\delta u dy$

$F_x = \rho B \int_0^\delta u (U - u) dy$ — ①

Integrating B/S.

$F_x = \rho B U^2 \delta \theta$
where θ is a function at boundary layer velocity distribution.

Now to find sheer stress.

$\tau = F_x / A = \frac{dF_x}{B dx} = \frac{dF_x}{B dx}$

$\tau_0 = \frac{\rho B U^2 \theta \, d\delta}{B dx} = \rho U^2 \times \frac{d\delta}{dx}$

Laminar boundary layer :-

$$\frac{u}{U} = f\left(\frac{y}{\delta}\right) \quad -①$$

$$y/\delta = \eta \rightarrow y = \delta\eta$$

$$dy = \delta d\eta \quad -②$$

$$\frac{u}{U} = f(\eta) \quad ,$$

$$du = U df(\eta) \quad -③$$

$$\tau_0 = \mu \frac{du}{dy} \quad -④$$

$$\tau_0 = \mu \frac{U df(\eta)}{\delta d(\eta)}$$

$$= \frac{\mu U B}{\delta} \quad -⑤$$

As we have $\tau_0 = \rho U^2 \times \frac{d\delta}{dx}$

Compare ~~with~~ both

$$\rho U^2 \alpha \frac{d\delta}{dx} = \frac{\mu U B}{\delta}$$

④

$$\delta d\delta = \frac{\mu B}{\rho U x} dx$$

integrating both sides

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho U x} x + C \qquad \therefore C = 0$$

$$\delta = \sqrt{\frac{2B}{d}} \cdot \sqrt{\frac{\mu 4}{\rho 4}}$$

$B = 1.68 \quad , d = 0.135$

$$\delta = \frac{4.91 x}{\sqrt{R_x}} \quad - ⑥$$

Where (R_x) is local reynold number

As we have

$$\tau_o = \frac{\mu U B}{\delta}$$

Put eq ⑥ in ⑤

$$\tau_o = 0.332 \frac{\mu U}{x} \sqrt{R_x}$$

$$F_f = B \int_0^l \tau_o \, dx$$

Where $\tau_o = 0.332 \frac{\mu U}{x} \cdot \sqrt{R_x}$

$$\therefore R_x = \frac{x U \rho}{\mu}$$

→ we get

$$F_f = C_f \rho \frac{V^2}{2} BL$$

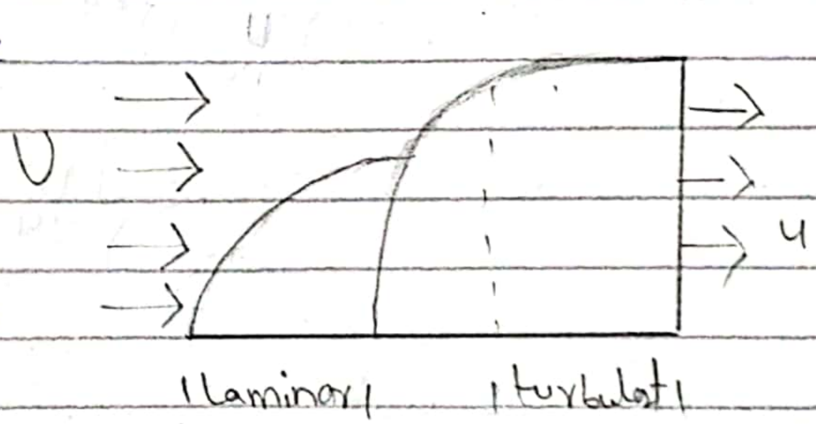
equating b/s

$$C_f = 1.328 \sqrt{\mu/\rho L U}$$

$$= \frac{1.328}{\sqrt{R_L}}$$

→ Turbulent boundary layer :-

This figure shows the velocity distribution of boundary layer which is steeper near walls and flatter through the remainder of layer.



The shear stress is greater in turbulent thus :-

$$\tau_0 = \rho f V^2 / 8$$

V = average velocity.

To obtain relation b/w average and max we have.

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$$\frac{V}{V_{max}} = \frac{1}{1+1.33\sqrt{f}} \quad ; \quad f = 0.028$$

$$U = 1.235\ V \quad , \quad V = \frac{U}{1.235}$$

→ and $f = \dfrac{0.316}{(R_x)^{3/4}}$

$$\tau_0 = f \rho V^2/8$$

$$\tau_0 = \frac{0.316}{\left(\left(\frac{D}{\nu}\right)\left(\frac{U}{1.235}\right)\right)^{3/4}} \cdot \frac{\rho}{8}\left(\frac{U}{1.235}\right)^2$$

$$\tau_0 = \frac{0.023\ \rho U^2}{\left(\frac{2\delta}{\nu}\right)^{3/4}} \quad - (1)$$

→ Genral equation

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dy} \quad - (2)$$

$$y = 0 \qquad \delta = 0$$

$$\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5} \left(\frac{\nu}{Ux}\right)^{3/5} \cdot x$$

$$\alpha = 0.0972$$

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$$\delta = \frac{0.371}{(R_x)^{1/5}} \cdot x \qquad -(8)$$

$$\tau_0 = 0.0587\, \rho \frac{U^2}{2} \left(\nu / Ux \right)^{1/5}$$

Now

$$F_f = B \int_0^L \tau_0 \, dx$$

$$= C_f \cdot \rho \frac{V^2}{2} BL$$

$$C_f = \frac{0.0735}{(R)}$$

For l_0 $R > 10^7$

$$= \frac{0.455}{(\log R)^{2.52}} \qquad \text{— Ans.}$$

Q#01 (b) (8)

→ Sol:-

Specific energy equation

$E = h + \frac{V^2}{2g}$ — ① h_1, h_2 = Alternate height

$Q = AV$ or h_c = Critical depth.

$V = Q/A$ → put in eq ①

$E = h + \frac{Q^2}{A^2 \cdot 2g}$ ∴ $A = b \times h$

$E = h + \frac{Q^2}{b^2 h^2 \cdot 2g}$ ∴ $q = Q/b$

$E = h + (Q/b)^2 \cdot 1/h^2 \cdot 2g$

So $E = h + \frac{Q^2}{2g \cdot h^2}$ → ②

derive eq ② w.r.t to h.

$\frac{dE}{dh} = \frac{d}{dh}\left(h_c + \frac{q^2}{2g} + h_c^{2}\right)$

$\frac{dE}{dh} = 1 + (-2) q^2 h_c^{-2-1}$

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$$0 = 1 - \frac{q^2}{h_c^3 g} \qquad \text{As } \frac{dE}{dh} = 0$$

$$h_c^3 \cdot g = q^2 \quad , \quad h_c^3 = \frac{q^2}{g}$$

$$h_c = \left(\frac{q^2}{g}\right)^{1/3}$$

→ Now for "V_c"

$$h^3_c g = q \qquad \frac{q \times b}{V} = V$$

As $q = AV/b$, $\frac{q\cancel{b}}{\cancel{b}h} = V$

So $V = q/h_c$, $q = Vh_c$

From eq (3)

$$q^2 = h^3_c \cdot g$$

put $q = V_c h_c$

$$V_c^2 h^2 = h_c^3 g$$

$$V_c^2 = g \cdot h_c \quad \text{or} \quad V_c = \sqrt{g \cdot h_c}$$

Therefore $h_c = \left(\frac{q^2}{g}\right)^{1/3}$

$$V_c = \sqrt{g \cdot h_c}$$

Q # 02

Given :

$d = ?$ $Q = 3.5\ m^3/sec$ $S_o = 0.0008$

$n = 0.0219$ width of bed $= 7228\ mm$ or $\boxed{7228\ m}$ — ID

$h_c = ?$

Sol:- Area $= 7.228 \cdot d = 7.228\,d$

Perimeter $= d + 7.228 + d = 7.228 + 2d$

Hydraulic radius $(R_h) = A/P$

$= 7.228d \,/\, 7.228 + 2d.$

→ Using manning equation

$Q = 1/n\ R_h^{2/3} \cdot S_o^{1/2}$ → Putting values

$$3.5 = 1/0.0219 \times 7.228\,d \left(\frac{7.228d}{2d+7.228}\right)^{2/3} \times (0.0008)^{1/2}$$

$d = 0.554\ m$

Area $= 7.228\,(0.554)$

$= 4.34\ m^2$

Perimeter $= 7.228 + 2(0.554)$

$= 8.94\ m^2$

$$Rh = \frac{4.34}{8.94} = 0.485 \text{ m}$$

→ Finding $hc = ycr = \left(\frac{q^2}{g}\right)^{1/3}$

As $q = Q/b = \frac{3.5}{7.822}$

$= 0.447 \text{ m}^2/\text{sec}$

$$ycr = \left(\frac{(0.447)^2}{9.81}\right)^{1/3}$$

$= 0.273$

As $y > ycr$

$0.554 > 0.273$

"So the flow is sub-critical".

Now $V_c = \sqrt{g\,hc}$

$= \sqrt{9.81 \times 0.447}$

$V_c = 2.094$ m/sec.

Q#03 (12)

Friction drag (Fd) = ?
Width (B) = 200mm = 0.2 m
length (L) = 800 mm = 0.8 m
Specific gravity = 0.89
Undisturbed velocity (u) = 5m/sec
Kinematic viscosity (ν) = 0.93×10^{-4} m^2/sec

Sol:- Checking wether flow is laminar or not by reynold number.

$$R = DV/\nu$$

For smooth flat plate D = L , V = U
So:

$$R = \frac{LU}{\nu} = \frac{0.8 \times 5}{0.93 \times 10^{-4}} = 43010$$

43010 < 500,000 → laminar

→ Using formula

$$F_f = C_f \cdot \rho \cdot \frac{v^2}{2} \cdot BL \qquad C_f = 0.0064$$

$$S = \frac{\rho_{oil}}{\rho_{water}} , \quad 0.89 = \frac{\rho_{oil}}{1000} , \quad 890 \ kg/m^3$$

$$F_f = 0.0064 \times 890 \times \frac{(5)^2}{2} \times 0.2 \times 0.8 = 11.39 N.$$

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