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7966

SECTION-B

NUMERICAL ANALYSIS

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QUESTION: 01:

Find the root of the equation

$$x^3 + 3.993 \times 10^{-4} = 0.165x^2$$

Use newton raphsan method with $x_0 = 0.02$.

Soln- Rearranging the equation

$$x^3 - 0.165x^2 + 0.00039 = 0$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = 3x^2 - 0.33x = 0$$

$$\rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.02 - \frac{0.00033}{-0.0054}$$

$$\boxed{x_1 = 0.081}$$

$f(10) = 0.00033$

$f'(x_0) = -0.0054$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.081 - \frac{(-0.00016)}{-0.0070}$$

$f(x_1) = -0.00016$

$f'(x_1) = -0.0070.$

$x_2 = 0.058$

$f(x_2) = 0.000030$

$f'(x_2) = 0.0090$

$$\rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.058 - \frac{0.000030}{(-0.0090)}$$

$x_2 = 0.061$

QUESTION : 02.

Use the number $x_0=2, x_1=2.75, x_2=4$ to find the lagrange interpolation polynomial for $f(x) = 1/x$ at $x=3$.

Solution:-

Putting the values in the function $y_0 = 0.5, y_1 = 0.364, y_2 = 0.25$

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)y_2}{(x_2-x_0)(x_2-x_1)}$$

$$= \frac{(3-2.75)(3-4)}{(2-2.75)(2-4)}(0.5) + \frac{(3-2)(3-4)}{(2.75-2)(2.75-4)} \times 0.364$$

$$+ \frac{(3-2)(3-2.75)}{(4-2)(4-2.75)}(0.25)$$

$$= -0.083 + 0.388 + 0.025$$

$$\rightarrow y = 0.33$$ Ans.

Q3 Complete the .... all this data.

y =	0.7651977	0.6200860	0.4554022
x =	1.0	1.3	1.6
y =	0.2818186		0.1103623
x =	1.9		2.2.

	x_i	$f(x_i)$	$f(x_{i-1}, x_i)$	$f(x_{i-2}, x_{i-1}, x_i)$	$f(x_{i-3}, x_i)$	$f(x_{i-4}, x_i)$
x_0	1	0.7651977				
			-0.4837056			
x_1	1.3	0.6200860		-0.108734		
			-0.548946		0.0658785	
x_2	1.6	0.4554022		-0.0494433		-0.0028049
			-0.578612		0.06251255	
x_3	1.9	0.2818186		0.006818		
			-0.571521			
x_4	2.2	0.1103623				

1) $f(x_0, x_1) = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$= \dfrac{0.6200860 - 0.7651977}{1.3 - 1}$

$f(x_0, x_1) = -0.4837056$

2) $f(x_1, x_2) = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{0.4554022 - 0.6200860}{1.6 - 1.3}$

$f(x_1, x_2) = -0.548946$

3) $f(x_2, x_3) = \dfrac{f(x_3) - f(x_2)}{x_3 - x_2}$

$= \dfrac{0.2818186 - 0.4554022}{1.9 - 1.6}$

$f(x_2, x_3) = -0.578612$

4) $f(x_3, x_4) = \dfrac{f(x_4) - f(x_3)}{x_4 - x_3}$

$= \dfrac{0.1103623 - 0.2818186}{2.2 - 1.9}$

$f(x_3, x_4) = 0.571521.$

Second D.D:

$$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$$

$$= \frac{-0.548946 - (-0.4837056)}{1.6 - 1}$$

$$= -0.108734$$

$$f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$$

$$= \frac{0.578612 - (-0.548946)}{1.9 - 1.3}$$

$$= 0.0994433$$

$$f(x_2, x_3, x_4) = \frac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2}$$

$$= \frac{0.571521 - (-0.578612)}{2.2 - 1.6}$$

$$= 0.006818$$

Third divided difference.

$$f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$$

$$= \frac{-0.0494433 - (-0.108734)}{1.9 - 1}$$

$$= \boxed{0.0658785}$$

$$f(x_1, x_2, x_3, x_4) = \frac{-f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1}$$

$$= \frac{0.006818 - (-0.049443)}{2.2 - 1.3}$$

$$\boxed{= 0.0625125 5}$$

4th divided difference

$$f(x_0, x_1, x_2, x_3, x_4) = \frac{0.0625125 5 - 0.0658785}{2.2 - 1}$$

$$\boxed{= 0.0028049}$$

$$f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2) + (x - x_0)(x - x_1)(x - x_2) f(x_0, x_1, x_2, x_3) + (x - x_0)(x - x_1)(x - x_2)(x - x_3) f(x_0, x_1, x_2, x_3, x_4)$$

$$= 0.7651977 + (x - 1)(-0.4837056) + (x - 1)(x - 1.3)(-0.108734) + (x - 1)(x - 1.3)(x - 1.6)(0.0658785) + (x - 1)(x - 1.3)(x - 1.6)(x - 1.9)(-0.0002804).$$