NAME:-

TANVEER ABBAS KHAN

I.D:-

7837

SUBJECT :-

DIFFERENTIAL EQUATION.

SEMS: SUMMER - FINAL.

Question No 01:-

$$f(t) = 1+t \qquad -\pi \le t \le \pi$$

Solution :-

$$f(t) = 1+t$$

Here we use the formula.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t \rightarrow eq\ (1)$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\, dt$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t)\, dt$$

$$a_0 = \frac{1}{2\pi} \left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(-\frac{\pi^2}{2} \right) \right)$$

$$a_0 = \frac{1}{2\pi} \left(2\pi + \frac{2\pi^2}{2} \right)$$

$$\boxed{a_0 = \frac{1}{2\pi} \left(2\pi + \pi^2 \right)}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t)(\cos nt)\, dt$$

3

$$a_n = \frac{1}{\pi}\left(\left.(1+t\ \frac{\sin nt}{n}\right|_{-\pi}^{\pi} - \int\left(\frac{\sin nt}{n}\frac{d}{dt}(1+t)\right)\right)$$

$$a_n = \frac{1}{\pi}\left(\left.(1+t)\ \cancel{\frac{\sin nt}{n}}^{0} - \frac{\cos nt}{n^2}\right|_{-\pi}^{\pi}\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(\cos n\pi - \cos n(-\pi)\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(-1-(-1)\right)$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)\sin nt\ dt$$

$$b_n = \frac{1}{\pi}\left((1+t)\int_{-\pi}^{\pi}\sin nt - \int\left(\int \sin nt\ \frac{d}{dt}(1+t)dt\right)\right)$$

$$b_n = \frac{1}{\pi}\left(\left.\frac{(1+t)(-\cos nt)}{n}\right|_{-\pi}^{\pi} - \int\left(\frac{-\cos nt}{n}(1)\right)\right)$$

$$b_n = \frac{1}{\pi}\left(\left.-\frac{(1+t)(\cos nt)}{n}\right|_{-\pi}^{\pi} + \left(\left.\cancel{\frac{\sin nx}{n^2}}^{0}\right|_{-\pi}^{\pi}\right)\right)$$

$$b_n = \frac{-1}{n\pi}\left((1+\pi)(\cos n\pi) - \left((1+(\pi)\right)\left(\cos n\pi\right)\right)$$

$$b_n = \frac{-1}{n\pi}\left(\cancel{\cos n\pi} + \pi\cos n\pi - \cos n\pi + \pi\cos n\pi\right)$$

$$b_n = \frac{-1}{n\cancel{\pi}}\left(2\cancel{\pi}\cos n\pi\right)$$

Here $\cos n\pi = \frac{(-1)^{n+1}}{n}$

$$b_n = \frac{2}{n}(-1)^{n+1}$$

So eq become .

$$\boxed{f(x = \frac{1}{2x}\left(2\pi + \pi^2\right) + 0 + \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin t}$$

ANS:

Question 02:-

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Eighen values = ?

Solution:-

we have

$$(A - \lambda I)x = 0$$

The characteristics equation is given by

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - \left|\text{sum of diagonal element}\right| \lambda^2 + \left|\text{sum of diagonal minors}\right| \lambda - |A| = 0 \quad \text{—} (B)$$

Sum of diagonal elements = 1+1+2 = 4

Sum of Diagonal minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= (-6)+(2)+(1)$

$= -6+2+1$

$= 3$

By Putting value in eq (B)

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \quad \text{—— (c)}$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$= 1(2-8) - 0 + 1(6-0)$

$= -6+6$

$= 0$

By Putting value in (c)

$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$

$\lambda^3 - 4\lambda^2 - 3\lambda = 0$

$\lambda(\lambda^2 - 4\lambda - 3) = 0$

$\lambda = 0$

$\lambda^2 - 4\lambda - 3 = 0$

Using Quadratic formula

$$\lambda = \frac{-b \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16+12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4+\sqrt{28}}{2}, \quad \lambda = \frac{4-\sqrt{28}}{2}$$

we have eigen values

$$\lambda = \left(0, \frac{4+\sqrt{28}}{2}, \frac{4-\sqrt{28}}{2}\right)$$ required solution.

Question 03:-

Solve the following system of linear equation:-

$$5x + 4z + 2m = 3$$

$$x - y + 2z + m = 1$$

$$4x + y + 2z = 1$$

$$x + y + z + m = 0$$

Solution:-

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \quad R_4 R_2$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1+4/5 \\ 0 & -1 & +6/5 & 4/5 \\ 0 & 2 & -1 & 0 \end{array}\right] \left[\begin{array}{c} 3 \\ 1 \\ 3/5 \\ -10 \end{array}\right] \quad -1/5 \times R_3$$

(9)

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 0 & 7/5 & 8/5 & 1/5 \end{array}\right] \quad 5\times R_3 \text{ and } 5\times R_2$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 5R_3 \text{ and } 5R_4$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 1/5\times R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -1 & 6/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_2\times 5$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_3 - R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad \begin{array}{l} R_3 \leftrightarrow R_4 \\ 1/7\times R_3 \\ 1/3\times R_4 \end{array}$$

Scanned with CamScanner

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad C_2 \times -5$$

$$\begin{bmatrix} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & -5 & 26/21 \\ 0 & 0 & 1 & 0 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{bmatrix} \quad \frac{3}{4} \times R_1$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 126/84 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$(x, y, z, m) = \left(3/4, 31/21, -11/21, 1/3\right)$$

$x = 3/4$

$y = 31/21$

$z = -11/21$

$m = 1/3$

Question No 04:-

$$u(x,t) = \sin(x+2t)$$

is a solution of the one-dimensional equation.

Solution:-

Given that

$$u(x,t) = \sin(x+2t)$$

Differentiat w.r.t x Partially.

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x} \sin(x+2t)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t) \frac{\partial}{\partial x} \cdot (x+2t)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)(1+0)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x} \cos(x+2t)$$

(13)

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t) \cdot \frac{\partial}{\partial x}(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)(1+0)$$

$$\boxed{\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)}$$

and $u(x,t) = \sin(x+2t)$

Differentiate w.r.t "t"

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t} \sin(x+2t)$$

$$\frac{\partial u}{\partial t} = \cos(x+2t)(0+2)$$

$$\frac{\partial u}{\partial t} = 2\cos(x+2t)$$

$$\frac{\partial^2 u}{\partial t^2} = (2) -\sin(x+2t)(0+2)$$

$$\boxed{\frac{\partial^2 u}{\partial t^2} = -4\sin(x+2t)}$$

we know that one-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$-4\sin(x+2t) = c^2[-\sin(x+2t)]$$

$$-4\sin(x+2t) = -c^2\sin(x+2t)$$

$$-4\sin(x+2t) + c^2\sin(x+2t) = 0$$

for the arbitrary constant = $c = \pm 2$

$$-4\sin(x+2t) + (\pm 2)^2\sin(x+2t) = 0$$

$$-4\sin(x+2t) + 4\sin(x+2t) = 0$$

$$0 = 0$$

Then it will be verified for the arbitrary constant.

$c = 2.$