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I.D NO 7614

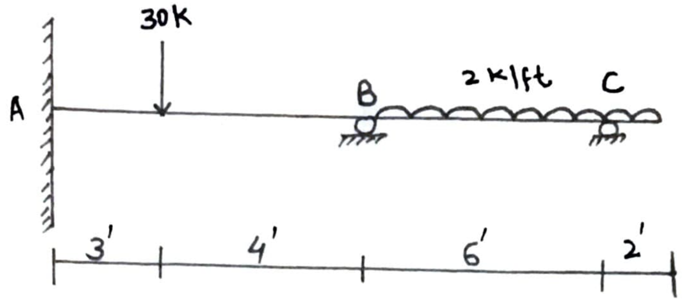
PAPER STRUCTURE - 2

SUBMITTED TO ENGR. ADEED

DATE 25/sep/2020

Q NO 1:

Analyze the beam shown in Fig-1 by stiffness method. Assume EI is constant.



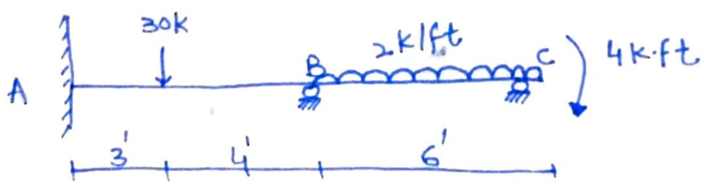
ANSWER: As we known that

STEP#01:

Determining kinematic Indeterminancy

$$K \cdot I = 5^{\circ}$$

So we have to reduce the extended portion.



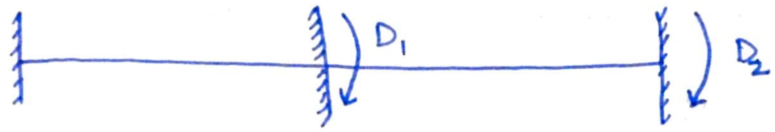
$$\Rightarrow \frac{2(2)}{1} = 4 \text{ k.ft}$$

Now;

$$K \cdot I = 2^{\circ}$$

STEP # 2:

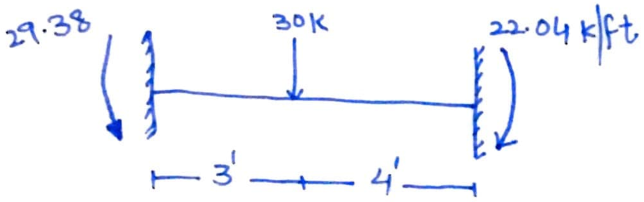
Determine unknown Joint displacement

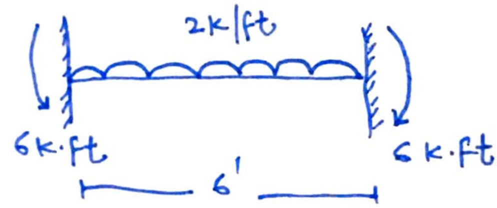


$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \qquad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

STEP #03:

Compute (ADL) Matrix





FROM POINTED LOAD (NOT AT MID):

For Left end:

$$\rightarrow \frac{Pab^2}{L^2} = \frac{(30)(3)(4)^2}{(7)^2} = 29.38 \text{ k.ft}$$

For Right end:

$$\rightarrow \frac{Pa^2b}{L^2} = \frac{(30)(3)^2(4)}{(7)^2} = 22.04 \text{ k.ft}$$

FOR UDL:

$$\frac{wL^2}{12} = \frac{(2)(6)^2}{12} = 6 \text{ k.ft}$$

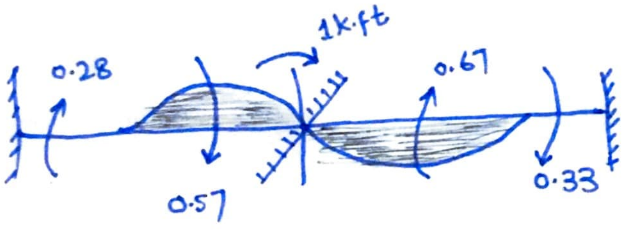
$ADL_1 = +22.04 - 6 = 16.04$ k.ft

$ADL_2 = 6$ k.ft

STEP #04: Compute [(S)] Matrix.

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

a): $D_1 = 1k$, $D_2 = 0$



$$\frac{4EI}{7} = 0.57 \qquad \frac{2EI}{6} = 0.33$$

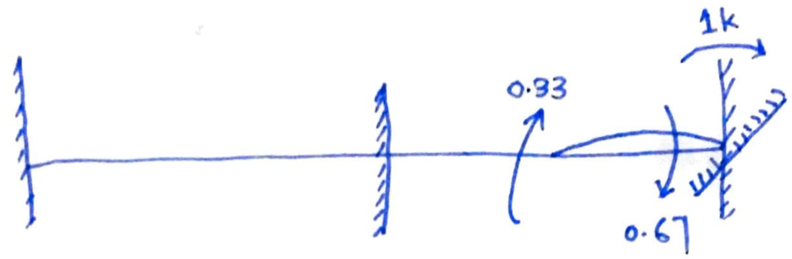
$$\frac{4EI}{6} = 0.67 \qquad \frac{2EI}{7} = 0.28$$

$S_{11} = 0.57 + 0.67$

$= 1.24$ EA

$S_{21} = 0.33$ EA

b): $D_1 = 0$, $D_2 = 1k$



$$\frac{4EI}{6} = 0.67$$

$$\frac{2EI}{6} = 0.33$$

$$S_{12} = 0.33$$

$$S_{22} = 0.67$$

$$S = \begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$

STEP #05:

Compute [D] Matrix

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \times \left(\begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} \right)$$

$$= \frac{1}{\begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}} \times \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix} \times \begin{bmatrix} 16.04 \\ 6 \end{bmatrix}$$

$$|S| = (1.24 \times 0.67) - (0.33 \times 0.33)$$

$$= 0.8308 - 0.1089$$

$$|S| = 0.7219$$

$$\text{Adj } A = \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}$$

Now;

$$\begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix} = \begin{bmatrix} 0 - 16.04 \\ 4 - 6 \end{bmatrix} = \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{\begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}}{0.7219} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

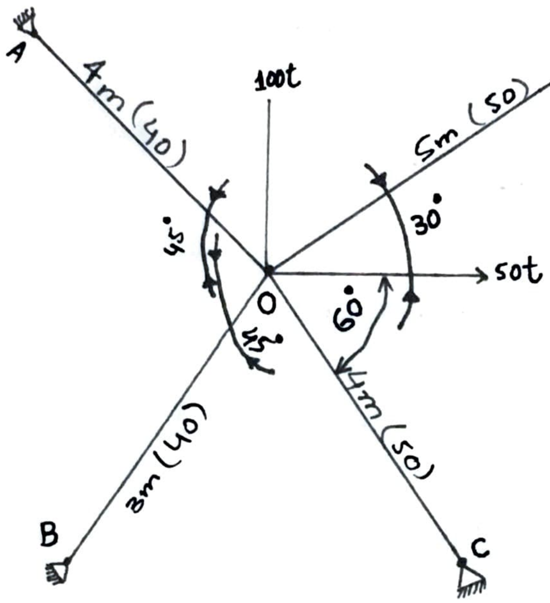
$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} -13.97 \\ 3.8902 \end{bmatrix}.$$

oR;

$$[D] = [-13.97].$$

QNO2:

Analyze the pin-jointed frame shown by stiffness method. Length of the members in "m" and cross-sectional area of the member in cm^2 are shown in FIG-3. Take $E = 2000\ t/cm^2$.



Solution:

FOR A:

$$\sin 45^\circ = \frac{P}{h} = \frac{P}{4}$$

$$\Rightarrow P = 2.828m$$

$$\cos 45^\circ = \frac{b}{4}$$

$$\Rightarrow b = 2.828m$$

FOR point B,

$$\sin 45^\circ = \frac{P}{3}$$

$$P = 2.12 m$$

$$\cos 45^\circ = \frac{b}{h}$$

$$b = 2.12 m$$

FOR point C:

$$\sin 60 = \frac{P}{h} = \frac{P}{5}$$

$$P = 4.330 m$$

$$\cos 60^\circ = \frac{b}{h}$$

$$b = 2.5 m$$

FOR POINT D:

$$\sin 30^\circ = \frac{P}{H} = P/5$$

$$P = 2.5$$

$$\cos 30^\circ = \frac{b}{5}$$

$$b = 4.33$$

So;

$$EA_{(A)} = 2000 \times 40 = 80000\,t$$

$$EA_{(B)} = 2000 \times 40 = 80000\,t$$

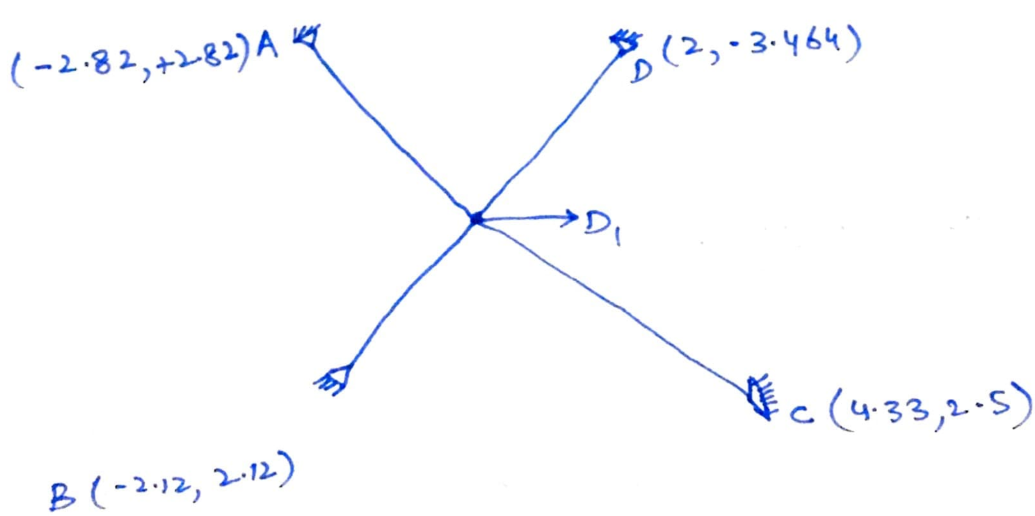
$$EA_{(C)} = 2000 \times 50 = 100{,}000\,t$$

$$EA_{(D)} = 2000 \times 50 = 100{,}000\,t$$

STEP #01: K.I

$$K.I = 2j - r$$
$$= 2(5) - 8$$
$$= 2^{\circ}$$

STEP #02: Unknown joint displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

STEP #03:

$$[AMD]_{4\times 2} \quad \text{and} \quad [S]_{2\times 2}$$

i) $D_1 = 1$, $D_2 = 0$

$$AMD = \frac{EA}{L^2}(x_k - x_j)$$

$$AMD_{11} = \frac{80,000}{(400)^2} \times (0+282) = 141$$

$$AMD_{21} = \frac{80,000}{(300)^2} \times (0+212) = 188.44$$

$$AMD_{31} = \frac{1,00,000}{(400)^2} \times (0-200) = 125$$

$$AMD_{41} = \frac{100000}{(500)^2} \times (0.433) = -173.2$$

NOW;

$$S_{11} = \sum_{i=1}^{m} \frac{EA}{L^3}(x_k - x_j)^2$$

$$= \frac{80,000}{(400)^3} \times (282)^2 + \frac{80,000}{(300)^3} \times (212)^2$$

$$+ \frac{100000}{(400)^3} \times (-200)^2 + \frac{100000}{(500)^3} \times (-433)^2$$

$$S_{11} = 99.405 + 133.107 + 62.5 + 149.991$$

$$\boxed{S_{11} = 445.062}$$

$$S_{12} = S_{21} = \sum_{i=1}^{m} \frac{EA}{L^3} \times (x_k - x_j)(Y_k - Y_j)$$

$$= \frac{80{,}000}{(400)^3} \times (282)(-282) + \frac{80{,}000}{(300)^3} \times (212)(212)$$

$$+ \frac{100000}{(400)^3} \times (-200)(0+346) + \frac{100000}{(500)^3} \times (-433)(0.250)$$

$$S_{12} = S_{21} = 12.237$$

ii) - $D_1 = 0$, $D_1 = 1K'$

$$AMD = \frac{EA}{L^2} (Y_k - Y_j)$$

$$AMD_{12} = \frac{80000}{(400)^2} (-282) = -141$$

$$AMD_{22} = \frac{80000}{(300)^2} (212) = 188.44$$

$$AMD_{32} = \frac{100000}{(400)^2} (346) = 216.25$$

$$AMD_{42} = \frac{100000}{(500)^2}(-250) = -100$$

NOW;

$$S_{22} = \sum_{i=1}^{m} \frac{EA}{L^3}\left(Y_k - Y_j\right)^2$$

$$= \frac{80000}{(400)^3}(-282)^2 + \frac{80000}{(300)^3}(212)^2 + \frac{100000}{(400)^3}(346)^2$$

$$+ \frac{100000}{(500)^2} \times (-250)^2$$

$$\boxed{S_{22} = 469.628}$$

STEP # 04:

$$[D] = [S]^{-1} \times [AD]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.063 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

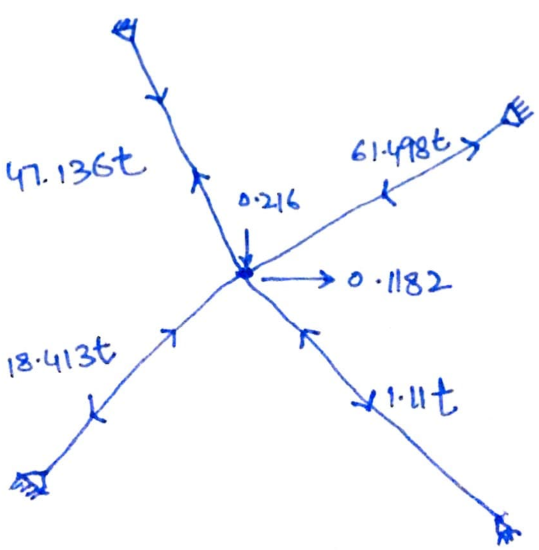
$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

STEP#5: [AM]

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -125 & 216.25 \\ -173.2 & -100 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

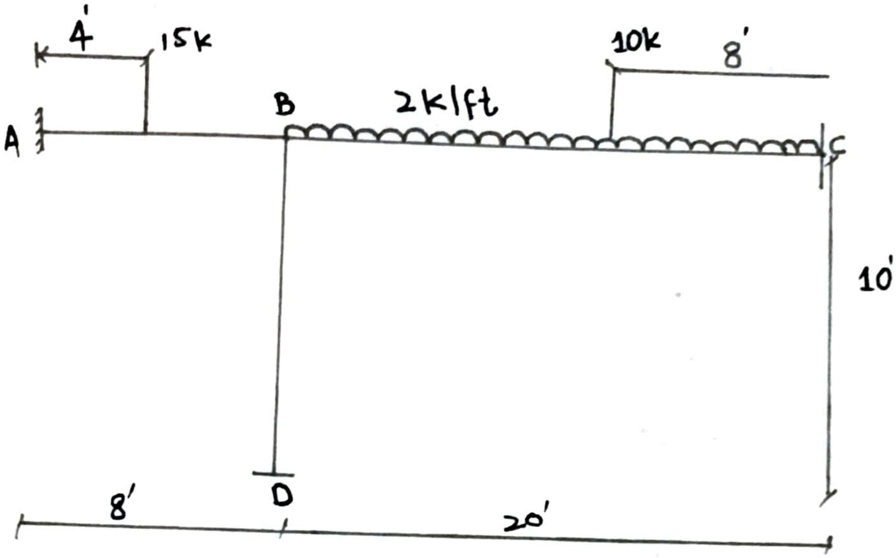
$$= \begin{bmatrix} 141 \times 0.1183 + (-141) \times (-0.216) \\ 188.44 \times 0.1183 + 188.44 \times (0.216) \\ -173.2 \times 0.1183 + (-100) \times (0.216) \\ -125 \times 0.1183 + (216.25) \times (0.216) \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 47.136t \\ -18.413t \\ -61.49t \\ 1.11t \end{bmatrix}$$



Q NO 3:

Analyze the rigid - Joint frame shown in FIG - 2 by stiffness method. Assume EI is constant.



Solution:

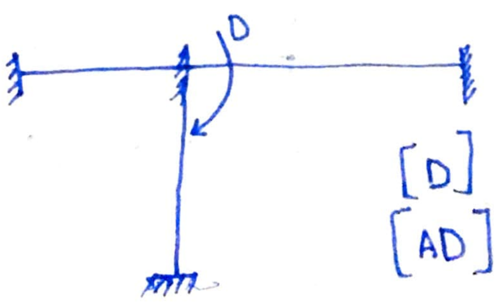
STEP#01:

Determine kinematic Indeterminancy

$$k.I = 1^{\circ}$$

STEP#02:

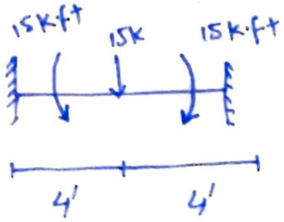
Determine unknown joint displacement.

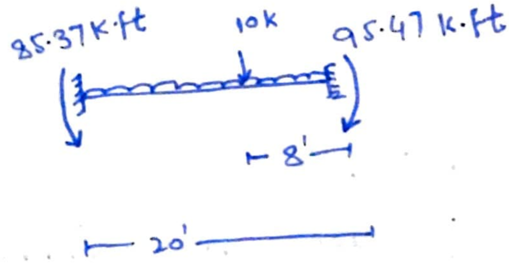


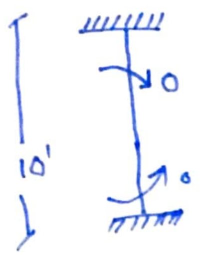
$$[D] = [?]$$
$$[AD] \quad [0]$$

STEP #03:

Compute [ADL] matrix.







⇒ Point Load at Centre:

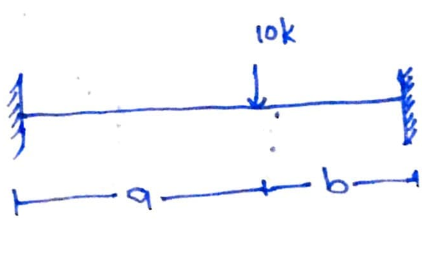
$$\frac{PL}{8} = \frac{(15)(8)}{(8)} = 15 \text{ kip.ft}$$

⇒ Uniform distributed Load:

$$\frac{wL^2}{12} = \frac{(2)(20)^2}{12} = 66.67 \text{ k.ft}$$

⇒ Point Load (Not at Mid):

Suppose



For Left End:

$$\frac{Pab^2}{L^2} = \frac{(10)(12)(8)^2}{(20)^2} = 19.2 \text{ k.ft}$$

For Right End:

$$\frac{Pa^2b}{L^2} = \frac{(10)(12)^2(8)}{(20)^2} = 28.8 \text{ k.ft}$$

Total moment at left end:

$$19.2 + 66.67 = 95.47 \text{ k.ft}$$

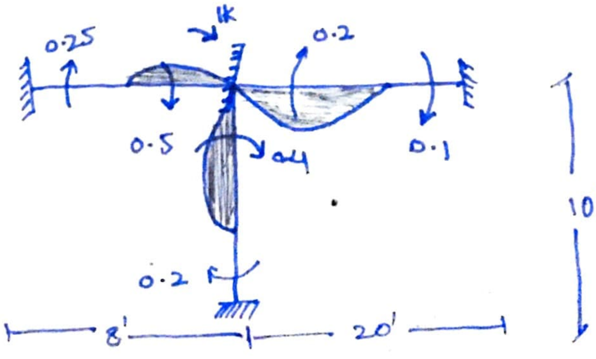
So $[ADL] = -85.87 + 15 = -70.87$ k.ft

STEP#04: Determine .[S] matrix.

$$[S] = [S_{11}]$$

Now;

$$D = 1K$$



$$\Rightarrow \frac{4EI}{8} = 0.5 \qquad \frac{2EI}{8} = 0.25$$

$$\Rightarrow \frac{4EI}{20} = 0.2 \qquad \frac{2EI}{20} = 0.1$$

$$\Rightarrow \frac{4EI}{10} = 0.4 \qquad \frac{2EI}{10} = 0.2$$

$$[S] = [0.5 + 0.4 + 0.2]\ EI$$

$$= 1.1\ EI$$

$$[S] = 1.1EI$$

STEP # 05 : Compute [D] matrix:

$$[D] = [S]^{-1} \times [AD] - [ADL]$$

$$[D] = \frac{1}{1.1} \times [0] - [-70.87]$$

$$= \frac{70.87}{1.1}$$

$$[D] = [64.42]\ 1/EI\ .$$