DISCRETE STRUCTURES

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DEPT # BSCS IIs

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Q1 :-

(A) → Let p be the Statement " DATAENDFLAG" is off," q the Statement "ERROR equals 0," and r the Statement "SUM is less than 1000" Express the following Sentence in Symbolic Notation,

a). DATAENDFLAG is off ERROR equals 0 and Sum is less than 1000.

b) DATAENDFLAG is off but

ERROR is not equal to 0.

c). DATAENDFLAG is off, however ERROR is not ~~equal~~ 0 or SUM is greater than or equal to 1000.

D). DATAENDFLAG is on, and ERROR equals 0 but SUM is greater than or equal to 1,000.

E). Eie ther DATAENDFLAG is on or it is the case that both ERROR equal 0 and SUM is less than 1,000.

A). $p \cdot q$ and r are Statements.

a). $p \wedge q \wedge r$

B). $p \sim q$

c). $p \sim q \vee \sim p$

d) $(\sim p \wedge q) \sim r$

E). $\sim p \vee \sim q \wedge r$: $\sim p \vee \sim q \wedge r$

B). Show That

ANS B): $p \vee q \to r \equiv (p \to r) \wedge (q \to r)$

B L·H·S

p	q	r	$p \vee q$	$p \vee q \to r$
F	F	F	F	F
F	F	T	F	T
F	T	F	T	F
F	T	T	T	T
T	F	F	T	F
T	F	T		T
T	T	F	T	F
T	T	T	T	T

R·H·S

$q \to r$

p	q	r	$p \to r$		
F	F	F	F	T	F
F	F	T	T	T	T
F	T	F	T	F	F
F	T	T	T	T	T
T	F	F	F	T	F
T	F	T	T	T	T
T	T	F	F	F	F
T	T	T	T	T	T

L·H·S = R·H·S

$(p \to r) \wedge (q \to r)$

Q1: Solution (A)

A): PART A:

a). p = "DATAENDFLAG" is off",

q = "ERROR equals 0"

r = "SUM is less than 1,000"

We can rewrite the
Given Sentence as -

"DATAENDFLAG is off, and ERROR equals 0 and SUM is less than 1000."

Replacing the Statements by p, q and r and replacing "and" By $\wedge$, We then Obtain

$p \wedge (q \wedge r)$.

B):

p = "DATAENDFLAG is off"

q = "ERROR equals 0"

r = "SUM is less then 1000"

Note that the word "but" In the Given Sentence Actually implies "and". Thus We can then rewrite the Given Sentence as:

"DATAENDFLAG is off and ERROR does not equal 0."

Replacing the Statement by p, q and r, replacing "and" by ∧ and replacing "Not" by "~"

we then Obtain

$p \wedge \sim q$

c).

p = "DATAENDFLAG is off"

q = "ERROR equal 0"

r = "SUM is less than 1000"

we can then rewrite the Given Sentence as:

"DATAENDFLAG is off and ERROR does not 0 or SUM is not less then 1000"

Replacing the Statement By p, q and r, replacing "And" By $\wedge$, replacing "or" by $\vee$ And replacing not by $\sim$, We then Obtain:

$$p \wedge (\sim q \vee \sim r)$$

Note:

Using De Morgan's law, we can rewrite $p \wedge (\sim q \vee \sim r)$ As $p \wedge \sim (q \wedge r)$

D).

p = "DATAENDFLAG is off"

q = "ERROR equals 0"

r = SUM is less than 1,000"

Note the word "but"
is the Given Sentence
Actually implies "and";
Thus we can rewrite the
Given Sentence as:

DATAENDFLAG is not off and
ERROR equals 0, and SUM is
Not less Then 1,000."

Replacing the Statements by
p, q and r, replacing "And"
by ∧ and replacing "Not by
by ~,

We ~~can~~ Then Obtain:

$$(\sim p \wedge q) \wedge \sim r$$

E):

p= "DATAENDFLAG is off"

q= "ERROR equals 0"

r = "SUM is less then 1,000"

We can then rewrite the Given Sentence as:

"DATAENDFLAG is not off or, ERROR equals 0 and SUM is less than 1000"

Replacing the Statements by p, q and r

Replacing "and" by ∧, replacing r by ∨

And replacing "Not" by ∼, we then Obtain

$$\sim p \vee (q \wedge r)$$

RESULTS:

(a) $p \wedge (q \wedge r)$, (b) $p \wedge \sim q$

(c) $p \wedge (\sim q \vee \sim r)$, (d). $(\sim p \wedge q) \wedge \sim r$

(e). $\sim p \vee (q \wedge r)$.

Q 2:: a) Write the Converse, Inverse and Contrapossitive of the following.

a). if Howard can Swim across the lake; then: Howard can Swim to the Island.

B). if today is Easter, then tommorrow is Monday.

Ans::

a). <u>Converse::</u> of (a)::

if Howard can Swim to the Island, then Howard can Swim across the lake.

a). Inverse:- of (a):-
if Howard Cannot Swim across The lake, Then Howard cannot Swim to The Island.

a). Contrapossitive of (a):-
Howard cannot Swim to The Island if Howard Cannot Swim across The lake.

B):- Inverse:- of (b):-
if today is not Easter, Then tomorrow is Not Monday.

Converse of (b) :-

if tomorrow is Monday, then today is Easter.

Contrapossitive of (b) :-

tomorrow is not Monday if today is not Easter.

Q 2 :·(B)

B). Use truthtable to determine weather the argument form are valid. Indicate which columns represent the premises and which represent the conclusion.

p

$p \rightarrow q$

$\sim q \vee r$

$\therefore r$

(A).

$p \wedge q \rightarrow \sim r$

$p \vee \sim q$

$\sim q \rightarrow p$

$\therefore \sim r$

(B).

Ans:. Q2 (~~B~~) :. PART B

~~B~~

a) p

$p \rightarrow q \quad \sim q \vee r \therefore r$

Valid vs Invalid argument:.

An Argument is Said to be valid if and only if statements are Subtituted in Such a way which Makes all the permises true

And thus Conclusion Column "FALSE" but all premises columns are "TRUE" then the Argument is said to be Invalid.

Q2 PART (B) → (b)

b) $p \wedge q \rightarrow \sim r$ $p \vee \sim q \rightarrow p \therefore \sim r$

<u>Calculation:</u>.

A Conjunction $p \wedge q$ is true, if both (sub)

Statements (p and q)

Are true.

A Negation $\sim p$ is true, if the (Sub) Statement p is false.

A Conditional Statement $p \rightarrow q$ is true, if p is false. or both (Sub) proportions are true.

Q 3:-

In the back of an Old Cupboard you discover a Note Signed by a pirate famous for his bezarre Sense of humor and love of logical puzzeles. In the note he wrote that he had hidden treasure Somewheare on the property. He listed five true Statement. (a-e below) and chalenged the reader to Use them to figure out the location of the treasure.

a):- if this house is next to a lake, then the treasur is not in the kitchen.

b)- if the tree in the front yard is an elm, then the treasure is in the kitchen.

c). this houses is next to A lake.

d). the tree in the front yard is an elm or the treasure is buried Under the flagpole.

e). if the tree in the back is an oak, then the treasure is in ~~burried~~ Under the Garage.

Where is the treasure hidden ?

Ans. 3).

p = this house is Next to a lake.

q = treasure is not in the Kitchen.

r = tree in the front yard is an elm.

s = treasure is burried under the flagpole.
t = tree in the back yard is an oak.
u = the treasure is in the garage.

$(p \rightarrow q) \longrightarrow 1$

$(r \rightarrow \neg q) \longrightarrow 2$

$(p) \longrightarrow 3$

$(r \vee s) \longrightarrow 4$

$(t \rightarrow u) \longrightarrow 5$

$(p \rightarrow q)$

(p)

$\therefore q \rightarrow$ 6 (from 1 and 3 by Using modus ponens)

$(r \rightarrow \neg q)$

(q)

$\therefore \neg r \rightarrow$ 7 (from 2 and 6 by Using modus tollens)

$(r \vee s)$

$(\neg r)$

$\therefore s \rightarrow$ 8 (from 4 and 7 by Using elemination.

Hence :-

Treasure is buried Under the Flagpole.