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Department : Bs (Civil Engineering)

Section : (A)

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Q11

The function $g(t)$ is defined by

$$g(t) = 0 \quad t < 0$$

$$t^2 \quad 0 \leq t \leq 3$$

$$2t + 3 \quad 3 < t \leq 4$$

$$\frac{1}{2} \quad t > 4$$

a) State any point of discontinuity.

b) Find if they exist

$$i, \quad \forall i \quad \lim_{t \rightarrow 3} g$$

Part (a)

State any point of discontinuity

Solution:-

At point $t = 3$ $g(t)$ is not

continuous because $g(3) = 9$

$$\text{and } \lim_{t \rightarrow 3^+} g(t) = \lim_{t \rightarrow 3^-} g(t)$$

So limit does not exist

In this case

Thus $t = 0$ is discontinuity point in the domain $g(t)$

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Q1) (B) Find if they exist

$$\lim_{t \rightarrow 3} g$$

Solution:

$$g(0) = 0 \quad \text{and}$$

$$\lim_{t \rightarrow 0} g(t) = 0$$

$$\text{and } \lim_{t \rightarrow 0} g(t) = 0$$

$$\text{i.e. } \lim_{t \rightarrow 0} g(t) = \lim_{t \rightarrow 0} g(t)$$

$$\Rightarrow \lim_{t \rightarrow 0} g(t) \text{ exist}$$

$$\text{and } \lim_{t \rightarrow 0} g(t) = g(0)$$

So $g(t)$ is continuous at point $t = 0$

Q No (a)

i) Find the Maclaurin's Series for

$$y(x) = x^2 + \sin x$$

Solution: $f(x) = x^2 + \sin x$

$$f(0) = 0 = f'(0) = 2x + \cos x$$

$$f'(0) = 2(0) + \cos(0)$$

$$f'(0) = 1$$

$$f''(x) = 2 - \sin x$$

$$f''(0) = 2 - \sin(0)$$

$$f''(0) = 2$$

$$f'''(x) = -\cos x$$

$$f'''(0) = -\cos(0)$$

$$f'''(0) = -1$$

Substituting these values in the formula

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

$$x^2 + \sin x = 0 + 1x + \frac{2x^2}{2!} + \frac{(-1)x^3}{3!} + \dots$$

$$x^2 + \sin x = x + x^2 - \frac{x^3}{6} + \dots$$

Q3, Part (a)

i) find y'' given

$$1 + xy = x^2 + y^2$$

$$\frac{d}{dx}(1) + \frac{d}{dx}xy = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$$

$$0 + x \frac{dy}{dx} + y = 2x - y$$

$$x \frac{dy}{dx} - 2y \frac{dy}{dx} = 2x - y$$

$$\frac{dy}{dx}(x - 2y) = 2x - y$$

$$\frac{dy}{dx} = \frac{2x - y}{x - 2y}$$

Now taking second derivation using quotient's formula

$$y'' = v \cdot u' - u \cdot v'$$

$$u = 2x - y$$

$$u' = 2$$

$$v = x - 2y$$

$$v' = 1$$

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put in formula

$$y'' = \frac{(x - 2y)(2) - (2x - y)(1)}{(x - 2y)^2}$$

$$y'' = \frac{2x - 4y - 2x + y}{(x - 2y)^2}$$

$$y'' = \frac{-3y}{x - 2y}$$

Q. NO (3) (Part B)

ii. Find y' by using logarithmic differentiation

$$y = x^3 (1+x)^9 e^{6x}$$

Taking \log on both side

$$\log y = \log x^3 (1+x)^9 e^{6x}$$

$$\frac{1}{y} \frac{d}{dx} = \frac{1}{x^3 (1+x)^9 e^{6x}} \frac{d}{dx} (x^3 (1+x)^9 e^{6x})$$

$$\frac{dy}{dx} = y' = \frac{y}{x^3 (1+x)^9 e^{6x}} \left[x^3 (1+x)^9 \frac{d}{dx} e^{6x} + x^3 \cdot \right.$$

$$\left. e^{6x} \frac{d}{dx} (1+x)^9 + (1+x)^9 e^{6x} \frac{d}{dx} x^3 \right]$$

$$\frac{y}{x^3 (1+x)^9 e^{6x}} \left[x^3 (1+x)^9 (6e^{6x}) + 9x^3 e^{6x} (1+x)^8 \right.$$

$$\left. + 3x^2 e^{6x} (1+x)^9 \right]$$

$$\frac{y}{x^3 (1+x)^9 e^{6x}} \left[6x^3 e^{6x} (1+x)^9 + 9x^3 e^{6x} (1+x)^8 + 3x^2 e^{6x} (1+x)^9 \right]$$

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$$\frac{y}{x^2(1+x)^9} e^{6x} \left(\cancel{3x^2} \cancel{e^{6x}} (1+x)^8 (2x(1+x)^9 + 3x + (1+x)) \right)$$

$$\frac{y}{x(1+x)} \left(3(2x(1+x)^9 + 4x + 1) \right)$$

$$\frac{3y [2x(1+x)^9 + 4x + 1]}{x(1+x)}$$