I,D # 13213

1a) DIFFERENTIAL EQUATION:

A differential equation is an equation that relates the unknown function to some of its derivatives, which, of course are not known either.

EXAMPLE OF DIFFERENTIAL EQUATIONS:

$$\theta'' + \sqrt{\frac{g}{2}} \sin\theta = 0,$$

$$R q' + \frac{1}{C} q = \sin\omega t,$$

$$p' = rp\left(1 - \frac{P}{k}\right),$$

$$T' = -h(T - Q).$$

b) SEPARATE DIFFERENTIAL (SD) EQUATION:-

A differential equation having the form.

$$x^t = f(x)\, g(t)$$

where the right side is the product of a function of a x & a function of t; is called a separate equation

i) SOLUTION:-

$$y' = \frac{xy^3}{\sqrt{1+x^2}}$$

$$\frac{1}{y^3}\frac{dy}{dx} = \frac{x}{\sqrt{1+x^2}}$$

$$\frac{1}{y^3}\, dy = \frac{x}{\sqrt{1+x^2}}\, dx$$

$$\int y^{-2}\, dy = \int \frac{x}{\sqrt{1+x^2}}\, dx$$

$$\int y^{-3}\, dy = \int x(1+x^2)^{-1/2}\, dx$$

$$\frac{y^{-3+1}}{-3+1} = \frac{1}{2}\int 2x\,(1+x^2)^{-\frac{1}{2}}\,dx$$

$$\frac{y^{-2}}{-2} = \frac{1}{2}\,\frac{(1+x^2)^{-\frac{1}{2}}}{-1/2+1} + C$$

$$\frac{-1}{2y^2} = \frac{1}{2}\,\frac{(1+x^2)^{1/2}}{1/2} + C$$

$$\frac{-1}{2y^2} = \cancel{2}\,\frac{1}{\cancel{2}}\,(1+x^2)^{1/2} + C$$

$$\frac{-1}{2y^2} = \sqrt{1+x^2} + C$$

$$2y^2 = \frac{-1}{\sqrt{1+x^2}} - \frac{1}{C}$$

$$y^2 = -1/2\,(1+x^2)^{-1/2} - \frac{1}{C}$$

$$y = \frac{-1}{2}\sqrt{(1+x^2)^{-1/2}} - \frac{1}{C}$$

$$y = i\,\frac{-1}{2}\,(1+x^2)^{\frac{1}{2}+\frac{1}{2}} - \frac{1}{C}$$

$$y = \frac{-1}{\sqrt{2}}(1) - \frac{1}{c}$$

$$y = \frac{-1}{\sqrt{2}} - \frac{1}{c} \longrightarrow A$$

Apply Initial Condition:-

$$y(0) = -1$$

$$-1 = \frac{-1}{\sqrt{2}} - \frac{1}{c}$$

$$-1 = -(\sqrt{2} + c)$$

$$1 = \sqrt{2} + c$$

$$1 = 2 + c$$

$c = -1$ Put in A

$$y(x) = \frac{-1}{2} - \frac{1}{-1}$$

$$\boxed{y(x) = \frac{-1}{\sqrt{2}} + 1}$$ Ans.

Pg # 05

ii) Solution

$$\frac{dx}{dt} = \frac{t}{x}$$

Separating the variable

$$x \frac{dx}{dt} = t$$

Integrating

$$\int x \frac{dx}{dt} dt = \int t \, dt,$$

$$\int x \, dx = \int t \, dt$$

or, Since $dx = \frac{dx}{dt} dt,$

$$\frac{1}{2} x^2 = \frac{1}{2} t^2 + C.$$

PROCEDURE FOR SOLVING A LINEAR Equation

STEP 1: Multiply both sides of the normal form of the equation

$$x' + p(t)x = q(t)$$

by the integrating factor

$$\mu(t) = e^{\int p(t)\,dt} = e^{P(t)}$$

STEP 2. Obtain

$$\left(e^{P(t)}x\right)' = e^{P(t)}q(t)$$

STEP 3:- Integrate (take the antiderivative) both sides to obtain

$$e^{P(t)}x(t) = \int e^{P(t)}p(t)\,dt + C$$

STEP 4:- Multiply by $e^{-P(t)}$ to obtain the general solution

$$x(t) = e^{-P(t)}\int e^{P(t)}p(t)\,dt + Ce^{-P(t)}$$

Ans SOLUTION:-

2(i)

$$\cos(x)\, y' + \sin(x)\, y = 2\cos^3(x)\sin(x) - 1$$

$$y' = \frac{\sin(x)}{\cos(x)}\, y = \frac{2\cos^3(x)\sin(x) - 1}{\cos x}$$

pg # 08

$y' \tan(x)\, y = 2\cos^2(x)\sin(x) - \sec(x) \longrightarrow$ (A)

(1) Compare eq (A) with

$y' + p(x)y = q(x)$ we obtain

$p(x) = \tan x,\ q(x) = 2\cos^2(x)\sin(x) - \sec(x)$

The integrating factor is

$\int p(x)\,dx = \int \tan x\,dx$

$u(t) = e = e$

$= -\cos(x)$

(2) Multiply $-\cos x$ to both sides of eq (A)

$-\cos(x)\, y' - \tan(x)\cos(x)\, y = -2\cos^3(x)\sin x + \sec x \cos(x)$

$-\cos(x)\, y' - \cancel{\tan(x)\cos(x)\, y}^{\sin} = -2\cos^3(x)\sin x + 1$

$(-\cos(x)y' \sin x = -2\cos^3(x)\sin(x) + 1$

(3) Integrate both sides

$\int (-\cos(x)\, y')\,dx = \int -2\cos^3(x)\sin(x)\,dx + \int 1\,dx$

$-\cos(x)y = -2\int \cos^3(x)\sin(x)\,dx + x + c$

$-\cos(x)y = 2\int \cos^3(x)(-\sin(x))\,dx + x + c$

Pg # 09

$$-\cos(x)\,y = \frac{2\cos^{3+1}(x)}{3+1} + x + C$$

$$-\cos(x)\,y = \frac{1}{2}\cos^4(x) + x + C$$

$$y = -\frac{1}{2}\cos^3(x) - \frac{x}{\cos x} - \frac{C}{\cos x} \longrightarrow (1)$$

Apply initial condition:

$$y[\pi/4] = 3\sqrt{2}$$

$$3\sqrt{2} = -\tfrac{1}{2}\cos^3\left(\frac{\pi}{4}\right) - \frac{\pi/4}{\cos\frac{\pi}{4}} - \frac{C}{\cos\left(\frac{\pi}{4}\right)}$$

$$3\sqrt{2} = -\tfrac{1}{2}\left(\sqrt{2}/2\right)^3 - \frac{\pi/4}{(\sqrt{2}/2)} - C/(\sqrt{2}/2)$$

$$3\sqrt{2} = -1/2\left(\frac{2\sqrt{2}}{8}\right) - \frac{2\pi}{4\sqrt{2}} - \frac{2C}{\sqrt{2}}$$

$$3\sqrt{2} = \frac{-\sqrt{2}}{8} - \frac{\pi}{2\sqrt{2}} - \frac{2C}{\sqrt{2}}$$

$$6 = \frac{-2}{8} - \frac{\pi}{2} - 2C$$

$$6 = -1/4 - \pi/2 = 2C$$

$$6 + 1/4 - \frac{\pi}{2} = 2C$$

$$24 + 1 - 2\pi/4 = 2C$$

$$\frac{25 - 2\pi}{4} = -2C$$

$$\frac{2\pi - 25}{8} = C$$

put in (1) equation

$$\boxed{y(x) = \frac{-1}{2}\cos^3(x) - \frac{x}{\cos^3} - \frac{(2\pi - 25)}{8\cos x}}$$ Ans

SOLUTION:

Consider the differential equation

$$x' + 2x = \sin t$$

We multiply the DE by the integrating factor

$$\mu(t) = e^{\int 2 dt} = e^{2t}$$

$$(xe^{2t})' = e^{2t} \sin t$$

Integrating both sides gives

$$xe^{2t} = \int e^{2t} \sin t \, dt + C$$

$$x(t) = e^{-2t} \int e^{2t} \sin t \, dt + Ce^{-2t}$$

The integral on the right side can be calculated using integration by parts (or consulting the integral table). In any case we obtain the general solution.

3) SOLUTION:

1) $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0, y(0) = -3$

$M = 2xy - 9x^2 \quad M_y = 2x$

$N = 2y + x^2 + 1 \quad N_y = 2x$

$= M$

$= N$

$= \int M\,dx$ OR $= \int N\,dy$

$(x,y) = \int 2xy - 9x^2\,dx = x^2y - 3x^3 + h(y)$

$y = x^2 + h'(y) = 2y + x^2 + 1 = B$

$h'(y) = 2y + 1$

$h(y) = \int 2y + 1\,dy = y^2 + y + k$

$(x,y) = x^2y - 3x^3 + y^2 + y + k = y^2 + (x^2+1)y - 3x^3 + k$

$y^2 + (x^2+1)y - 3x^3 + k = C$

$y^2 + (x^2+1)y - 3x^3 = C - k$

Pg #

$y^2 + (x^2+1)y - 3x^3 = c$

$(-3)^2 + (0+1)(-3) - 3(0)^3 = c$ $\boxed{c = 6}$

$y^2 + (x^2+1)y - 3x^3 - 6 = 0$

Now, as we in the seperable differential equation section, this is quadratic in y & so we can solve the y(x) by using quadratic formula:

$$y(x) = \frac{-(x^2+1) \pm \sqrt{(x^2+1)^2 - 4(1)(-3x^3-6)}}{2(1)}$$

$$= \frac{-(x^2+1) \pm \sqrt{x^4+12x^3+2x^2+25}}{2}$$

$$-3 = y(0) = \frac{-1 \pm \sqrt{25}}{2}$$

$$= \frac{-1 \pm 5}{2}$$

$$= -3, 2$$

$$y(x) = \frac{-(x^2+1) - \sqrt{x^4+12x^3+2x^2+25}}{2}$$

$$= x^4+12x^3+2x^2+25 = 0$$

Pg # 13

(ii) SOLUTION:-

$$\frac{2ty}{t^2+1} - 2t - (2 - \ln(t^2-1)) y' = 0$$

$$= \frac{2ty}{t^2+1} - 2t + (\ln(t^2+1) - 2) y' = 0$$

$$M = \frac{2ty}{t^2+1} - 2t, \quad M_y = \frac{2t}{t^2+1}$$

$$N = \ln(t^2+1) - 2, \quad N_t = \frac{2t}{t^2+1}$$

$$(t,y) = \int \frac{2ty}{t^2+1} - 2t \, dt = y \ln(t^2+1) - t^2 + h(y)$$

$$y = \ln(t^2+1) \, h'(y) = \ln(t^2+1) - 2 = N$$

$$h'(y) = (-2) \Rightarrow h(y) = -2y$$

$$(t,y = y \ln(t^2+1) - t^2 - 2y$$

$$y \ln(t^2+1) - t^2 - 2y = C$$

$$-25 = C$$

$$y \ln(t^2+1) - 2 - t^2 = -25$$

$$y(t) = \frac{t^2-25}{\ln(t^2+1)-2}$$

$$\ln(t^2+1) - 2 = 0$$

$$\ln(t^2+1) = 2$$

$$t^2+1 = e^2$$

$$t = \pm\sqrt{e^2-1}$$

Possible interval of validity

$$-\infty < t < -\sqrt{e^2-1}$$

$$-\sqrt{e^2-1} < t < \sqrt{e^2-1}$$

$$\boxed{\sqrt{e^2-1} < t < \infty}$$ Ans.