

Sol-
$$\begin{matrix} 40 & & 50 \\ \begin{matrix} P & E \\ A & E \end{matrix} & \begin{matrix} P & P \\ A & E \end{matrix} & \begin{matrix} P & P \\ P & A \end{matrix} \\ b_1 & b_2 & b_3 \\ 1:2:1 & 2:1:1 & 2:0:2 \end{matrix}$$

So,

Let x, y, z be the cost/kg in Pak, Egypt and American.

According to given conditions:

$$\left. \begin{aligned} \frac{1}{4}x + \frac{2}{4}y + \frac{1}{4}z &= 40 \\ \frac{2}{4}x + \frac{1}{4}y + \frac{1}{4}z &= 50 \\ \frac{2}{4}x + \frac{2}{4}z &= 60 \end{aligned} \right\} \rightarrow (1)$$

$$\left. \begin{aligned} 1x + 2y + 1z &= 160 \\ 2x + 1y + 1z &= 200 \\ 1x + 1z &= 120 \end{aligned} \right\} \rightarrow (2)$$

in

matrices;

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

$$\text{Let } A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, B = \begin{pmatrix} 160 \\ 200 \\ 120 \end{pmatrix}$$

We know $Ax = B$

$$X = A^{-1}B$$

$$\text{and } A^{-1} = \frac{\text{adj}(A)}{|A|}$$

$$\Rightarrow A_1 = \begin{pmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{pmatrix}, A_2 = \begin{pmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{pmatrix}$$

Now $|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$

expand by R_1

$$|A| = 1 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= 1(1) - 2(2-1) + 1(2-1)$$

$$= -1 + 2 + 1$$

$$|A| = 2$$

Now

$$A_1 = \begin{pmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{pmatrix}$$

Expand with R_1

$$= 160 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} + 1 \begin{vmatrix} 200 & 1 \\ 120 & 0 \end{vmatrix}$$

$$= 160 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} + 1 \begin{vmatrix} 200 & 1 \\ 120 & 0 \end{vmatrix}$$



$$= 160(1) - 2(200 - 120) + 1(200 - 120)$$

$$|A| = -120$$

Same process for A_2 and A_3 .

$$|A_2| = \begin{vmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{vmatrix} \text{ expand with } R_1,$$

$$= 1 \begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} - 160 \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 200 \\ 1 & 120 \end{vmatrix}$$

$$= 1(200 - 120) - 160(2 - 1) + 1(240 - 200).$$

$$A_2 = -40$$

$$|A_3| = \begin{vmatrix} 1 & 20 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{vmatrix} \text{ expand}$$

$$= 1 \begin{vmatrix} 1 & 200 \\ 0 & 120 \end{vmatrix} - 2 \begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} + 160 \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= -1120$$

According to Cramer's Rule.

$$x = \frac{|A_1|}{|A|} = \frac{120}{-2} = 60$$

$$y = \frac{|A_2|}{|A|} = \frac{-40}{-2} = 20$$

$$z = \frac{|A_3|}{|A|} = \frac{-1120}{-2} = 60$$

→

$$(x, y, z) = (60, 20, 60).$$

path ; 60

egyptian ; 20

American ; 60