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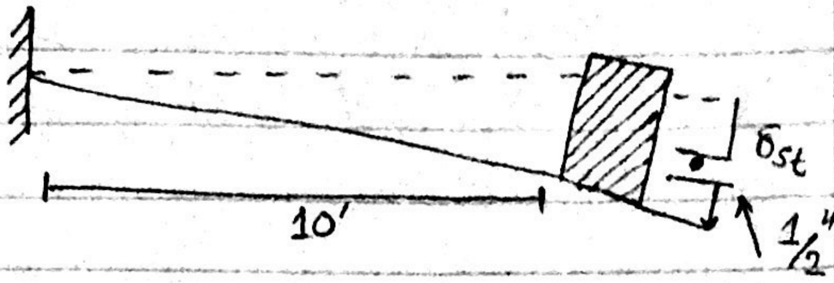
Assignment ; Introduction to earth quake engineering

Date ; 29/06/2020.

Question No: 1

Q.No:1 Given data



$E = 29,000$ KSI

$I = 150$ inch^4

Load $= 7686$ lb

Δ (Deflection) $= 1/2''$

* Solution;

* The general E.O.M for SDOF system is

$$ku + c\dot{u} + m\ddot{u} = P(t)$$

* In our case system is undamped ($c = 0$) undergoing free vibration ($P(t) = 0$)
* Hence general EOM become

$$ku + m\ddot{u} = 0 \quad \text{--- (1)}$$

$$k = \frac{3 \times 29000 \text{ k/inch}^2 \times 150 \text{ inch}^4}{(10 * 12)^3}$$

$$\boxed{k = 7.55 \text{ k/inch}}$$

Question # 01

* In order to eliminate the chances of mistake during calculation, it is more appropriate to use fundamental units like lb, ft sec or kg, m, sec.

$$K = 7.55\ k/inch = 90625\ lb/ft$$

$$m = \frac{7686\ lb/sec^2}{32.2\ ft} = 238.69\ Slug$$

$$\omega_n = \sqrt{K/m} = \sqrt{\frac{90625}{238.69}}$$

$$\omega_n = \sqrt{379.68}$$

$$\omega_n = 19.49\ rad/sec$$

$$T_n = \frac{2\pi}{\omega_n} = \frac{2 \times 3.14}{19.49} = 0.322\ Sec$$

Substituting the corresponding values in eq-1

$$90625u + 238.69\ddot{u} = 0$$

Question # 1

* Where "K" is in lb/ft and "m" is in lb.sec/ft²

* General solution to the EOM for undamped free vibration is

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n \sin(\omega_n t)}$$

$$u(0) = \frac{1}{2^4} = \frac{1}{24}\ ft \text{ and } \dot{u}(0) = 0.$$

$$u(t) = \left(\tfrac{1}{24}''\right)\cos(19.49) + 0 = \tfrac{1}{24}(\cos(19.49))$$

* Equivalent static force at any time "t" is

$$f_s(t) = K.u(t) = \frac{90625 \times \cos(19.49)}{24.}$$

$$f_s(t) = 3776\cos(19.49)$$

* Amplitude of dynamic displacement u_o for undamped free vibration is.

$$u_o = \sqrt{\left[(u(0))^2 + \left(\frac{\dot{u}(0)}{\omega_n}\right)^2\right]} = \sqrt{\left(\frac{1}{24}\right)^2 + 0}$$

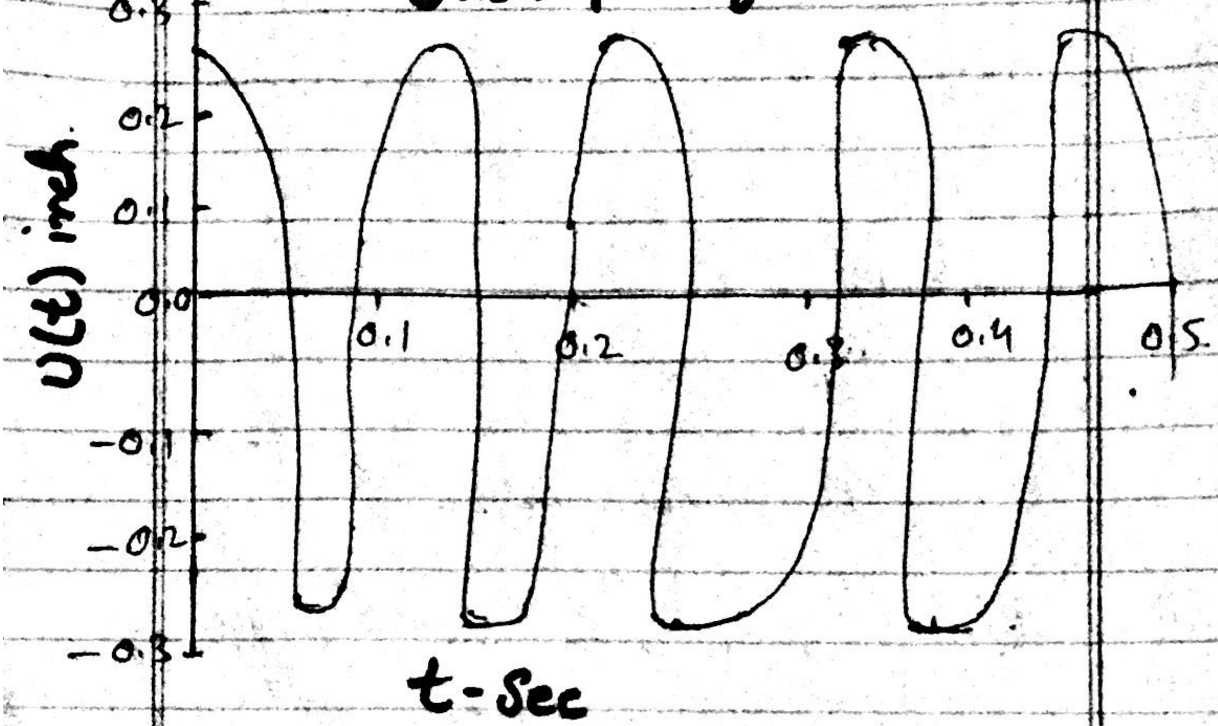
$$u_o = 1/24$$

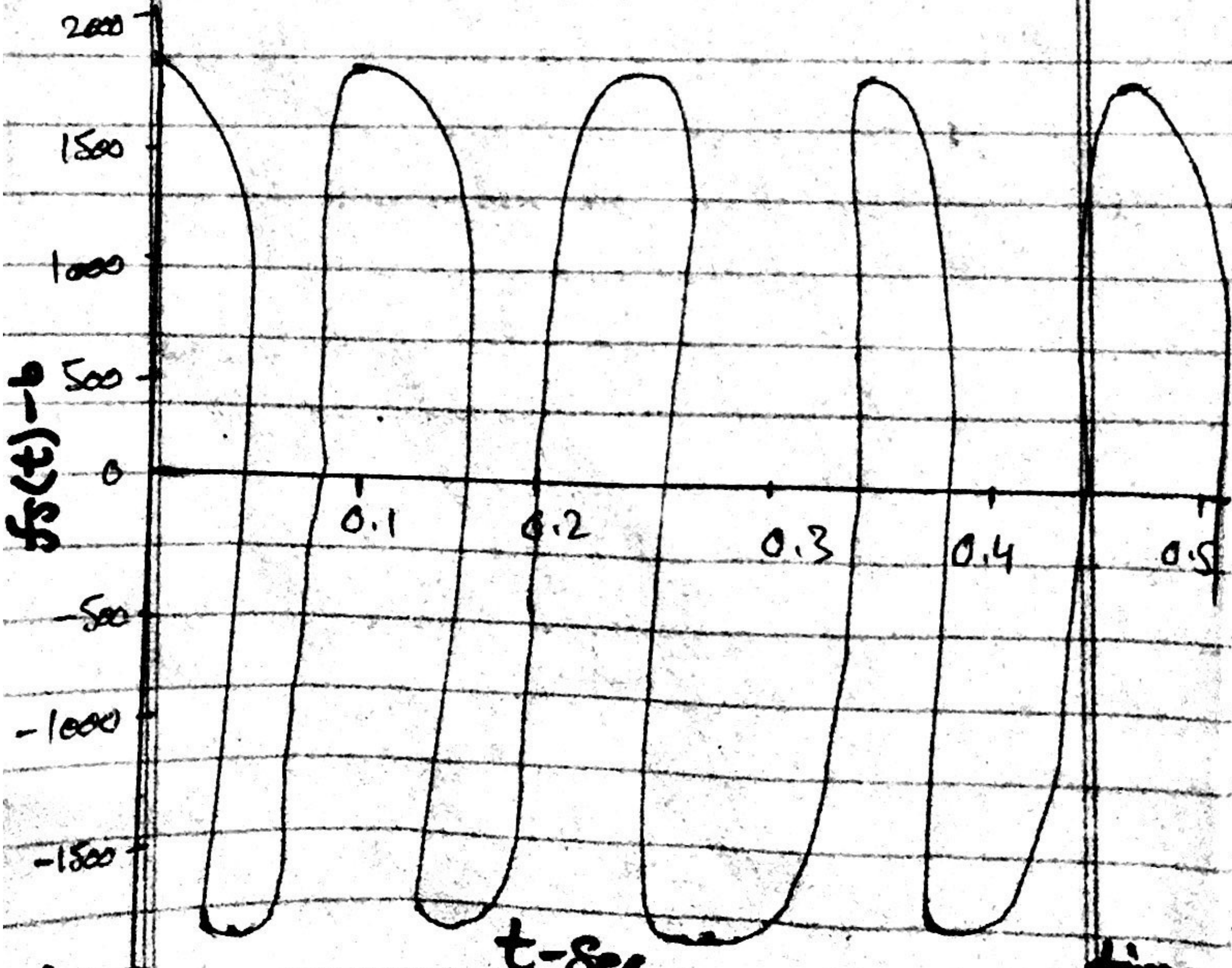
* Amplitude of equivalent force is, f_{so}

$$K u_o = 90625 \times 1/24 = 3776\ lb.$$

Question #01

Undamped free vibration





variation of equivalent static force with time

Question # 02

Page # 05

* Given data

$\Delta = 1/2''$

load = 7686 lb

$E = 29,000$ ksi

$I = 150$ inch^4

$\zeta = 2.5\%$

* Solution;

* E.O.M for damped free vibration is:

$ku + c\dot{u} + m\ddot{u} = 0$ —— (1)

* It is known from problem 1 that

$k = 90625$ $m = 238.69$ slug

$\omega_n = 19.49$ rad/sec.

$C = \zeta \times 2m\omega_n$

$C = 2 \times 0.025 \times 238.69 \times 19.49$

$C = 232.60$ lb.sec/ft.

* By substituting values of k, c and m in eq (1).

Question # 02

$$90625\,u + 232.60\,\dot{u} + 238.69\,\ddot{u}$$

* Solution of the E.O.M for damped free vibration is

$$u(t) = e^{-\xi \omega n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\dot{u}(0) + u_0 \xi \omega_n\right] \cdot \sin(\omega_D t)\right]$$

$$u(t) = e^{-0.025 \times 19.49t}\left(\frac{1}{24}\cos(19.49t) + \frac{1}{19.49}\left(0 + \frac{1}{24} \times 0.025 \times 19.49 \sin(19.49t)\right)\right)$$

$$u(t) = e^{-0.487}\left(0.042\cos(19.49t) + 0.0513\left(0.001 \times 19.49 \sin(19.49t)\right)\right)$$

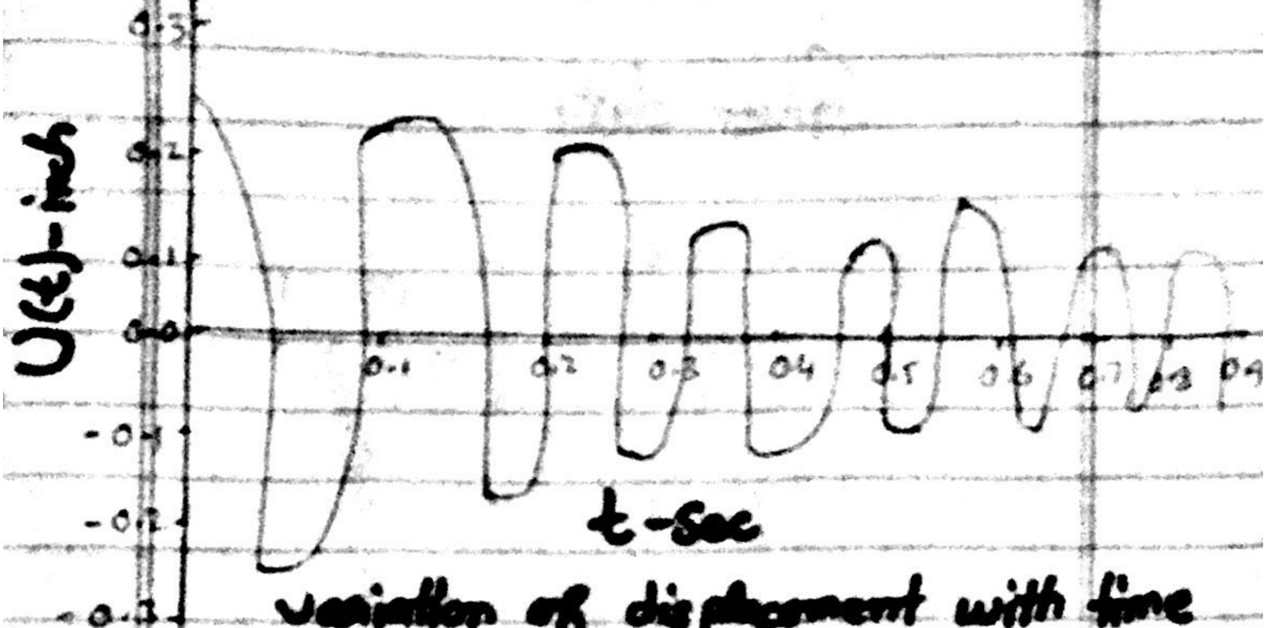
$$u(t) = e^{-0.487}\left(0.042\cos(19.49t) + 0.0513\left(0.0195 \sin(19.49t)\right)\right)$$

$$u(t) = e^{-0.487}\left(0.042\cos(19.49t) + 0.001\sin(19.49t)\right)$$

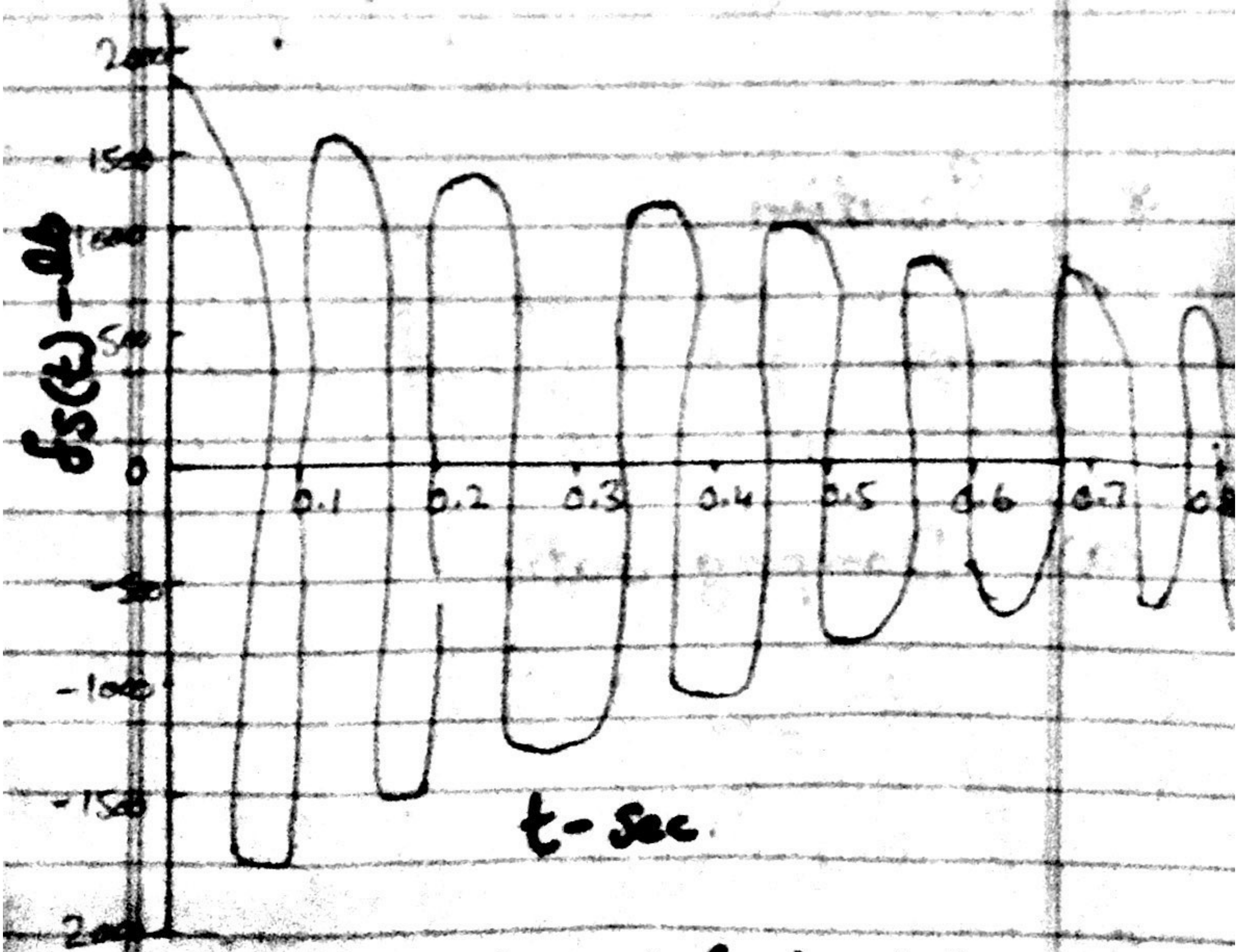
$$f_s(t) = K \cdot u(t) = 90625\, u(t)$$

$$f_s(t) = e^{-0.487}\left(3806.25\cos(19.49t) + 90.63\sin(19.49t)\right)$$

Question #02



variation of displacement with time

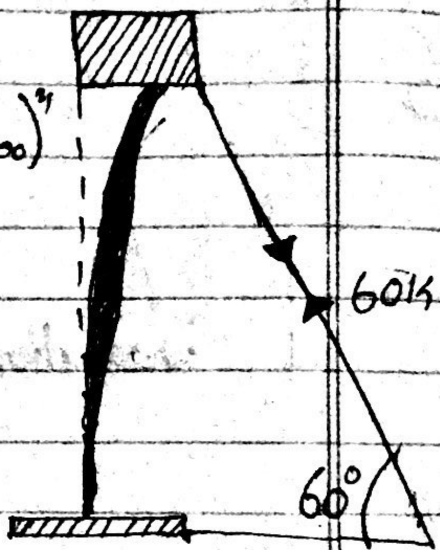


variation of equivalent static forces with time

Question # 03

Given data

Force = 60 kips



Displace the tanke = $(^{ID}/_{1000})''$

$= \left(\frac{7686}{1000}\right)'' = 7.69''$

Cycles = 7

Time = 3.57 sec

Amplitude of displacement = 2.286 cm.

* Solution

$U_1 = 7.69''$

After $j = 7$ $U_{j+1} = U_8 = 2.286 = 0.9''$

1 inch = 2.54 cm.

(a) Damping ratio

$$j = \frac{1}{2\pi\delta} \ln\left(\frac{U_1}{U_{j+1}}\right)$$

$$7 = \frac{1}{(2)(3.14)(\delta)} \ln\left(\frac{7.69}{0.9}\right)$$

$$7 = \frac{1}{6.28\,\delta} \ln(8.54)$$

Question # 03

$$7 = \frac{1}{6.28\,\delta}(2.145)$$

$$7 \times 6.28\,\delta = 2.145$$

$$43.96\,\delta = 2.145$$

$$\delta = \frac{2.145}{43.96}$$

$$\delta = 0.049 = 4.9\%$$

(b) $T_n = ?$

* 7 cycles of vibrations are completed in 3.57 sec

* Time required to complete one cycle $= \frac{3.57}{7} = T_D$

$$T_D = \frac{3.57}{7} = 0.51\ \text{sec}$$

Question #03

Now

$$\omega_D = \omega_n \sqrt{(1-\delta^2)}$$

$$\frac{2\pi}{\omega_D} = \frac{2\pi}{\omega_n \sqrt{1-\delta^2}}$$

$$T_D = \frac{T_n}{\sqrt{1-\delta^2}}$$

$$T_n = T_D \times \sqrt{1-\delta^2}$$

$$T_n = 0.51 \times \sqrt{1-(0.049)^2}$$

$$T_n = 0.51 \times \sqrt{1-0.0024}$$

$$T_n = 0.51 \times \sqrt{1}$$

$$T_n = 0.51 \times 1$$

$$T_n = 0.51$$

Question # 03

(C) $K = ?$

$$K = \frac{60 \cos 60^0}{2} = \frac{60(0.5)}{2} = 15 \; K/\text{inch}$$

$K = 18000 \; lb/ft.$

(D) Weight of tank, $W = ?$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{(W/g)}} = \sqrt{\frac{K \times g}{W}}$$

$$\omega_n^2 = \frac{K \times g}{W}$$

$$W = \frac{K \times g}{\omega_n^2} \quad \text{Also } \omega_n = 2\pi/T_n$$

$$W = Kg \Big/ \left(\frac{4\pi^2}{T_n^2}\right) = Kg \times \frac{T_n^2}{4\pi^2}$$

$$W = \frac{18000 \frac{lb}{ft} \times 32.2 \frac{ft}{sec^2} (0.51 \, sec)^2}{4\pi^2}$$

[redacted]

$$W = \frac{150753.96}{39.44}$$

$W = 3822.36 \; lbs$

Question #03

(c) $C = ?$

* It is known that $\zeta = \frac{C}{2m \times \omega_n}$

$$C = \zeta \times 2m(2\pi/T_n)$$

$$C = 4\zeta m \pi/T_n$$

$$C = \frac{4 \times 0.049 \times 3.14\left(\frac{3822.36}{32.2}\right)}{0.51}$$

$$C = \frac{4 \times 0.049 \times 3.14(118.71)}{0.51}$$

$$C = 143.253 \text{ lb.sec/ft}$$

(b) No. of cycles to reduce displacement amplitude from 2 inch to 0.5 inch, j=?

$$j = \frac{1}{2\pi\zeta} \ln\left(\frac{U_1}{U_{j+1}}\right)$$

$$j = \frac{1}{2 \times 3.14 \times 0.049} \ln\left(\frac{7.69}{0.9}\right)$$

$$j = \frac{1}{0.307} \ln(8.54)$$

$$j = \frac{2.145}{0.307} = 7 \text{ Cycles}$$