Department of Electrical Engineering
Sessional Assignment
Date: 01/06/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:	Pir Meher Ali Shah	**Total Marks:**	20

Student Details

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Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$ To the input $x(n) = 4^n u(n)$.	Marks 6
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) = 0.6y(n-1) - 0.8y(n-2) + x(n)$$	
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	Marks 6
	(b)	Determine the partial fraction expansion of the following proper function $$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$	
Q.3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_0}{(1-pz^{-1})^2}$$ Determine the values of b_0 and p such that the frequency response H(ω) satisfies the condition H(0) = 1 and $\left\|H(\frac{\pi}{4})\right\|^2 = \frac{1}{2}$.	Marks 4

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Q: 1 (a)
Ans:

Solution:

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1) \quad -(i)$$

$$y_h(n) = c_1 \lambda_1^n + c_2 \lambda_2^n$$

$$y_h(n) = c_1(-1)^n + c_2(4)^n \quad (a)$$

$$y_p(n) = K(4)^n u(n) \quad -(ii)$$

Now substitution of (ii) into (i) we obtain

$$Kn(4)^n u(n) - 3K(n-1)(4)^{n-1} u(n-1) - 4K(n-2)(4)^{n-2} u(n-2) = (4)^n u(n) + 2(4)^{n-1} u(n-1)$$

To determin K we evaduate this equation for any $n \geq 2$, where none of the unit step term fanish. To simplify the arithematic, we select $n = 2$ from which we obtain $K = 6/5$ therefore

$$y_p(n) = 6/5\, n(4)^n u(n) \quad (iii)$$

So Add (a) & (iii)

$$y(n) = c_1(-1)^n + c_2(4)^n + 6/5\, n(4)^n \quad n \geq 0 \quad (iv)$$

where the constant c_1 & c_2 are determined such that the initial conditions are satisfied.

$y(0) = 3y(-1) + 4y(-2) + 1$

$y(1) = 3y(1) + 4y(-1) + 6$

$= 13y(-1) + 12y(-2) + 9$

equation (iv) evaluated at $n=0$ & $n=1$

$y(0) = C_1 + C_2$
$y(1) = -C_1 + 4C_2 + 24/5$

$C_1 + C_2 = 1$
$-C_1 + 4C_2 + \frac{24}{5} = 9$

Hence $C_1 = -\frac{1}{25}$ & $C_2 = \frac{26}{25}$ finally we have the zero state response to the forcing function $x(n) = 4^n u(n)$ in the form

$y_{zs}(n) = -\frac{1}{25}(-1)^n + \frac{26}{25}(4)^n + 6/5\, n (4)^n, \quad n \geq 0$

Q:1(b)

Ans: Solution: $y(n) = 0.6y(n-1) - 0.8(n-2) + x(n)$

Consider the differential equation

$y(n) = 0.6y(n-1) - 0.8(n-2) + x(n)$

$y(n) - 0.6(n-1) + 0.8y(n-2) = x(n)$

To obtain the homogeneous equ set input $x(n) = 0$

$y(n) - 0.6(n-1) + 0.8(n-2) = 0$

Determine the solution to the homogenous equ

$y_h(n) = \lambda^n$

$\lambda^n - 0.6\lambda^{n-1} + 0.8\lambda^{n-2} = 0$

$\lambda^{n-2}(\lambda^2 - 0.6\lambda + 0.8) = 0$

$\lambda^2 - 0.6 + 0.8 = 0$

$(\lambda - 0.2)(\lambda - 0.4) = 0$

They have two roots

$\lambda_1 = 0.2, \ \lambda_2 = 0.4$

The General from of homogeneous equation as,

$y_n(n) = C_1 (\alpha_1)^n + C_2 (\alpha_2)^n$

So Put the value of α_1 & α_2

$y(n) = C_1 (0.2)^n + C_2 (0.4)^n$ —(i)

$\alpha_1 = 0.2$, $\alpha_2 = 0.4$

Hence

$$y_n(n) = C_1 \frac{1^n}{5} + C_2 \frac{2^n}{5}$$

with $x(n) = \delta(n)$ the initial condition are,

$y(0) = 1$

$y(1) - 0.6y(0) = 0$

$y(1) = 0.6$

Hence

$C_1 + C_2 = 1$ & $\frac{1}{5} C_1 + \frac{2}{5} = 0.6$

$\Rightarrow C_1 = -1$, $C_2 = 3$

Therefore

$$h(n) = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right] u(n)$$

Step response is

$$S(n) = \sum_{K=0}^{n} h(n-K) , n > 0$$

$$= \sum_{K=0}^{n} \left[2\left(\frac{2}{5}\right)^{n-K} - \left(\frac{1}{5}\right)^{n-K}\right]$$

$$= \left\{\frac{1}{0.12}\left[\frac{2}{5}^{n+1} - 1\right] - \frac{1}{0.16}\left[\frac{1}{5}^{n+1} - 1\right]\right\} u(n)$$

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Q:2 a

Ans

Solution:

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

By partial fraction method

$$\frac{1}{(1-2z^{-1})(1-z^{-1})} = \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$$= \frac{A(1-z^{-1})^2 + B(1-2z^{-1}) + Cz^{-1}(1-2z^{-1})}{(1-2z^{-1})(1-z^{-1})^2}$$

$$1 = A(1-z^{-1})^2 + B(1-2z^{-1})(1-z^{-1}) + Cz^{-1}(1-2z^{-1}) \quad \text{— (i)}$$

Put $z = 1$

$$1 = A(1-1)^2 + B(1-2(1)(1-1) + C1(1-2(1))$$

$$1 = A(0) + B(-1)(0) + C(1)(-1)$$

$$1 = 0 + 0 - C$$

$$1 = -C$$

$$C = -1$$

Put $z = 2$ in equ (i)

$$1 = A\left(1-\frac{1}{2}\right)^2 + B\left(1-\frac{2}{2}\right)\left(1-\frac{1}{2}\right) + C\left(\frac{1}{2}\right)\left(1-\frac{2}{2}\right)$$

$$1 = A\left(\frac{1}{2}\right)^2 + B(0)\left(\frac{1}{2}\right) + C\left(\frac{1}{2}\right)(1-1)$$

$$1 = \frac{A}{4} + B(0) + C(0)$$

$$1 = \frac{A}{4} + 0 + 0$$

$$A = 4$$

Put $z = 3$ in equ (i)

$$1 = A\left(1-\frac{1}{3}\right)^2 + B\left(1-\frac{2}{3}\right)\left(1-\frac{1}{3}\right) + C\left(\frac{1}{3}\right)\left(1-\frac{2}{3}\right)$$

$$1 = A\left(\frac{4}{9}\right) + B\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) + C\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)$$

$$1 = \frac{A4}{9} + \frac{2}{9}B + \frac{1}{9}C$$

$$1 + \frac{1}{9} - \frac{16}{9} = \frac{2}{9}B$$

Taking L.C.M

$$\frac{9+1-16}{9} = 2/9\, B$$

$$\frac{\overset{3}{\cancel{-6}}}{\cancel{9}} \times \frac{\cancel{9}}{\cancel{2}_1} = B$$

$$B = -3$$

Hence $x[n] = 4(2)^n - 3 - n]\, u(n)$

Q:2 (b)

Ans: Solution: First we eliminate the negative powers by ×ing both numerator & denominator by z^2.

$$x(z) = \frac{1}{1-1.5z^{-1}+0.5z^{-2}}$$

$$x(z) = \frac{z^2}{z^2-1.5z+0.5}$$

the poles of $X(z)$ are $P_1 = 1$ & $P_2 = 0.5$

$$\frac{x(z)}{z} = \frac{z}{(z-1)(z-0.5)}$$

$$= \frac{A_1}{z-1} + \frac{A_2}{z-0.5}$$

A_1 & A_2 multiply the eqn by the denominator term $(z-1)(z-0.5)$ we obtain

$$z = (z-0.5)A + (z-1)A_2 \quad -(i)$$

Now $z = p_1 = 1$, we eliminat the term involving A_2.

Hence

$$1 = (1-0.5)A_1$$

$A_1 = 2$ Next we retur eqn (i) & set $z = P_2 = 0.5$ then eliminating A_1.

$$0.5 = (0.5-1)A_2$$

Hence

$$A_2 = -1$$

Therefore the result of the partial expansion is

$$\boxed{\frac{X(Z)}{Z} = \frac{2}{Z-1} - \frac{1}{Z-0.5}}$$

Required Answer.

Q:3 (a)
Ans:

Solution:

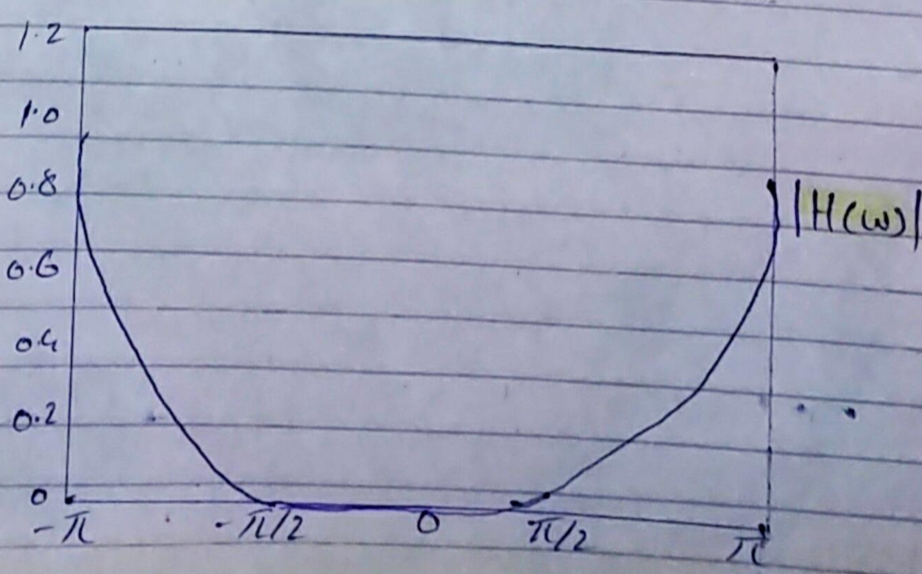
At $\omega = 0$ we have

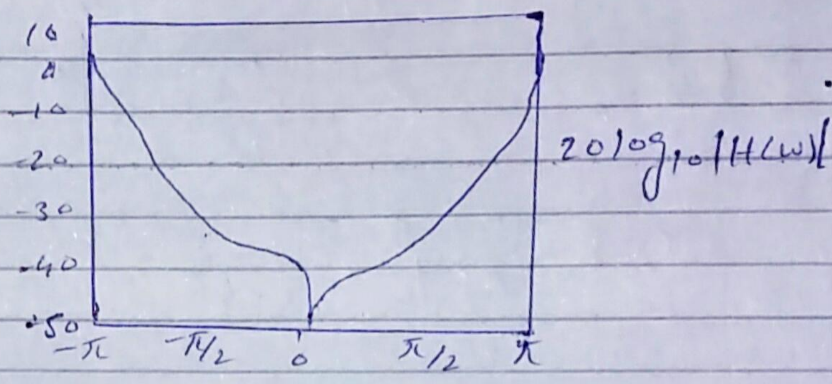
$$H(Z) = \frac{b_0}{(1-PZ^{-1})^2}$$

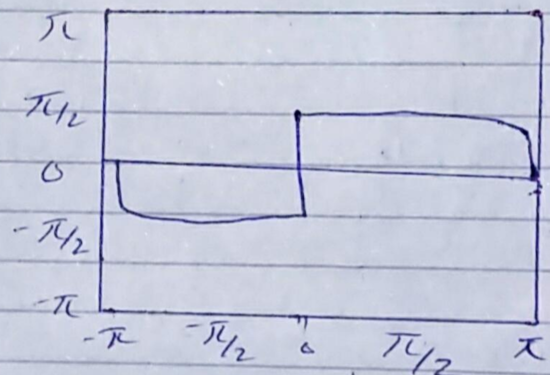
$$H(0) = \frac{b_0}{(1-P)^2} = 1$$

Hence

$$b_0 = (1-P)^2$$







At $\omega = \pi/4$.

$$H(\pi/4) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{(1-P\cos(\pi/4) + jP\sin(\pi/4))^2}$$

$$= \frac{(1-P)^2}{(1-P/\sqrt{2} + jP/\sqrt{2})^2}$$

Hence $\boxed{\dfrac{(1-P)^2}{[(1-P/\sqrt{2})^2 + P^2/2]^2} = \dfrac{1}{2}}$

Required Answer.

Q:3 (B)

Ans:

Solution:

$$P_{1/\sqrt{2}} = re^{4\pi/2}$$

& zeros at $z=1$ & $z=-1$

consquently, the system function is

$$H(z) = G \frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G \frac{z^2-1}{z^2+r^2}$$

the gain factor is determaind by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$, thus we have

$$H(\pi/2) = G \frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

the value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$. thus we have

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^2+2r^2\cos(8\pi/9)}$$

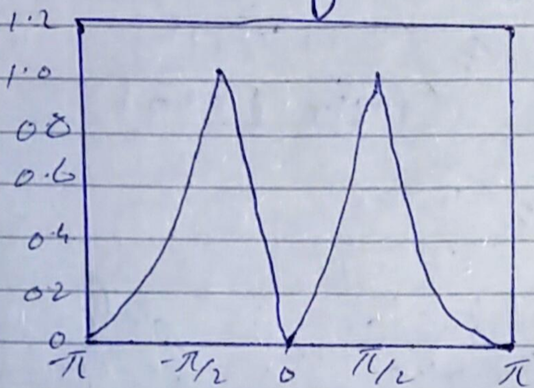
$$= \frac{1}{2}$$

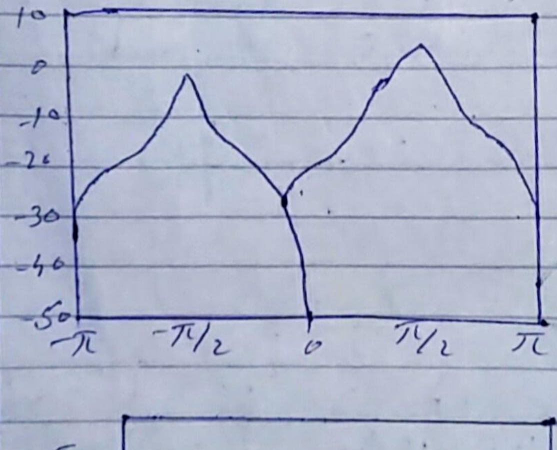
or equivalently.

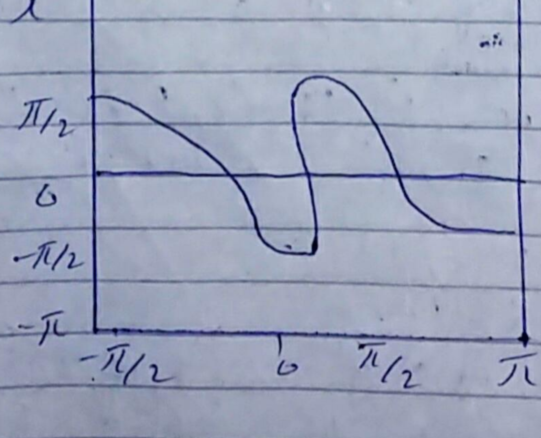
the value of $r^2 = 0.7$ satisfies this equation. therefore the system function for the desired filter is

$$H(z) = 0.15 \frac{1 - z^{-2}}{1 + 0.7 z^{-2}}$$

its frequency response is illustrated







Q: 4 (a)

Ans:

The fourier transform of this sequence is

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}}$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

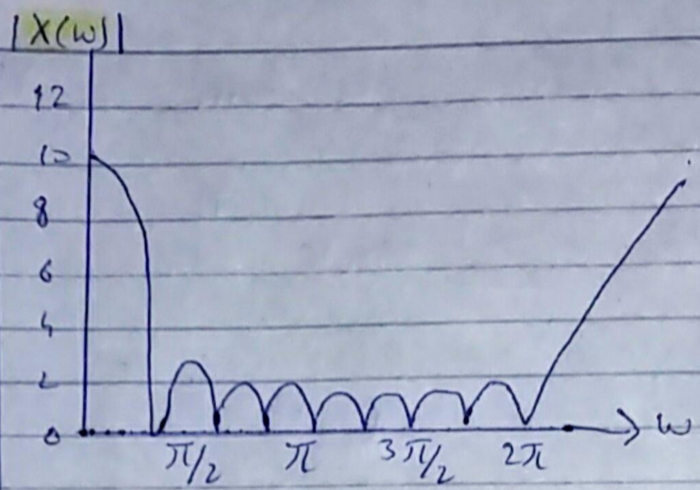
the magnitude & Phase of $X(\omega)$ are illustrated are the following figure for $L = 10$. the N-Point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the Set of N equallay spaced frequiencies $\omega_K = 2\pi K/N$

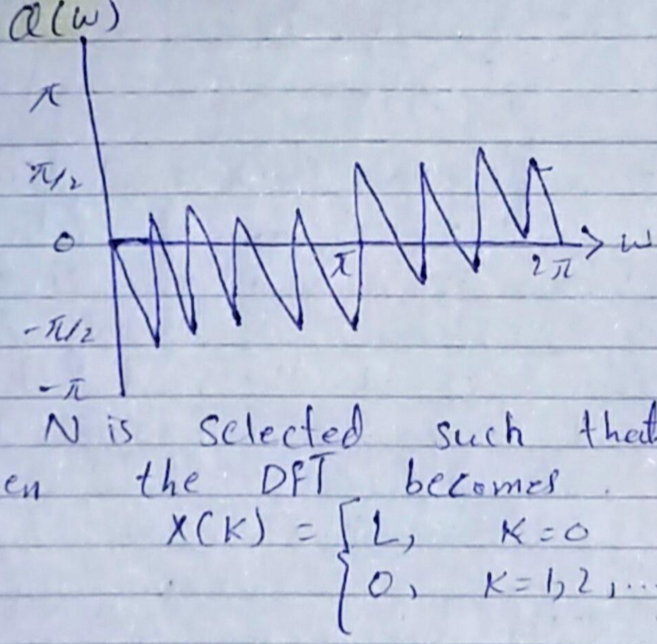
$K = 0, 1, \cdots, N-1$ Hence.

$$X(K) = \frac{1 - e^{-j2\pi KL/N}}{1 - e^{-j2\pi K/N}}$$

$K = 0, 1, \cdots, N-1$

$$= \frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$





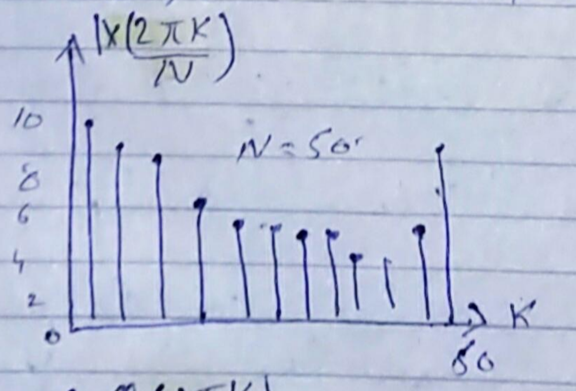
if N is selected such that N = L then the DFT becomes

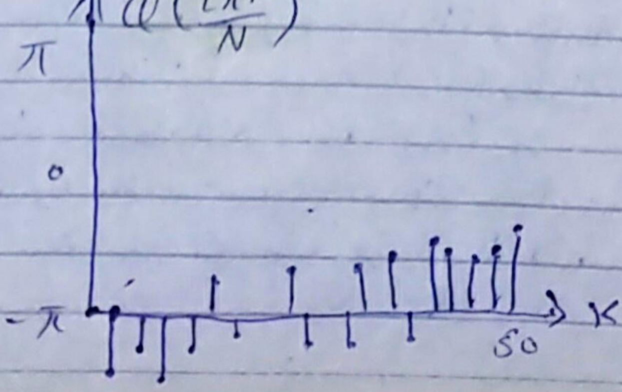
$$X(K) = \begin{cases} L, & K = 0 \\ 0, & K = 1, 2, \ldots L-1 \end{cases}$$

thus there is only one nonzero value in the DFT. this is apparent from observation of $X(\omega)$, Since $X(\omega) = 0$ at the frequiencies $\omega_K = 2\pi K / L$, $K \neq 0$ the reader should verify that $x(n)$ can be recovered from $X(K)$ by performing an L-point IDFT.

Although the L-Point DFT is sufficient to uniquely represent the Sequence $x(n)$ in the frequency domain, it is apparent that it does not provide sufficient detail to yield a good picture of the spectral characteristics of $x(n)$ if we wish to have a better pic we must evaluate (interpolate) $X(\omega)$ at more closely spaced frequency say $\omega_k = 2\pi k/N$ where $N > L$ in effect. L points to N point by appending $N-L$ zeros to the sequence $x(n)$ that is zero padding then the n-point DFT provides finer interpolation than the L-point DFT.

$N = 50$, $L = 10$ & $N = 100$





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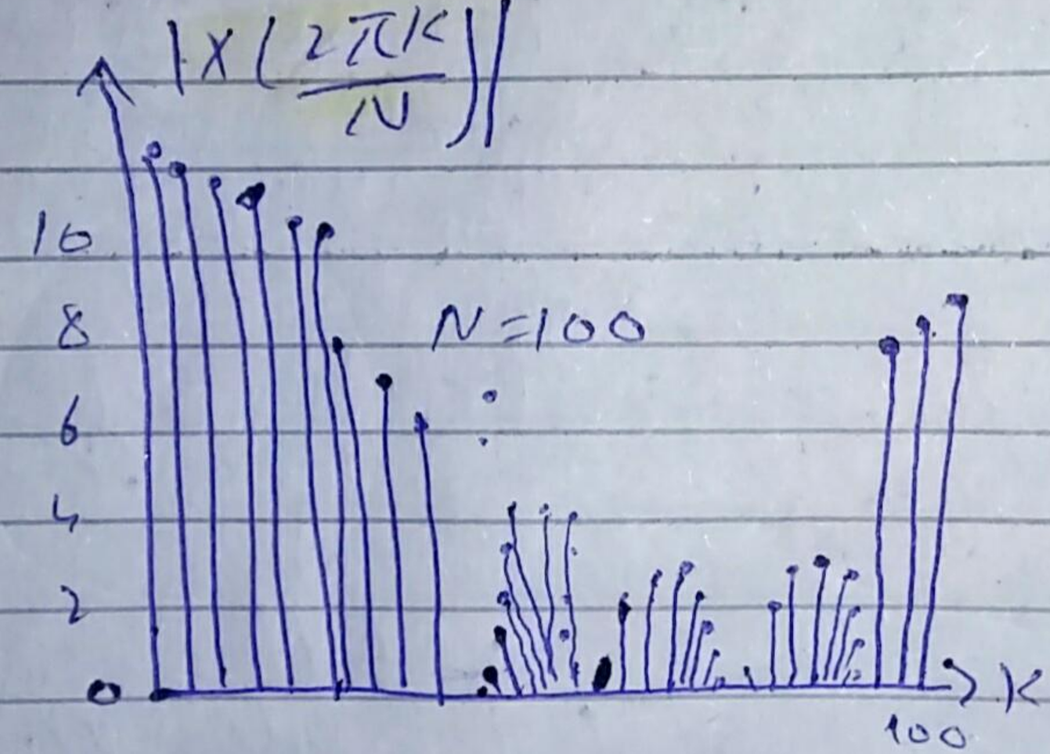

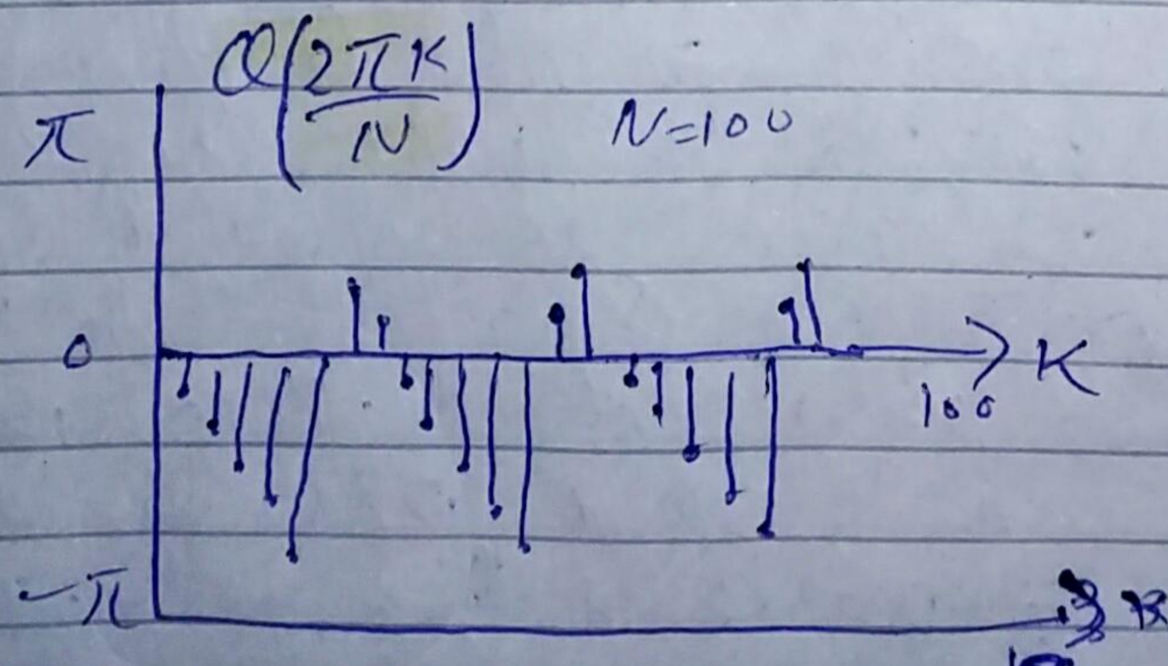

Q:4 (b)

Ans:

Solution: The first step is to determine the matrix W_4. By exploiting the periodicity property of W_4 & the symmetry property

$$W_N^{k+N/2} = -W_N^k$$

The matrix W_4 may be expressed

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

then

$$X_4 = W_4 x_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

The IDFT of X_4 may be determined the conjugate of the elements in W_4 to obtain W_4^* the applying formula.

$$x_N = \frac{1}{N} W_N^* X_N$$