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Assignment # 01

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Application of ODE's and PDE's for Engineering:

⇒ Application of ODE's:-
An equation contains only ordinary derivates of one or more dependent variables of a single independent variable.

For Example:

$$dy/dx + 5y = ex, \; (dx/dt) + (dy/dt) = 2x + y$$

Application of ODE's:-

(1) Newton's law of cooling
(2) Beam
(3) physical Application
(4) Radio Active Elements
(5) Electrical circuits
(6) modelling free mechanical oscilations.
(7) No Damping
(8) Light Damping
(9) Heavy Damping.
(10) Computer excercise OR activity.
(11) modelling with first-order Equations.

⇒ Physical Application of ODE :-

(i) ⇒ its velocity $(v) = \frac{dx}{dy}$

(ii) its acceleration $(a) = \frac{dv}{dt}$ or

d^2x/dt^2 or $v\frac{dv}{dx}$

if however. the body be moving along a curve, then

(i) its velocity $(v) = \frac{ds}{dt}$ or

$v\frac{dv}{ds}$ or d^2s/dt^2

⇒ Newton's law of second :-

The rate of change of ~~movem~~ momentum Encountered by a moving objects is equal to the NET force Applied To it. In mathematics terms,

$$F = \frac{d(mv)}{dt} \rightarrow m\frac{dv}{dt} + v\frac{dm}{dt} \rightarrow F = m\frac{dv}{dt}$$

$$F = ma$$

Newton law of Cooling :-

law :- The rate of change of the temperature of an object is proportional to the difference between its own temperature and the temperature of its surroundings.

Therefore.

$d\phi/dt = EA(\phi - \phi_r)$: E-A Contant as depends upon the object A- surface area

ϕ-A Certain temperature : ϕ_r-Room/ambient temperature or the temperature of the surroundings.

Radioactive Half-life :

=> Radioactive (random) process.

(•) A stochastic (random) process.

(•) The rate of decay is dependent upon the number of molecules/atoms that are there

(•) Negative because the number is decreasing

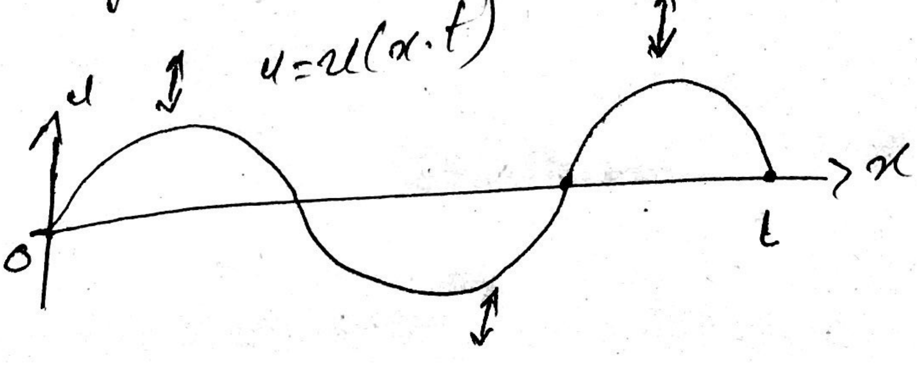
(•) K is the constant of proportionality.

$$dN/dt = KN$$

⇒ Application of PDE :-

Introduction :- In this section we discuss briefly some of the most important PDE's that arise in various branches of science and engineering, we shall see that some Equation can be used to describe a variety of different situations.

(1) wave length :- Equation :- The simplest situation to give rise to the one dimensional wave equation is the motion of a strected string specially the transverse vibration of a string such as a string of a musical instrument. Assume that a string place along the x-axis is stretched and then fixed at end $x=0$ and $x=L$: its The deflected and at some instant which we call $t=0$ is released and allow to vibrate the quantity of interest is the deflection u of the string at any point x. $0 \le x \le L$. and at any time $t>0$, we write $u=u(x,t)$



Thus we must be the given conditions:

$u(x,0) = f(x) \quad 0 \le x \le L$ (Initial equation possition)

$\frac{\partial u}{\partial t}(x,0) = g(x) \quad 0 \le x \le L$ (Initial velocity)

where $f(x)$ and $g(x)$ are known

(2) Heat Conduction equation:

Consider a long thin bar or wire of constant cross-section and of homogeneous material obtained along the x-axis (see figure)

Imagine that the bar is thermally insulated laterally and is sufficiently thin that heat flow (by conduction) only in the x-direction. Then the temperature u at any point bar in depends only on the x-cordinates of the point at the time t = By applying the principle of conservation of energy it can be shown that $u(x,t)$ satisfied the PDE

$$\partial u / \partial t = k \frac{\partial^2 u}{\partial x^2} \quad \begin{matrix} 0 \le x \le l \\ t > 0 \end{matrix}$$

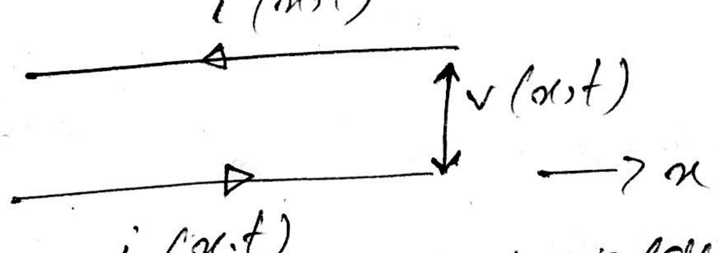
where k is a possitive Constant, in fact k, some time called the Thermal diffusivity of the bar, is given by:

$$k = K/s\rho$$

where K = thermal conductivity of the bar.
s = specific heat capacity of the material of the bar.
ρ = density of the material of the bar.

The Initial condition would have to form $u(x,y,0) = f(x,y)$ where f is a given function.

(3) Transmission <u>line</u> equations:-
In a long electrical cable or a telephone wire both the current and voltage depend upon possition along the wire as well as the time.



it is possible to show using a basic law of Electrical circuits theory. that the electrical current $i(x,t)$ satisfied the PDE

$$\partial^2 i/\partial x^2 = LC\,\partial^2 i/\partial t^2 + (RC + GL)\frac{\partial i}{\partial t} + RGi$$

$$\Rightarrow \frac{\partial^2 i}{\partial x^2} = RC \frac{\partial i}{\partial t}$$

which is called the submarine equation or telegraph equation.

$$\frac{\partial^2 i}{\partial x^2} = LC \frac{\partial^2 i}{\partial t^2}$$

which is called the high frequency line equation

(4) laplace's equation:
if you look back at the two dimensional heat conduction equation.

$$\partial u/\partial t = k\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

it is clear that if the heat flow is steady i.e time independent, then $\partial u/\partial t = 0$
so the temperature $u(x,y)$ is a solution of

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

it is the two dimensional laplace equation
Both at this Three dimensional counterpart.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

(5) Other Important PDE in science and Engineering

(1) Poisson's equation:-

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f(x,y) \quad \text{(two dimensional form)}$$

where $f(x,y)$ is a given function.

(2) Helmholtz's equation:-

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+k^2 u=0 \quad \text{(two dimensional form)}$$

which is arises in wave theory

(3) Schrodinger's equation:-

$$-\frac{h^2}{8\pi^2 m}\left(\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}+\frac{\partial^2 \psi}{\partial z^2}\right)=E\psi$$

which is arises in quantum mechanics
(h is plancle's constants.)

(4) Transverse vibration's equation.

$$a^2\frac{\partial^4 u}{\partial x^4}+\frac{\partial^2 u}{\partial t^2}=0$$

for a homogeneous rod, where $u(x,t)$ as the displacement at time t of the Crosssection through x.