CALCULUS AND ANALYTICAL GEOMETRY

Examination: Final Paper

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①

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a)

Q) Sol

Reqi

GIVEN:-

$$X = ?$$

$$X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix}$$

Subtracting $\begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$ on both sides.

$$X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$$

(or)

$$X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} + \left(- \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} \right) = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} + \left(- \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} \right)$$

$$X + \begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} -3 & +1 \\ -2 & -2 \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -3 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 1 \\ -2 & -2 \end{bmatrix}$$

$$X + \begin{bmatrix} 3+(-3) & -1+1 \\ 2+(-2) & 2+(-2) \end{bmatrix} = \begin{bmatrix} 5+(-3) & 1+1 \\ -3+(-2) & 1+(-2) \end{bmatrix}$$

$$X + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 5-3 & 2 \\ -3-2 & 1-2 \end{bmatrix}$$

$$X + 0 = \begin{bmatrix} 2 & 2 \\ -5 & -1 \end{bmatrix}$$

$$\boxed{X = \begin{bmatrix} 2 & 2 \\ -5 & -1 \end{bmatrix}}$$

③

(b) Sol

Req;   X = ?

GIVEN:-

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -2 & 0 \end{bmatrix}$$

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2+(-4) & 6+(-8) \\ 1+(-2) & 5+0 \end{bmatrix}$$

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2-4 & 6-8 \\ 1-2 & 5 \end{bmatrix}$$

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix}.$$

Subtracting $\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$ on both sides.

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

OR

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} + \left(- \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\right) = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} + \left(- \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\right)$$

$$X + \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -1 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$X + \begin{bmatrix} -1+1 & 0+0 \\ 0+0 & 2+(-2) \end{bmatrix} = \begin{bmatrix} -2+1 & -2+0 \\ -1+0 & 5+(-2) \end{bmatrix}$$

$$X + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$X + 0 = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$\boxed{X = \begin{bmatrix} -1 & -2 \\ -1 & 3 \end{bmatrix}}$$

Rogi:

$X = ?$

GIVEN:-

$$X + 2I = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

Put $I = (\text{Identity matrix})_{2\times2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$X + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

$$X + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix}$$

Subtracting $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ o. b. S.

$$X + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$X + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \left(-\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right) = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} + \left(-\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right)$$

$$X + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

$$X + \begin{bmatrix} 2+(-2) & 0+0 \\ 0+0 & 2+(-2) \end{bmatrix} = \begin{bmatrix} 3+(-2) & -1+0 \\ 1+0 & 2+(-2) \end{bmatrix}$$

$$X + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 3-2 & -1 \\ 1 & 0 \end{bmatrix}$$

$$X + 0 = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

$$\boxed{X = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}}$$

Q5) b) ⑤

Req; $A^2 + BC = ?$

<u>1st $A^2 = ?$</u>

$$A^2 = A \times A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \quad RC$$

$$A^2 = \begin{bmatrix} (1 \times 1) + (4 \times 2) & (1 \times 4) + (4 \times 1) \\ (2 \times 1) + (1 \times 2) & (2 \times 4) + (1 \times 1) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 + 8 & 4 + 4 \\ 2 + 2 & 8 + 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} \longrightarrow ①$$

<u>2nd $BC = ?$</u>

$$BC = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \quad RC$$

$$BC = \begin{bmatrix} (-3 \times 1) + (2 \times 0) & (-3 \times 0) + (2 \times 2) \\ (4 \times 1) + (0 \times 0) & (4 \times 0) + (0 \times 2) \end{bmatrix}$$

$$BC = \begin{bmatrix} -3 + 0 & 0 + 4 \\ 4 + 0 & 0 + 0 \end{bmatrix}$$

$$BC = \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix} \longrightarrow ②$$

Navi   P.f.V.

$$A^2 + BC = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9+(-3) & 8+4 \\ 4+4 & 9+0 \end{bmatrix}$$

$$= \begin{bmatrix} 9-3 & 12 \\ 8 & 9 \end{bmatrix}$$

$$A^2 + BC = \begin{bmatrix} 6 & 12 \\ 8 & 9 \end{bmatrix}$$

⑦

Q2) sol (a)

Req; Integral = I = ?

GIVEN:-

let $I = \int \dfrac{1}{\sqrt{x^5}} \, dx$

(OR)

$I = \int \dfrac{1}{(x^5)^{1/2}} \, dx$

$= \int \dfrac{1}{x^{5/2}} \, dx$

(OR)

$= \int x^{-5/2} \, dx$

$\int x^n \, dx = \dfrac{x^{n+1}}{n+1} + C$

$I = \dfrac{x^{-\frac{5}{2}+1}}{-\frac{5}{2}+1} + C$ $\therefore \ -\frac{5}{2}+\frac{1}{1} = \dfrac{-5+2}{2}$
$= -3/2$

$I = \dfrac{x^{-3/2}}{-3/2} + C$

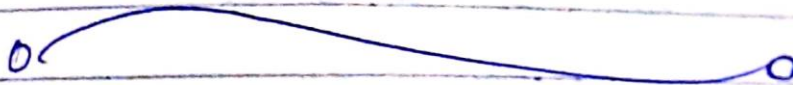(OR) $I = \dfrac{-2}{3} x^{-3/2} + C$

$$I = \frac{-2}{3\,x^{3/2}} + C.$$

$$I = \frac{-2}{3(x^3)^{1/2}} + C$$

$$\boxed{I = \frac{-2}{3\sqrt{x^3}} + C.}$$

Ans

(b)Sol ⑨

Req;   Integral = $I = ?$

GIVEN:-

Let

$$I = \int \frac{1}{(8x+7)^8} \, dx$$

(OR)

$$I = \int (8x+7)^{-8} \, dx \longrightarrow ①$$

Let

$$U = 8x + 7 \longrightarrow ②$$

Differentiating w.r.t x o.b.s.

$$\frac{dU}{dx} = \frac{d}{dx}(8x+7)$$

$$= \frac{d}{dx} 8x + \frac{d}{dx} 7$$

$$Cf \xrightarrow{d/dx} c \, d/dx \, f \cdot \qquad C \xrightarrow{d/dx} 0$$

$$= 8 \frac{d}{dx} x + 0$$

$$\frac{dU}{dx} = 8$$

(OR) $\dfrac{dU}{\div} = 8\ dx.$

$$\boxed{\dfrac{dU}{8} = dx} \quad\longrightarrow\ 0\ \text{③}$$

Put eq ② & ③ in eq ① ⇒

$$I = \int U^{-8} \cdot \dfrac{dU}{8}$$

$$\underset{2.}{\int} cf\ dx = c\int f\ dx$$

$$I = \dfrac{1}{8} \int U^{-8}\ dU$$

$$\int x^n\ dx = \dfrac{x^{n+1}}{n+1} + C$$

$$I = \dfrac{1}{8}\left[\dfrac{U^{-8+1}}{-8+1}\right] + C$$

$$I = \dfrac{1}{8}\left[\dfrac{U^{-7}}{-7}\right] + C$$

$$I = \dfrac{1}{-56}\ U^{-7} + C$$

(OR) ⑪

$$I = \frac{-1}{56} \, U^{-7} + C.$$

(OR) $$I = \frac{-1}{56\, U^7} + C$$

Put $U = 8x+7 \longrightarrow$ eq ② ⁻⁾

$$\boxed{I = \frac{-1}{56(8x+7)^7} + C.}$$

Q.7) a) Sol
obj:-
$$\frac{d}{dx} f(x) = ?$$

GIVEN:- Let
$$f(x) = \frac{3x^4 - 2x^3 + 5}{x^3 + 1}$$

(Differentiating w.r.t x o.b.s)

$$\frac{d}{dx} f(x) = \frac{d}{dx}\left[\frac{3x^4 - 2x^3 + 5}{x^3 + 1}\right]$$

↙.

Quotient Rule.
$$\frac{U}{V} \xrightarrow{\, d/dx \,} \frac{V \cdot \frac{d}{dx} U - U \cdot \frac{d}{dx} V}{V^2}$$

$$f'(x) = \frac{(x^3+1) \frac{d}{dx}(3x^4 - 2x^3 + 5) - (3x^4 - 2x^3 + 5) \cdot \frac{d}{dx}(x^3+1)}{(x^3+1)^2}$$

$$= \frac{(x^3+1)\left[\frac{d}{dx} 3x^4 - \frac{d}{dx} 2x^3 + \frac{d}{dx} 5\right] - (3x^4 - 2x^3 + 5)\left[\frac{d}{dx} x^3 + \frac{d}{dx} 1\right]}{(x^3+1)^2}$$

$$c f \xrightarrow{d/dx} c \frac{d}{dx} f^0$$

(13)

$$= \frac{(x^3+1)\left[3\frac{d}{dx}x^4 - 2\frac{d}{dx}x^2 + 0\right] - (3x^4-2x^2+5)\left[\frac{d}{dx}x^3+0\right]}{(x^3+1)^2}$$

Power rule

$$x^n \xrightarrow{d/dx} nx^{n-1} \cdot \frac{d}{dx}x$$

$$= \frac{(x^3+1)\left[3(4x^{4-1}\cdot\frac{d}{dx}x) - 2(3x^{3-1}\cdot\frac{d}{dx}x)\right] - (3x^4-2x^3+5)\left[3x^{2}\cdot\frac{d}{dx}x\right]}{(x^3+1)^2}$$

$$= \frac{(x^3+1)\left[12x^3 - 6x^2\right] - \left[(3x^4-2x^3+5)(3x^2)\right]}{(x^3+1)^2}$$

$$= \frac{(x^3+1)(12x^3-6x^2) - 3x^2(3x^4-2x^3+5)}{(x^3+1)^2}$$

Simplify

$$= \frac{(12x^6 - 6x^5 + 12x^3 - 6x^2) - (9x^6 - 6x^5 + 15x^2)}{(x^3+1)^2}$$

$$= \frac{12x^6 - 6x^5 + 12x^3 - 6x^2 - 9x^6 + 6x^5 - 15x^2}{(x^3+1)^2}$$

By Re-arranging)

$$= \frac{12x^6 - 9x^6 + 12x^3 - 6x^2 - 15x^2}{(x^3+1)^2}$$

$$= \frac{3x^6 + 12x^3 - 21x^2}{(x^3+1)^2} \quad \sim \text{(Common)}$$

$$\boxed{f'(x) = \frac{3x^2(x^4 + 4x - 7)}{(x^3+1)^2}}$$

(b) Sol

Req; $\dfrac{d}{dx} f(x) = ?$

GIVEN:-

Let
$$f(x) = \dfrac{(x^3+1)^2}{x^3-1}$$

1st Simplifying :-

$(a+b)^2 = a^2+b^2+2ab$

$$f(x) = \dfrac{(x^3)^2 + (1)^2 + 2(x^3)(1)}{x^3-1}$$

$$f(x) = \dfrac{x^6 + 2x^3 + 1}{x^3-1}$$

Differentiating w.r.t x o.b.s

$$\dfrac{d}{dx} f(x) = \dfrac{d}{dx}\left(\dfrac{x^6 + 2x^3 + 1}{x^3-1}\right)$$

Quotient Rule

$$\dfrac{U}{V} \xrightarrow{d/dx} \dfrac{V\cdot\frac{d}{dx}U - U\cdot\frac{d}{dx}V}{V^2}$$

$$f'(x) = \dfrac{(x^3-1)\frac{d}{dx}(x^6+2x^3+1) - (x^6+2x^2+1)\frac{d}{dx}(x^3-1)}{(x^3-1)^2}$$

$$f'(x) = \frac{(x^3-1)\left[\frac{d}{dx}x^6 + \frac{d}{dx}2x^3 + \frac{d}{dx}1\right] - (x^6+2x^3+1)\left[\frac{d}{dx}\cdot x^3 - \frac{d}{dx}1\right]}{(x^3-1)^2}$$

$$Cf \xrightarrow{d/dx} c\frac{d}{dx}f \qquad C \xrightarrow{d/dx} 0$$

$$C \xrightarrow{d/dx} 0$$

$$f'(x) = \frac{(x^3-1)\left[\frac{d}{dx}x^6 + 2\frac{d}{dx}x^3 + 0\right] - (x^6+2x^3+1)\left[\frac{d}{dx}x^3 - 0\right]}{(x^3-1)^2}$$

Power rule.

$$x^n \xrightarrow{d/dx} n\, x^{n-1} \cdot d/dx\, x.$$

$$f'(x) = \frac{(x^3-1)\left[6x^{6-1}\cdot\frac{d}{dx}x + 2\left(3x^{3-1}\cdot\frac{d}{dx}x\right)\right] - (x^6+2x^3+1)\left[3x^{3-1}\cdot\frac{d}{dx}x\right]}{(x^3-1)^2}$$

$$= \frac{(x^3-1)\left[6x^5 + 6x^2\right] - (x^6+2x^3+1)(3x^2)}{(x^3-1)^2}$$

Simplify

$$= \frac{(6x^8 + 6x^5 - 6x^5 - 6x^2) - (3x^8 + 6x^5 + 3x^2)}{(x^3-1)^2}$$

$$= \frac{6x^8 - 6x^2 - 3x^8 - 6x^5 - 3x^2}{(x^3-1)^2}$$

(17)

(By Re-arranging)

$$= \frac{6x^8 - 3x^8 - 6x^5 - 6x^2 - 3x^2}{(x^3-1)^2}$$

$$= \frac{5x^8 - 6x^5 - 9x^2}{(x^3-1)^2} \quad\text{----(Common)}$$

(OR)

$$\boxed{f'(x) = \frac{x^2 \left[5x^6 - 6x^3 - 9\right]}{(x^3-1)^2}}$$

Q3) a) If

$$I = ?$$

Proj.
method :- Partial fraction.

GIVEN :- Let :-

$$I = \int \frac{-x+9}{2x^2-8x+6} \, dx \longrightarrow ①$$

By taking :-

$$= \frac{-x+9}{2x^2-8x+6} \longrightarrow ②$$

As: Degree of Ⓝ < Degree of Ⓓ

$$1 \quad < \quad 2.$$

It is Proper rational fraction.

Now make factors of :-

$$= 2x^2 - 8x + 6$$

By Sum & Product

$$= 2x^2 - \boxed{6x} - \boxed{2x} + 6$$

(Common)

$$= 2x(x-3) - 2(x-3)$$

(Common)

$$= (x-3)(2x-2) \quad (\text{common})$$

$$= \boxed{2(x-3)(x-1).}$$

Now; ② ⇒ becomes.

$$\frac{-x+9}{2x^2-8x+6} = \frac{-x+9}{2(x-3)(x-1)} = \frac{1}{2}\left[\frac{-x+9}{(x-3)(x-1)}\right].$$

↳③

Let;

$$\frac{-x+9}{(x-3)(x-1)} = \frac{A}{x-3} + \frac{B}{x-1} \quad ④$$

Xing $(x-3)(x-1)$ o.b.s. ⇒

$$-x+9 = A(x-1) + B(x-3) \quad ⑤$$

Put $\underline{x-3=0}$ in of ⑤ ⇒

$\boxed{x=3}$

$$-3+9 = A(3-1) + B(3-3)$$

$$6 = A(2) + 0$$

$$6 = 2A$$

$$\Rightarrow \quad 2A = 6$$

$$A = \cancel{6}/\cancel{2}^{3}$$

$$\boxed{A = 3}$$

Now put $x - 1 = 0 \Rightarrow x = 1$ in eq $\textcircled{5} \Rightarrow$

$$-1 + 9 = A(\overset{0}{\cancel{1-1}}) + B(1-3)$$

$$8 = 0 + (-2B)$$

$$\underset{\div}{8} = -2B$$

$$\Rightarrow \quad B = \cancel{8}^{4}/-\cancel{2}$$

$$\boxed{B = -4.}$$

Put-v of $\textcircled{A}$ & $\textcircled{B}$ in eq $\textcircled{4} \Rightarrow$

$$\frac{-x+9}{(x-3)(x-1)} = \frac{3}{x-3} + \frac{(-4)}{x-1}$$

$$= \boxed{\frac{3}{x-3} - \frac{4}{x-1}} \Rightarrow \textcircled{6}$$

(21)

Put eq.6 in eq.③ = 0

$$= \boxed{\frac{1}{2}\left[\frac{3}{x-3} - \frac{4}{x-1}\right]} \cdot ⑦$$

Put eq.⑦ in eq.① = 0

$$I = \int \frac{1}{2}\left[\frac{3}{x-3} - \frac{4}{x-1}\right] dx.$$

$$\int cf \frac{d(dx)}{dx} \, c\int f \, dx$$

$$I = \frac{1}{2} \int \left(\frac{3}{x-3} - \frac{4}{x-1}\right) dx$$

(OR)

$$I = \frac{1}{2}\left[\int \frac{3}{x-3} dx - \int \frac{4}{x-1} dx\right].$$

$$\int cf \, dx = c\int f \, dx$$

$$= \frac{1}{2}\left[3\int \frac{1}{x-3} dx - 4\int \frac{1}{x-1} dx\right]$$

$$\int \frac{1}{x} = \ln x + c$$

$$= \frac{1}{2}\left[\frac{3 \ln|x-3|}{2} - 4\ln|x-1| \right] + C.$$

$$n\ln m = \ln m^n$$

$$= \frac{1}{2}\left[\frac{\ln|x-3|^3}{2} - \ln|x-1|^4 \right] + C. \qquad \#$$

$$\ln m - \ln n = \ln \frac{m}{n}.$$

$$= \frac{1}{2}\left[\frac{\ln|x-3|^3}{|x-1|^4} \right] + C. \qquad \#$$

(BR) $$n\ln m = \ln m^n$$

$$= \left[\ln\left| \frac{(x-3)^3}{(x-1)^4} \right| \right]^{1/2} + C.$$

(b) Sol

Req,i $\qquad$ $I = ?$

method:- $\qquad$ Partial fraction

Given:- let

$$I = \int \frac{4x^2 + 8x}{(x^2+1)(x^2+2x+3)} \, dx$$

$$\quad \longrightarrow ①$$

By taking:-

$$= \frac{4x^2 + 8x}{(x^2+1)(x^2+2x+3)} \quad — ②$$

As

Degree of ⓝ $<$ Degree of ⓞ

$$2 \quad < \quad 4.$$

It is Proper rational fraction.

of ② ⇒

$$\frac{4x^2 + 8x}{(x^2+1)(x^2+2x+3)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+2x+3} \quad — ③$$

X'ng $(x^2+1)(x^2+2x+3)$ o.b.s.
we get:-

$$4x^2 + 8x = (Ax+B)(x^2+2x+3) + (Cx+D)(x^2+1)$$

$$= Ax^3 + 2Ax^2 + 3Ax + Bx^2 + 2Bx + 3B + Cx^3 + Cx + Dx^2 + D$$

(By Re-arranging)

$$4x^2+8x = Ax^3 + Cx^3 + 2Ax^2 + Bx^2 + Dx^2 + 3Ax + 2Bx + Cx + 3B + D$$

(common)

$$4x^2+8x = (A+C)x^3 + (2A+B+D)x^2 + (3A+2B+C)x + (3B+D)$$

By Compairing co-efficients

for x^3:-

$$x^3 = x^3$$

$$\boxed{A+C = 0} \longrightarrow (i)$$

for x^2:-

$$x^2 = x^2$$

$$\boxed{2A+B+D = 4} \longrightarrow (ii)$$

for x:-

$$\boxed{3A+2B+C = 8} \longrightarrow (iii)$$

for Const:-

$$3B+D = 0 \longrightarrow (iv)$$

Subtracting (ii) & (iv)

$$2A + B + \cancel{C} = 4.$$

$\ominus$

$$\underline{\quad 3B + \cancel{C} = 0.}$$

$$2A - 2B = 4.$$

$$2(A - B) = 4.$$

$$A - B = 4/2$$

$$\boxed{A - B = 2.} \quad \longrightarrow \circ \text{(V)}$$

Now put $A + C = 0 \Rightarrow C = -A$ in eq (iii).

$$3A + 2B + (-A) = 8.$$

$$3A - A + 2B = 8$$

$$2A + 2B = 8$$

$$2(A + B) = 8.$$

$$A + B = 8/2$$

$$\boxed{A + B = 4} \quad \longrightarrow \circ \text{(Vi)}$$

Adding eq (V) & eq (Vi) we get :-

$$\begin{aligned} A - B &= 2. \\ \oplus \quad A + B &= 4. \\ \hline 2A &= 6 \end{aligned}$$

(26)

$$\frac{2A}{2} = \frac{6}{2}$$

$$A = 6/2$$

$$\boxed{A = 3}$$

of (V) ⇒ $A - B = 2.$

$$⇒ B = A - 2 \quad \text{put } A = 3$$

$$B = 3 - 2$$

$$\boxed{B = 1}$$

put $A = 3$ & $B = 1$ in of (iii) ⇒

$$3(3) + 2(1) + C = 8.$$

$$9 + 2 + C = 8$$

$$11 + C = 8$$

$$C = 8 - 11$$

$$\boxed{C = -3}$$

Put $A = 3$ & $B = 1$ in of (ii) ⇒

$$2(3) + 1 + D = 4.$$

$$6 + 1 + D = 4.$$

$$7 + D = 4$$

$$D = 4 - 7.$$

$$\boxed{D = -3.}$$

Put value of (A), (B), (C) & (D) in eq. ③ ⇒

$$\frac{4x^2 + 8x}{(x^2+1)(x^2+2x+3)} = \frac{3x+1}{x^2+1} + \frac{(-3)x + (-3)}{x^2+2x+3.}$$

$$\boxed{= \frac{3x+1}{x^2+1} - \frac{3x+3}{x^2+2x+3}}$$

Put in eq ① ⇒

$$I = \int \left(\frac{3x+1}{x^2+1} - \frac{3x+3}{x^2+2x+3} \right) dx$$

$$I = \int \left[\frac{3x}{x^2+1} + \frac{1}{x^2+1} - \frac{3x+3}{x^2+2x+3} - \frac{1}{x^2+2x+(1)+2} \right]$$

$$I = \int \frac{3x}{x^2+1} dx + \int \frac{1}{x^2+1} dx - \int \frac{3x+3}{x^2+2x+3} dx - \int \frac{1}{x^2+2x(1)+2} dx$$

Mug & ÷ng by 2

$$I = \frac{3}{2} \int \frac{2x}{x^2+1} dx + \int \frac{1}{x^2+1} dx - \frac{3}{2} \int \frac{2x+2}{x^2+2x+3} dx - \int \frac{1}{x^2+2x+1+2} dx$$

$$I = \frac{3}{2} \ln \left| x^2 + 1 \right| + \ln \left| x^2 + 1 \right| + \frac{3}{2} \ln \left| x^2 + 2x + 3 \right| - \int \frac{1}{(x+1)^2 + (\sqrt{2})^2} dx$$

$$n \ln m = \ln m^n$$

$$I = \ln \left| (x^2 + 1)^{3/2} \right| + \ln \left| x^2 + 1 \right| + \ln \left| (x^2 + 2x + 3)^{3/2} \right| - \frac{1}{\sqrt{2}} \tan^{-1} \frac{x+1}{\sqrt{2}} + C$$

$$\ln m + \ln n + \ln o = \ln (m \times n \times o)$$

$$I = \ln \left| \left[(x^2 + 1)(x^2 + 2x + 3) \right]^{3/2} (x^2 + 1) \right| - \frac{1}{\sqrt{2}} \tan^{-1} \frac{x+1}{\sqrt{2}} + C$$

: END :-