IQRA NATIONAL UNIVERSITY

Structural Analysis 2

Final Term Examination
(Summer 2020)

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Section = (A)

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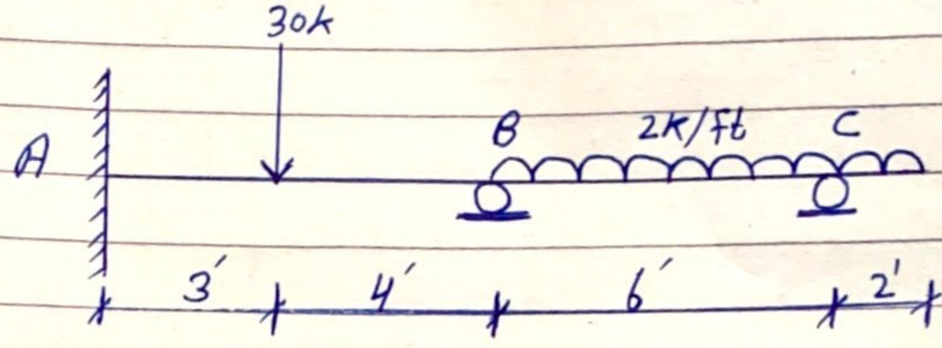
Dated = 25/09/2020

Date: 25/09/20

Q 1:- Analyze the beam - - - -
- - - - - . - . - EI is constant.



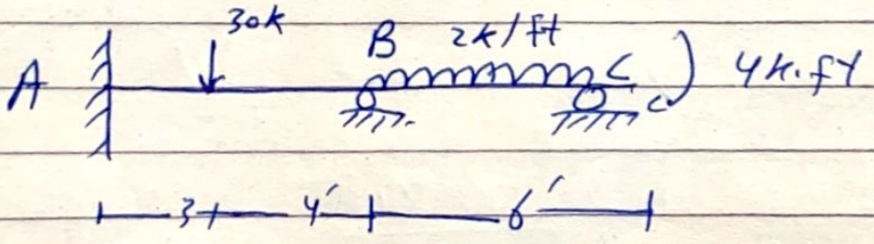
Sol:-

Step# 1

Determine kinetic Indeterminancy

$K.I = 5^\circ$

So, we have to reduce the extended portion



$$\Rightarrow \frac{2(2)}{1} = 4\ k.ft$$

Now

$K.I = 2^\circ$

Step#2.

Determine unknown joint displacement.

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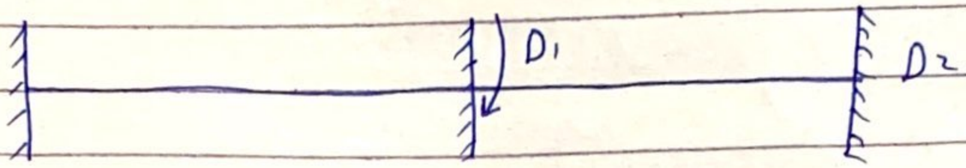
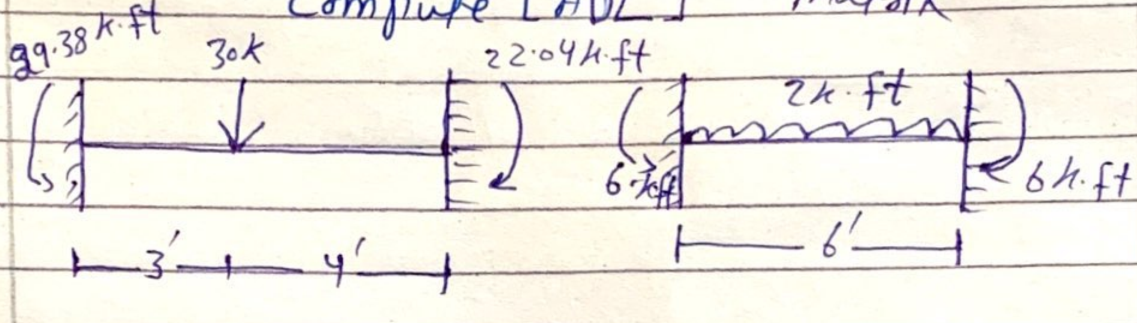


$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \qquad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step # 3 :-

Compute [ADL] Matrix



=> For pointed load (not at mid)

=> For left end:-

$$= \frac{Pab^2}{L^2} = \frac{(30)(3)(4)^2}{(7)^2} = 29.38\ k.ft$$

=> For Right end:-

$$= \frac{Pa^2b}{L^2} = \frac{(30)(3)^2(4)}{(7)^2} = 22.04\ k.ft$$

=> For UDL

$$\frac{wl^2}{12} \Rightarrow \frac{(2)(6)^2}{12} = 6\ k.ft$$

$$ADL_1 = +22.04 - 6 = 16.04\ k.ft$$

$$ADL_2 = 6\ k.ft$$

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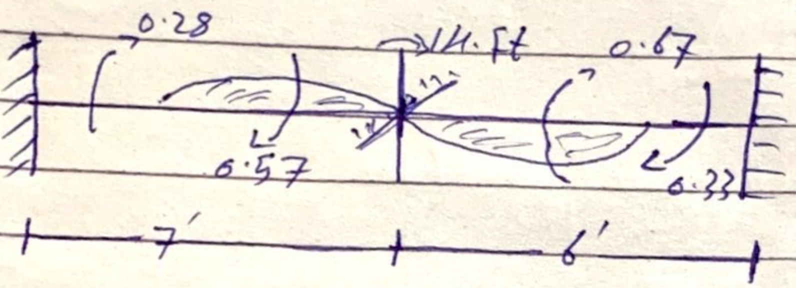
Step# 4 :-

Compute [S] Matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

(a) $D_1 = 1k$, $D_2 = 0$



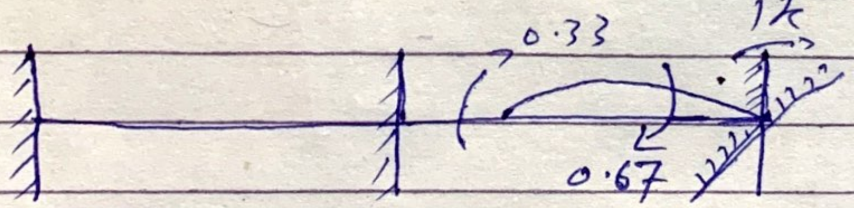
$\frac{4EI}{7} = 0.57$ | $\frac{2EI}{6} = 0.33$

$\frac{4EI}{6} = 0.67$ | $\frac{2EI}{7} = 0.28$

$S_{11} = 0.57 + 0.67$
$= 1.24\ EA$

$S_{21} = 0.33\ EA$

(b) $D_1 = 0$, $D_2 = 1k$



$\frac{4EI}{6} = 0.67$

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$\frac{2EI}{6} = 0.33$

$S_{12} = 0.33$

$S_{22} = 0.67$

$$S = \begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$

Step # 5:-

Compute [D] matrix

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \times \left[\begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix} \right]$$

$$= \frac{1}{\begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}} \times Adj\ A \times \begin{pmatrix} ADL1 \\ AD22 \end{pmatrix}$$

$$|S| = (1.24 \times 0.67) - (0.33 \times 0.33)$$
$$= 0.8308 - 0.1089$$
$$|S| = 0.7219$$

$$Adj\ A = \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}$$

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Now:

$$\begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix} = \begin{bmatrix} 0 - 16.04 \\ 4 - 6 \end{bmatrix} = \begin{bmatrix} -16.04 \\ -2 \end{bmatrix} E$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{\begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}}{0.7219} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} -13.97 \\ 3.8902 \end{bmatrix}$$

$$\begin{bmatrix} D_1' \\ D_2 \end{bmatrix} = 13.971$$

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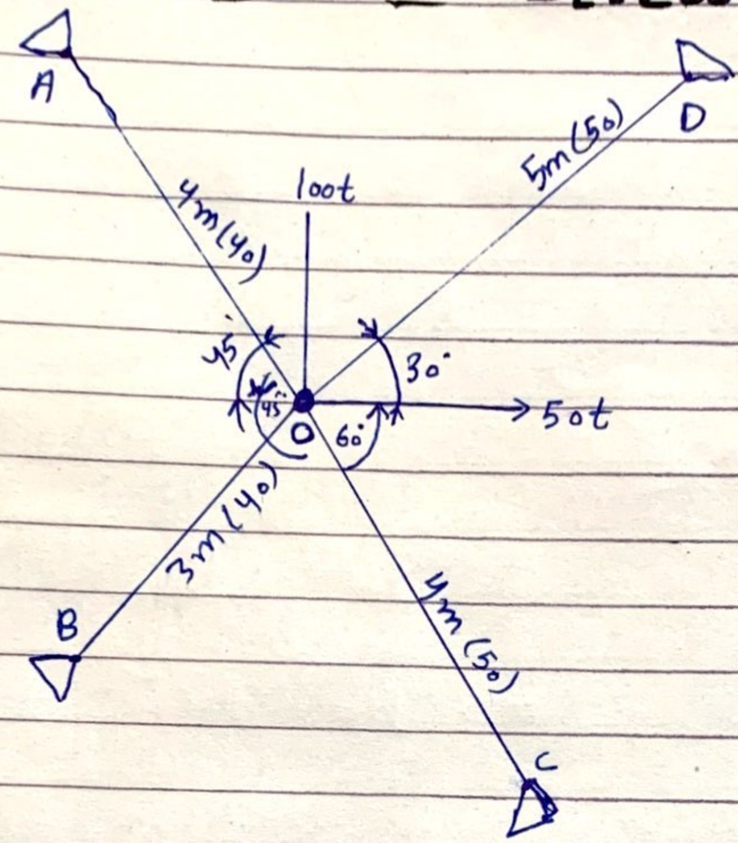
Q2:- Analyze the Pin-jointed frame shown — — — — — — — — — — — — — — — E = 2000 t/cm²



Sol:-

For A:

$$\sin 45^\circ = \frac{P}{h} = \frac{P}{4}$$

$$\Rightarrow P = 2.828 \, m$$

$$\cos 45^\circ = \frac{b}{4}$$

$$\Rightarrow b = 2.828 \, m$$

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For B:

$$\sin 45^\circ = \frac{p}{3}$$

$$\Rightarrow p = 2.12m$$

$$\cos 45^\circ = \frac{b}{h}$$

$$\Rightarrow b = 2.12m$$

For C:

$$\sin 30^\circ = \frac{P}{h=5}$$

$$\Rightarrow P \Rightarrow 2.5m$$

$$\cos 30^\circ = \frac{b}{5}$$

$$\Rightarrow b = 4.33m$$

Now
$EA(A) = 2000 \times 40 = 80,000t$
$EA(B) = 2000 \times 40 = 80,000t$
$EA(C) = 2000 \times 50 - 100,000t$
$EA(D) = 2000 \times 50 - 100,000t$

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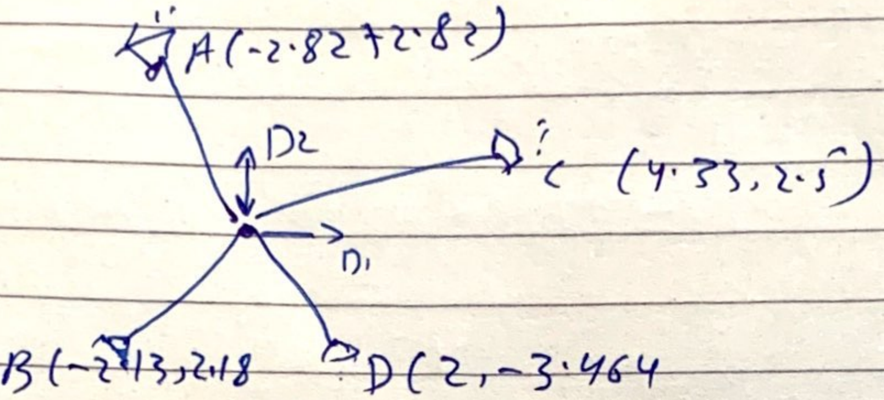
Step#1: K.I

$K.I = 2j - r$

$= 2(5) - 8 = 2^\circ$

Step#2: Select unknown joint displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad \begin{bmatrix} AD_1 \\ AD_1 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

Step #3: $[AMD]_{4\times2}$ $\quad \mathcal{E}[S]_{2\times2}$

i) $D_1 = 1$, $D_2 = 0$

$$AMD = \frac{EA}{L^2}(X_k - X_j)$$

$$AMD_{11} = \frac{80{,}000}{(400)^2} \times (0 + 282) = 141$$

$$AMD_{21} = \frac{80{,}000}{(300)^2} \times (0 + 212) = 188.44$$

$$AMD_{31} = \frac{100{,}000}{(500)^2} \times (0 - 433) = -173.2$$

$$AMD_{41} = \frac{100{,}000}{(400)^2} \times (0 - 200) = -125$$

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Now $S_{11} = \sum_{1 \to 1}^{m} \frac{EA}{L^3} (Xk - Xj)^2$

$= \frac{80,000}{(400)^3} \times (282)^2 + \frac{80,000}{(300)^3} \times (212)^3 +$

$\frac{100,000}{(500)^3} \times (-433)^2 + \frac{100,000}{(400)^3} \times (-200)^2$

$S_{11} = 99.405 + 133.107 + 149.991 + 82.5$

$\boxed{S_{11} = 445.063}$

$S_{12} = S_{21} = \sum_{i,i}^{m} \frac{EA}{L^3} \times (Xk - Xj)(Yk - Yi)$

$= \frac{80,000}{(400)^3} \times (282)(-282) + \frac{80,000}{(300)^3} \times (212)(212)$

$+ \frac{100,000}{(500)^3} \times (-433)(0-250) + \frac{100000}{(400)^3} \times (-200)(0+346)$

$\boxed{S_{12} = S_{21} = 12.237}$

(ii) $D_1 = 0$, $D_1 = 1k'$

$AMD = \frac{EA}{L^2} (Yk - Yj)$

$AMD_{12} = \frac{80,000}{400^2} (-282) = -141$

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$$AMD_{22} = \frac{80,000}{300^2}(212) = 188.44$$

$$AMD_{32} = \frac{100,000}{500^2}(-250) = -100$$

$$AMD_{42} = \frac{100,000}{400^2}(346) = 216.25$$

Now, $S_{22} = \sum_{281}^{m} \frac{EA}{L^3}(Y_k - Y_j)^2$

$$= \frac{800,00}{400^3}(-282)^2 + \frac{80,000}{300^3}(212)^2 +$$

$$\frac{100,000}{500^3}(-250)^2 + \frac{100,000}{400^3}(346)^2$$

$$\boxed{S_{22} = 469.628}$$

Step# 4:

$$[D] = [S]^{-1} \times [AD]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.003 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

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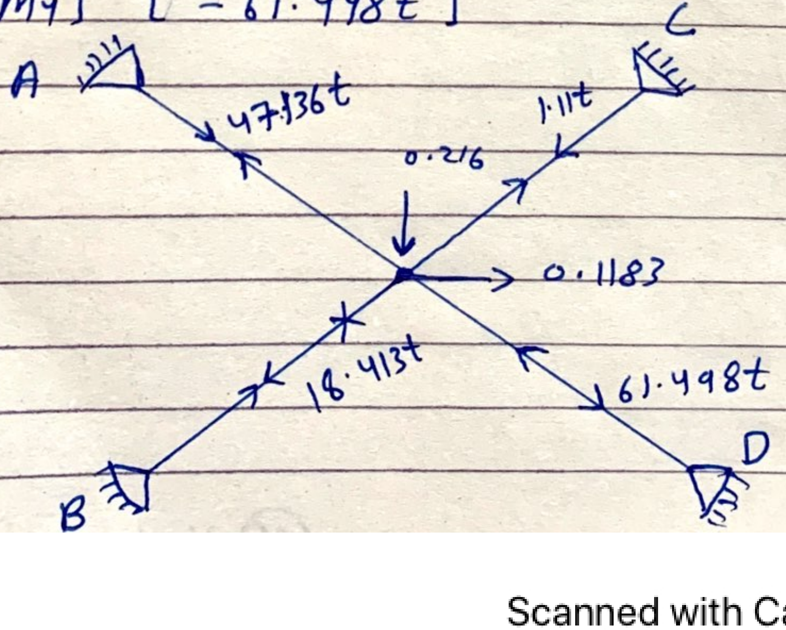
Step# 5:- [AM]

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

$$\begin{bmatrix} 141 \times 0.1183 + (-141) \times (-0.216) \\ 188.44 \times 0.1183 + 188.44 \times (-0.216) \\ -173.2 \times 0.1183 + (-100) \times (-0.216) \\ -125 \times 0.1183 + 216.25 \times (0.216) \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 16.68 + 30.46 \\ 22.29 - 40.70 \\ -20.49 + 21.6 \\ -14.79 - 46.71 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 47.136t \\ -18.413t \\ 1.11t \\ -61.498t \end{bmatrix}$$



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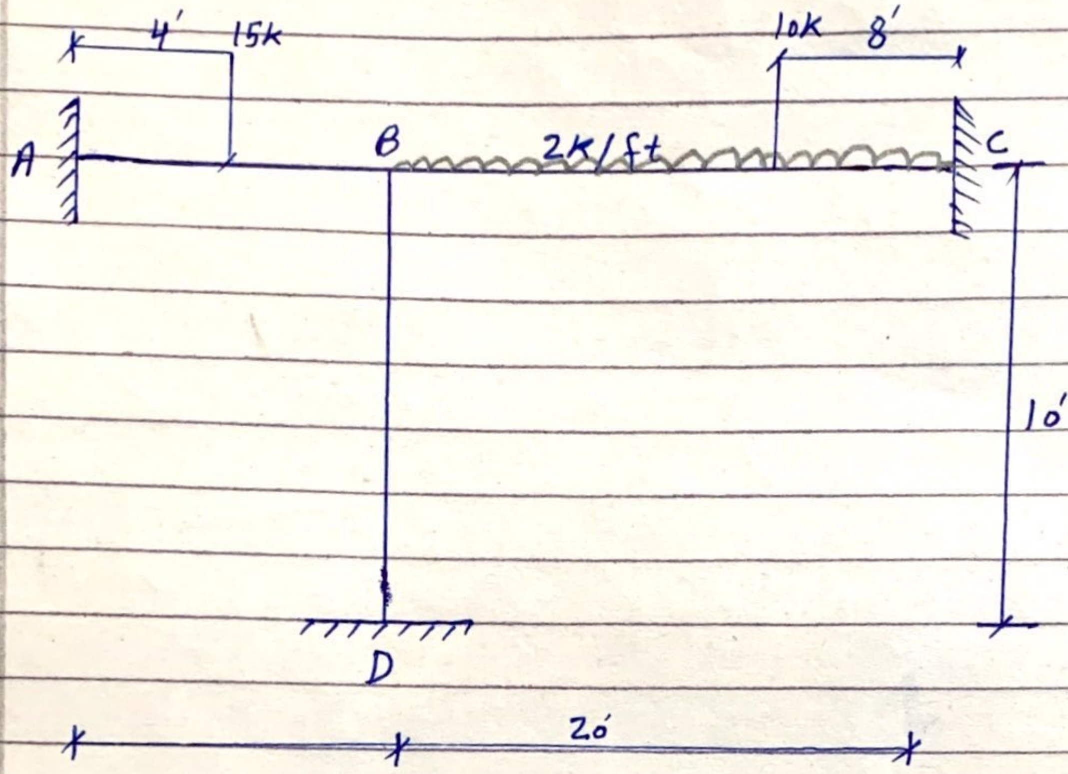
Q 3:- Analyze the rigid - joint - - -
- - - - - - - - constant.



Sol:-

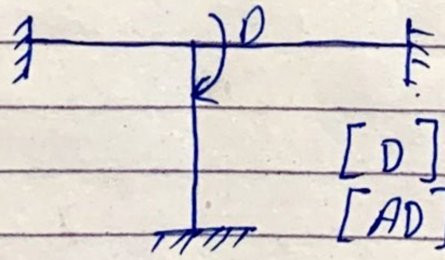
Step # 1

Determine Kinetic Indeterminancy

K.I = 1°

Step # 2

Determine Unknown joint displacement.



[D] = [?]

[AD] = [0]

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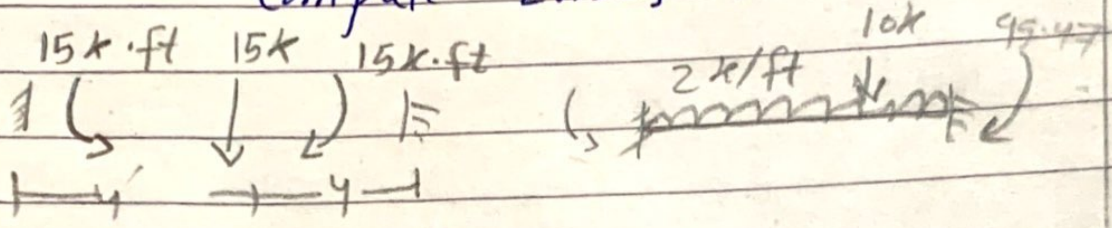
Step#3:-

Compute [ADL] Matrix.



=> Point Load at center:-

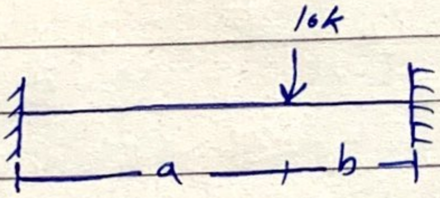
$$\frac{PL}{8} \Rightarrow \frac{(15)(8)}{8} = 15 \text{ kip.ft}$$

=> Uniformly distributed load

$$\frac{wL^2}{12} \Rightarrow \frac{(2)(20)^2}{12} = 66.67 \text{ k.ft}$$

=> Point load (Not at mid) :-

Suppose



For left End:-

$$\frac{Pab^2}{L^2} \Rightarrow \frac{(10)(12)(8)^2}{(20)^2} = 19.2 \text{ k.ft}$$

For Right End:-

$$\frac{Pa^2b}{L^2} = \frac{(10)(12)^2(8)}{(20)^2} = 28.8 \text{ k.ft}$$

So Total moment at left end.

$19.2 + 66.67 = 85.87$ k.ft

Similarly at right end:-

$28.8 + 66.67 = 95.47$ k.ft

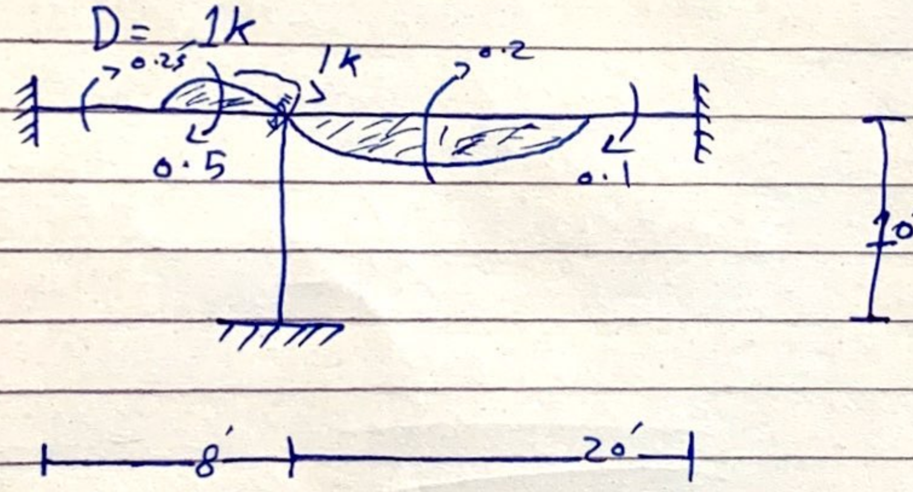
So [ADj] = $-85.87 + 15 = -70.87$ k.ft

Step# 4:-

Determine [S] Matrix.

$[S] = [S_{11}]$

Now



$\Rightarrow \frac{4EI}{8} = 0.5$ $\qquad \frac{2EI}{8} = 0.25$

$\Rightarrow \frac{4EI}{20} = 0.2$ $\qquad \frac{2EI}{20} = 0.1$

$\Rightarrow \frac{4EI}{10} = 0.4$ $\qquad \frac{2EI}{10} = 0.2$

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$[S] = (0.5 + 0.4 + 0.2) EI$

$= 1.1 \quad EI$

$[S] = 1.1 \, EI$

Step # 5:

Compute [D] matrix

$[D] = [S]^{-1} \times [AD] - [ADL]$

$[D] = \frac{1}{1.1} \times [0] - [-70.87]$

$\frac{70.87}{1.1}$

$[D] = [64.42] \, ^1/_{EI}$