Course : Signal and System

Module : 4th

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Q1 Part (a):-

show with a help of an equation that the differentiation of a function is time domain results in the multiplication by jw in frequency domain.

Ans:-

Fourier Transform of differention Integration of Continuous-time. let $n(t)$ be a Continuos-time Signal with a fourier Transform of $n(jw)$

i.e

$$n(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(jw)\, e^{jwt}\, dw$$

Differentiating both Side with respect to (t)

$$\frac{dn}{dt}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(jw)\, \frac{d}{dt}\{e^{jwt}\}\, dw$$

$$\frac{dx}{dt}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(j\omega)\left\{e^{j\omega t}\cdot j\omega\right\} d\omega$$

$$\frac{dx}{dt}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left\{j\omega\, x(j\omega)\right\} e^{j\omega t}\, d\omega$$

$$\Rightarrow F\left\{\frac{d}{dt}x(t)\right. = j\omega\, x(j\omega)$$

Result:

we concluded that if a function is differentiated in time domain. it is multiplied by $j\omega$ in frequency domain.

Q1
Part B:-
if $x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$
$h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$

Produce Yz and $y[n]$

Solution
$$Y[z] = H(z)\,x(z)$$

Find $y[n]$

$$x[z] = 2 - 4z^{-2} + 2z^{-3}$$

$$H(z) = 3 + 1z^{-1} + 2z^{-2}$$

Now
$$\cancel{Y}\; Y(z) = H(z) * x(z)$$

$$= (2-4z^{-2}+2z^{-3})(3+z^{-1}+2z^{-2})$$

$$= 6 + 2z^{-1} + 4z^{-2} - 12z^{-2} - 4z^{-3} - 8z^{-4} + 6z^{-3} + 2z^{-4} + 4z^{-5}$$

$$= 6 + 2z^{-1} - 8z^{-2} - 2z^{-3} + 6z^{-4} + 4z^{-5}$$

To find $y[n]$ use the delay property

$$y[n] = 6\delta[n] + 2\delta[n-1] - 8\delta[n-2] - 2\delta[n-3]$$
$$+ 6\delta[n-4] + 4\delta[n-5].$$

Q5 :-

Apply Fourier Transform on the Signal, $x(t) = e^{-a|t|} u(t)$ where $u(t)$ is a unit step function. $a > 0$

$x(t) = e^{-a|t|}$ $\quad x(j\omega) = ?$

Solution

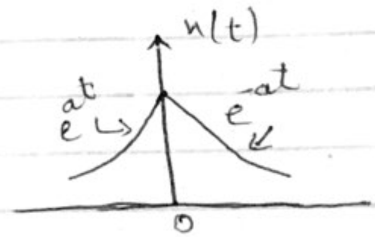
Fourier Transform of the given function $x(t)$ is given by

$$x(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|}\, e^{-j\omega t}\, dt$$

Note

$$e^{-a|t|} = \begin{cases} e^{-at} & \text{for } t \geq 0 \\ e^{-a(-t)} = e^{at} & \text{for } t < 0 \end{cases}$$



$\therefore$

$$x(j\omega) = \int_{-\infty}^{0} e^{at}\, e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$$\Rightarrow \int_{-\infty}^{0} e^{(a-jw)t}\,dt + \int_{0}^{\infty} e^{-(a+jw)t}\,dt$$

$$\Rightarrow \left.\frac{e^{(a-jw)t}}{a-jw}\right|_{-\infty}^{0} + \left.\frac{e^{-(a+jw)}}{-(a+jw)}\right|_{0}^{\infty}$$

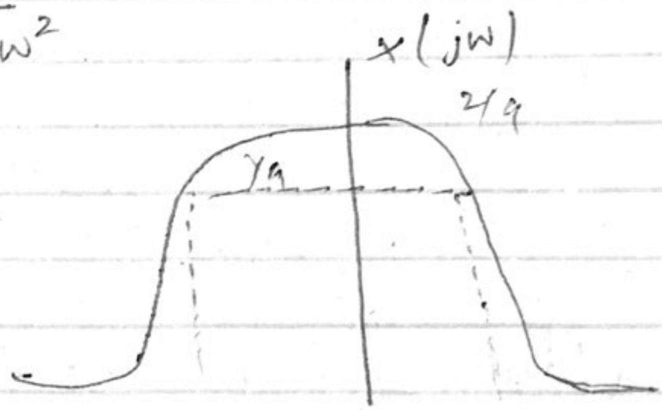
$$\Rightarrow \frac{1}{(a-jw)}\left[e^{0}-e^{-\infty}\right] - \frac{1}{(a+jw)}\left[e^{-\infty}-e^{0}\right]$$

$$\Rightarrow \frac{1}{(a-jw)}\left[1-0\right] - \frac{1}{(a+jw)}\left[0-1\right]$$

$$\Rightarrow \frac{1}{a-jw} + \frac{1}{a+jw}$$

$$\Rightarrow \frac{a+jw+a-jw}{a^2-(jw)^2}$$

$$\Rightarrow \frac{2a}{a^2+w^2}$$



Q2:-

$$f(x) = \begin{cases} -\pi/2 & -\pi \le x \le 0 \\ \pi/2 & 0 \le x \le \pi \end{cases}$$

Retrieve the fourier Series for the given function.

Ans

$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx\right]$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0} -\frac{\pi}{2}\,dx + \int_{0}^{\pi} \pi/2\,dx$$

$$= \frac{1}{2\pi}\left[-\frac{\pi}{2}\int_{-\pi}^{0} 1\,dx + \pi/2\int_{0}^{\pi} 1\,dx\right.$$

$$= \frac{1}{2\pi}\left[-\frac{\pi}{2}\, x\Big|_{-\pi}^{0} + \pi/2\, x\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[\pi/2\left[0-(-\pi)\right] + \pi/2\left[\pi-0\right]\right]$$

$$= \frac{1}{2\pi}\left(\pi/2(\pi) + \pi/2\left[\pi\right]\right.$$

$$= \frac{1}{2\pi}\left[\frac{\pi^2}{2} + \frac{\pi^2}{2}\right.$$

$= \frac{1}{2\pi}\left[\frac{0}{2}\right] =$ $(a_0 \Rightarrow 0)$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx\,dx$

$= \frac{1}{\pi}\int_{-\pi}^{0} f(x) \cos nx\,dx + \int_{0}^{\pi} f(x) \cos nx\,dx$

$= \frac{1}{\pi}\int_{-\pi}^{0} -\frac{\pi}{2} \cos nx\,dx + \int_{0}^{\pi} \frac{\pi}{2} \cos nx\,dx$

$= \frac{1}{\pi}\ \frac{\pi}{2}\ \frac{\sin nx}{n}\Big|_{-\pi}^{0} + \frac{\pi}{2}\frac{\sin nx}{n}\Big|_{0}^{\pi}$

$= \frac{1}{n\pi}\left[-\frac{\pi}{2} \sin n(0) - \sin n(-\pi)\right]$

$+ \frac{\pi}{2}\left[\sin n(\pi) - \sin n(0)\right]$

$= \frac{1}{n\pi}\left[-\frac{\pi}{2}(0) + \frac{\pi}{2}(0)\right]$

$= \frac{1}{n\pi}(0)$

$a_n = 0$

Now:

$= \frac{1}{\pi}\int_{-\pi}^{\pi} \sin nx\,dx$

$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x) \sin nx\,dx + \int_{0}^{\pi} f(x) \sin nx\,dx\right.$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} \frac{-\pi}{2} \sin nx\,dx + \int_{0}^{\pi} \frac{\pi}{2} \sin nx\,dx\right.$$

$$= \frac{1}{\pi}\left[\frac{-\pi}{2}\int_{-\pi}^{0} \sin nx\,dx + \frac{\pi}{2}\int_{0}^{\pi} \sin nx\,dx\right.$$

$$= \frac{1}{\pi}\left[\frac{-\pi}{2} \cdot \frac{-\cos nx}{n}\Big|_{-\pi}^{0} + \frac{\pi}{2} \cdot \frac{-\cos nx}{n}\Big|_{0}^{\pi}\right.$$

$$= \frac{1}{n\pi}\left[\frac{-\pi}{2}\left[-\cos n(0) + \cos n(-x) + \frac{\pi}{2}\left[-\cos(\pi) + \cos n(0)\right]\right.\right.$$

$$= \frac{1}{n\pi}\left[\frac{-\pi}{2}\left[-1 + \cos n \cdot (-\pi) + \frac{\pi}{2}\left[-\cos n\pi + \cos n(0)\right]\right.\right.$$

$$\frac{\pi}{2} = \frac{1}{n\pi}\left[-1\left(-1 + \cos n(-\pi)\right) + 1\left[-\cos nx + 1\right]\right.$$

$$= \frac{1}{2n}\left[1 - \cos n\pi - \cos n\pi + 1\right]$$

$$= \frac{1}{2n}\left[2 - 2\cos n\pi\right.$$

Now

$$b_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{4}{2n} & \text{if } n \text{ is odd.} \end{cases}$$

$$\left\{ bn = \frac{4}{2n} \right\}$$

$$f(x) = a_0 + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \cdots$$

$$+ b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \cdots$$

$$f(x) = (0) + (0)\cos x + 0\cos(2x) + 0\cos 3x + \cdots$$

$$= \frac{4}{2}\sin x + (0)\sin^2 x + \frac{4}{3(2)} \sin 3x + \cdots$$

$$\left\{ \frac{4}{2}\sin x + \frac{4}{6}\sin 3x + \text{——} \right\}$$

Q4:-
Express the transfer function using the given data.

$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 2 \end{bmatrix}$ $D = [0]$

Solution:-
we know that

$$\frac{Y(s)}{u(s)} = H(s)$$

$$H[s] = C(sI - A)^{-1} B + D$$

Puting the value

$$H[s] = \begin{bmatrix} 1 & 2 \end{bmatrix}\left[s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}\right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0]$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\left[\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}\right]^{-1}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 \end{bmatrix}\begin{bmatrix} s+2 & +1 \\ -1 & s \end{bmatrix}^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$Adj = S(S+2)+1 = S^2+2S+1$$

$$\Rightarrow S^2+2S+1$$

$$H[S] = [1 \;\; 2]\begin{bmatrix} S & -1 \\ 1 & S+2 \end{bmatrix} \times \frac{1}{S^2+2S+1}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

So

$$H[S] = [1 \;\; 2] \times \frac{1}{S^2+2S+1}\begin{bmatrix} S & +0 \\ 1 & \end{bmatrix}$$

$$H[S] = \frac{[1 \;\; 2]\begin{bmatrix} S \\ 1 \end{bmatrix}}{S^2+2S+1}$$

$$H[S] = \frac{S+2}{S^2+2S+1}.$$

Q 3:- if $x(z) = 2z^2 + 2z / (z^2 + 2z - 3)$

Retrieve $x[n]$ using inverse z-transform method.

Sol:-

$$x(z) = \frac{2z\,(z+1)}{z^2+2z-3}$$

$$\frac{x(z)}{2z} = \frac{z+1}{z^2+2z-3}$$

$$\frac{x(z)}{2z} = \frac{z+1}{(z-1)(z+3)}$$

As

$$\frac{z+1}{(z-1)(z+3)} = \frac{A}{(z-1)} + \frac{B}{(z+3)} \quad \text{— (A)}$$

Now

$$z+1 = A(z+3) + B(z-1) \quad \text{— (B)}$$

Now put $z=1$ in (B)

$1+1 = A(1+3) + B(1-1)$

$\cancel{2} = \cancel{4}A$

$$\boxed{\frac{1}{2} = A} \text{ ——— (i)}$$

Now put $z = -3$ in (B)

$-3+1 = A(-3+3) + B(-3-1)$

$+\cancel{2} = +\cancel{4}B$

$$\boxed{\frac{1}{2} = B} \text{ ——— (ii)}$$

So put (i) & (ii) in (A)

$$\frac{z+1}{(z-1)(z+3)} = \frac{1/2}{(z-1)} + \frac{1/2}{z-3}$$

$$X(z) = \frac{1}{2}\frac{z}{z-1} + \frac{1}{2}\frac{z}{z+3}$$

So the inverse z-transform is

$x[n] = \frac{1}{2}u[n] + \frac{1}{2}(3).$