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paper : mid term.

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Q 1. (a) P# (1)

Soln: $y' = (x+2)y^2$

$$y' = (x+2)y^2$$

$$\frac{dy}{dx} = (x+2)y^2$$

$$\int \frac{1}{y^2}\,dy = \int (x+2)\,dx.$$

$$\int y^{-2}\,dy = \int (x+2)\,dx.$$

$$\frac{y^{-2+1}}{-2+1} = \frac{x^2}{2} + 2x + C.$$

$$\frac{y^{-1}}{-1} = x^2/2 + 2x + C$$

$$y^{-1} = -\left(x^2/2 + 2x + C\right)$$

$$\boxed{y = -\left(\frac{1}{x^2/2 + 2x + C}\right)} \text{ Ans}$$

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Q # 9 (2)

(b)

$$y' = (y+9x)^2$$

Soln:

$$y' = (y+9x)^2 \quad \text{--- (i)}$$

let.

$$y + 9x = u$$

$$\frac{\partial y}{\partial x} + 9 = \frac{\partial u}{\partial x}.$$

$$\frac{\partial y}{\partial x} = \frac{\partial u}{\partial x} - 9.$$

So, eq (i) become.

$$\frac{\partial u}{\partial x} - 9 = u^2.$$

$$\frac{\partial u}{\partial x} = u^2 + 9.$$

$$\int \frac{1}{u^2+9}\, du = \int dx.$$

$$\int \frac{1}{(3)^2 + (u)^2}\, \partial u = \int \partial x.$$

$$\frac{1}{3}\tan^{-1}\left(\frac{u}{3}\right) = x + C_1$$

$$\tan^{-1}\left(\frac{u}{3}\right) = 3x + 3C_1$$

$$\tan^{-1}\left(\frac{u}{3}\right) = 3x + C$$

$$\frac{u}{3} = \tan(3x + C)$$

$$u = 3\tan(3x + C)$$

$$y + 9x = 3\tan(3x + C)$$

$$\boxed{y = -9x + 3\tan(3x + C)}$$

$$\boxed{y = -9x + 3\tan(x + C)}$$

x ——— x ——— x. Ans

(4)

Q#2 $x^3 dx + y^3 dy = 0$

Soln:

$$x^3 dx + y^3 dy = 0.$$

$$M dx + N dy = 0.$$

$$M = x^3 \quad , \quad N = y^3$$

$$\frac{\partial M}{\partial y} = 0 \qquad \frac{\partial N}{\partial x} = 0$$

$$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$ so exact.

$$u = \int M dx + k(y)$$

$$u = \int x^3 dx + k(y)$$

$$u = \frac{x^4}{4} + k(y) \quad \text{—— (1)}$$

$$\frac{\partial u}{\partial y} = 0 + \frac{d}{dy} k(y)$$

(5)

$$\frac{\partial u}{\partial y} = \frac{d}{dy} k(y)$$

Since we know that.

$$\frac{\partial u}{\partial y} = N = y^3$$

$$y^3 = \frac{d}{dy} k(y) \Rightarrow \int y^3 = \int \cancel{d}\, k(y)$$

$$\Rightarrow k(y) = y^4/_4 + C. \quad \text{Put in (1)}$$

$$(1) \Rightarrow u = \frac{x^4}{4} + \frac{y^4}{4} + C_1$$

$$C_2 = \frac{x^4}{4} + \frac{y^4}{4} + C_1$$

$$C_2 - C_1 = \frac{x^4}{4} + \frac{y^4}{4}$$

$$\boxed{C = \frac{x^4}{4} + \frac{y^4}{4}} \quad \text{Ans.}$$

X —— X —— X

(b)

Q#3 (a)

$$4y'' - 20y' + 25y = 0$$

Soln:

$$4y'' - 20y' + 25y = 0$$

Here

$$a = -20, \quad b = 25.$$

So,

$$4\lambda^2 - 20\lambda + 25 = 0$$

$$4\lambda^2 - 10\lambda - 10\lambda + 25 = 0$$

$$2\lambda(2\lambda - 5) - 5(2\lambda - 5) = 0$$

$$(2\lambda - 5)(2\lambda - 5) = 0$$

$$2\lambda - 5 = 0 \Rightarrow 2\lambda = 5$$

$$\Rightarrow \lambda = 5/2.$$

So, roots are real and equal.

$$y = (c_1 + c_2 x)e^{\lambda x}$$

$$\boxed{y = (c_1 + c_2 x)e^{5/2 x}}$$ Ans

CE #3

(b)

$$4y'' - 6y' - 7y = 0.$$

Soln:

$$4y'' - 6y' - 7y = 0$$

Assume $y(x) = e^{\lambda x}$

Put in eq.

$$\Rightarrow 4\frac{d^2 y(x)}{dx^2} - 6\frac{d}{dx}y(x) - 7y(x) = 0$$ eq (i).

$$\Rightarrow \frac{d^2}{dx^2}(e^{\lambda x}) = \lambda^2 e^{\lambda x} \quad \text{— (A)}.$$

$$\Rightarrow \frac{d}{dx}(e^{\lambda x}) = \lambda e^{\lambda x} \quad \text{— (B)}$$

Put (A) and (B) in eq (i).

$$4\lambda^2 e^{\lambda x} - 6\lambda e^{\lambda x} - 7e^{\lambda x} = 0$$

$$\Rightarrow (4\lambda^2 - 6\lambda - 7)\, e^{\lambda x} = 0$$

$$\Rightarrow \lambda = 3/4 - \sqrt{37}/4$$

$$\Rightarrow \lambda = 3/4 + \sqrt{37}/4.$$

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$\Rightarrow y(x) = y_1(x) + y_2(x)$

$\Rightarrow y(x) = C_1 e^{(3/4 - \sqrt{37}/4)x}$

$C_2 e^{(3/4 + \sqrt{37}/4)x}$.

Ans.

THE END.