

NUMERICAL ANALYSIS



Submitted by

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Section B

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Q1) Find the root of the equation

$$x^3 + 3.993 \times 10^{-4} = 0.165x^2$$

use newton raphson method with

$$x_0 = 0.02$$

Solution:~

Rearranging the equation

$$x^3 - 0.165x^2 + 0.00039 = 0$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = 3x^2 - 0.33x = 0$$

$$\begin{aligned} \Rightarrow x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\ &= 0.02 - \frac{0.00033}{-0.0054} \end{aligned}$$

$$x_1 = 0.081$$

$$\begin{aligned} x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\ &= 0.081 - \frac{(-0.00016)}{-0.0070} \end{aligned}$$

$$x_2 = 0.058$$

$$\begin{aligned} f(x_0) &= 0.00033 \\ f'(x_0) &= -0.0054 \end{aligned}$$

$$\begin{aligned} f(x_1) &= -0.00016 \\ f'(x_1) &= -0.0070 \end{aligned}$$

$$\rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.058 - \frac{0.000030}{(-0.0090)}$$

$$x_3 = 0.061$$

$$f(x_2) = 0.000030$$

$$f'(x_2) = -0.0090$$

②

22) use the number $x_0 = 2$, $x_1 = 2.75$, $x_2 = 4$ to find the lagrange interpolation Polynomial for $f(x) = \sin x$ at $x = 3$.

Solution:~

Putting values in the function

$$y_0 = 0.5, y_1 = 0.364, y_2 = 0.25$$

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

$$= \frac{(3-2.75)(3-4)}{(2-2.75)(2-4)} (0.5) + \frac{(3-2)(3-4)}{(2.75-2)(2.75-4)} \times 0.364$$

$$+ \frac{(3-2)(3-2.75)}{(4-2)(4-2.75)} (0.25)$$

$$= -0.083 + 0.388 + 0.025$$

$$\Rightarrow \boxed{y = 0.33}$$

Q3) Complete the divided difference table for the given and construct the interpolating polynomial that uses all this data.

x = 1.0 1.3 1.6 1.9 2.7
Y = 0.7651977 0.6200860 0.4554022 0.2818186 0.1103623

x_i	$f(x_i)$	$f(x_{i-1}, x_i)$	$f(x_{i-2}, x_{i-1}, x_i)$	$f(x_{i-3}, \dots, x_i)$	$f(x_{i-4}, \dots, x_i)$
x_0 1	0.7651977	-0.4837656			
x_1 1.3	0.6200860	-0.548946	-0.108734	0.0658785	
x_2 1.6	0.4554022	-0.573612	-0.0494433		-0.0028049
x_3 1.9	0.2818186	-0.571521	0.006818	0.06251255	
x_4 2.2	0.1103623				

⑤

$$\begin{aligned} 1) f(x_0, x_1) &= \frac{f(x_1) - f(x_0)}{x_1 - x_0} \\ &= \frac{0.6200860 - 0.7651977}{1.3 - 1} \end{aligned}$$

$$f(x_0, x_1) = -0.4837056$$

$$\begin{aligned} 2) f(x_1, x_2) &= \frac{f(x_2) - f(x_1)}{x_2 - x_1} \\ &= \frac{0.4554022 - 0.6200860}{1.6 - 1.3} \end{aligned}$$

$$f(x_1, x_2) = -0.548946$$

$$\begin{aligned} 3) f(x_2, x_3) &= \frac{f(x_3) - f(x_2)}{x_3 - x_2} \\ &= \frac{0.2818186 - 0.4554022}{1.9 - 1.6} \end{aligned}$$

$$f(x_2, x_3) = -0.578612$$

$$\begin{aligned} 4) f(x_3, x_4) &= \frac{f(x_4) - f(x_3)}{x_4 - x_3} \\ &= \frac{0.1103623 - 0.2818186}{2.2 - 1.9} \end{aligned}$$

$$f(x_3, x_4) = -0.571521$$

Second divided difference

$$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$$

$$= \frac{-0.548946 - (-0.4837056)}{1.6 - 1}$$

$$= -0.108734$$

$$f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$$

$$= \frac{-0.578612 - (-0.548946)}{1.9 - 1.3}$$

$$f(x_1, x_2, x_3) = -0.0494433$$

$$f(x_2, x_3, x_4) = \frac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2}$$

$$= \frac{-0.571521 - (-0.578612)}{2.2 - 1.6}$$

$$= 0.006818$$

Third divided difference

(7)

$$\begin{aligned}f(x_0, x_1, x_2, x_3) &= \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0} \\&= \frac{-0.0494433 - (-0.108734)}{1.9 - 1} \\&= \boxed{0.0658785}\end{aligned}$$

$$\begin{aligned}f(x_1, x_2, x_3, x_4) &= \frac{f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1} \\&= \frac{0.006818 - (-0.049443)}{2.2 - 1.3} \\&= \boxed{0.06251255}\end{aligned}$$

4th divided difference

$$\begin{aligned}f(x_0, x_1, x_2, x_3, x_4) &= \frac{0.06251255 - 0.0658785}{2.2 - 1} \\&= \boxed{0.0028049}\end{aligned}$$

⑧

$$\begin{aligned} f(x) &= f(x_0) + (x-x_0)f(x_0, x_1) + (x-x_0)(x-x_1)f(x_0, x_1, x_2) + (x-x_0) \\ &\quad (x-x_1)(x-x_2)f(x_0, x_1, x_2, x_3) + (x-x_0)(x-x_1)(x-x_2) \\ &\quad (x-x_3)f(x_0, x_1, x_2, x_3, x_4) \\ &= 0.7651977 + (x-1) - 0.4837056 + (x-1)(x-1.3)(-0.108734) \\ &\quad + (x-1)(x-1.3)(x-1.6)(0.0658785) + (x-1)(x-1.3) \\ &\quad (x-1.6)(x-1.9)(-0.0028049) \end{aligned}$$