



Assignment
Date: 20/4/2020

Course Code: MTH 102 Course Title: Calculus and analytic geometry
 Prerequisite: _____ Instructor: HIMAYATULLAH
 Module: 3 Program: BEE Total Marks: 30

Q1.	(a)	. Identify $\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$	Marks 5
			CLO1 C1
	(b)	Find the first order derivatives of the function $y = \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x} + 1\right)$	Marks 5
			CLO1 C1
Q2	(a)	. A dynamite blast blows up a heavy rock with launch velocity of 160m/sec reaches a height of $s = 160t - 16t^2$ ft after t sec, (i) How high does the rock go (ii) Find the velocity and speed of the rock when it is 256 ft above the ground on the way up and down (iii) find the acceleration of the rock at time 5sec	Marks 10
			CLO2 C2
Q3	(a)	Does the curve $y = x^4 - 2x^2 + 2$ have nay horizontal tangent if so where ?	Marks 10
			CLO1 C1

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Q No 1
Part (A) Identify Limit $\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$

Sol: $\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$

$$\lim_{h \rightarrow 0} \frac{\sqrt{2+0} - \sqrt{2}}{h}$$

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$$\sqrt{2+h} + \sqrt{2}$$

$$h. \lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} \times \frac{\sqrt{2+h} + \sqrt{2}}{\sqrt{2+h} + \sqrt{2}}$$

$$\lim_{h \rightarrow 0} \frac{(\sqrt{2+h})^2 - (\sqrt{2})^2}{h(\sqrt{2+h} + \sqrt{2})}$$

$$\lim_{h \rightarrow 0} \frac{2+h-2}{h(\sqrt{2+h} + \sqrt{2})}$$

$$\lim_{h \rightarrow 0} \frac{h}{h(\sqrt{2+h} + \sqrt{2})}$$

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$$\lim_{h \rightarrow 0} \frac{1}{\sqrt{2+h} + \sqrt{2}}$$

$$\lim_{h \rightarrow 0} = \frac{1}{\sqrt{2+0} + \sqrt{2}}$$

$$= \frac{1}{\sqrt{2} + \sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} \text{ Ans}$$

Past B Find the first order derivatives of the following function.

$$y = \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x} + 1\right)$$

Sol: $y = \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x} + 1\right)$

Taking Derivative.

$$\frac{dy}{dx} = \left(x + x^{-1}\right) \frac{d}{dx} \left(x - x^{-1} + 1\right) + \left(x - x^{-1} + 1\right) \frac{d}{dx} \left(x + x^{-1}\right)$$

$$= \left(x + x^{-1}\right) \left(1 - x^{-2}\right) + \left(x - x^{-1}\right) + \left(1 - x^{-2}\right) \left(x + \frac{1}{x}\right) \left(1 + \frac{1}{x^2}\right) + \left(x - \frac{1}{x}\right) \left(1 - \frac{1}{x^2}\right)$$

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(3)

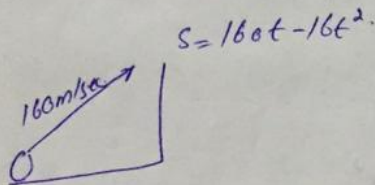
$$= x + x \frac{1}{x^2} + \frac{1}{x} + \frac{1}{x^3} + x - x \frac{1}{x^2} - \frac{1}{x} + \frac{1}{x^3} + 1 - \frac{1}{x^3}$$

Q No 2

A dynamite blast blows up a heavy rock. with launch velocity of 160 m/sec reaches a height of $S = 160t - 16t^2$ ft after t sec.

- i) How high does the rock go.
- ii) Find the velocity & speed of the rock when it is 256 ft above the ground on the way up & down.
- iii) Find the acceleration of the rock at time sec.

Sol:-



$$i) S = 160t - 16t^2$$

$$\begin{aligned} v(t) &= 160 - 32t \\ &= 160 - 64 \\ &= 96 \text{ m/sec.} \end{aligned}$$

(3)

$$(ii) \quad \frac{ds}{dt} = v.$$

$$s = 256 = 160t - 16t^2.$$

$$16t^2 - 160t + 256 = 0.$$

$$16(t^2 - 10t + 16) = 0.$$

$$t^2 - 10t + 16 = 0.$$

$$t^2 - 8t - 2t + 16 = 0.$$

$$t(t-8) - 2(t-8) = 0.$$

$$(t-2)(t-8) = 0.$$

$$\boxed{t = 2, 8}$$

iii) Find the acceleration of the rock at time 5 sec?

$$s'(t) = 160 - 32t.$$

$$s''(t) = -32 \text{ ft/sec}^2.$$

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Q3 Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangent if so where.

$$y = x^4 - 2x^2 + 2.$$

Sol: Taking Derivative.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} [x^4 - 2x^2 + 2] \\ &= \frac{d}{dx} (x^4) - 2 \frac{d}{dx} (x^2) + \frac{d}{dx} (2)\end{aligned}$$

$$\frac{dy}{dx} = 4x^3 - 4x + 0$$

$$\frac{dy}{dx} = 4x^3 - 4x$$

If the tangent is horizontal then $\frac{dy}{dx} = 0$.

$$\begin{aligned}\text{So, } 4x^3 - 4x &= 0 \\ 4x(x^2 - 1) &= 0 \\ 4x = 0, x^2 - 1 &= 0\end{aligned}$$

$$\text{So } x = 0, 1, -1$$

The corresponding points in $y = x^4 - 2x^2 + 2$ are given below.

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$$\text{For } x=0 \quad y = 0 - 0 + 2$$

$$\Rightarrow y = 2.$$

$$\text{For } x=1$$

$$y = 1 - 2(1) + 2.$$

$$\Rightarrow y = 1$$

$$\text{For } x=-1$$

$$y = -1 - 2 + 2$$

$$y = 1$$

Hence $(0, 2)$ $(1, 1)$ & $(-1, 1)$ Ans