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Section # A

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Subject # Differential Equation

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Q# 1

i) $W = \sin(x+ct) + \cos(2x+2ct)$

Given $\dfrac{\partial^2 w}{\partial t^2} = c^2 \dfrac{\partial^2 w}{\partial x^2} \rightarrow ①$

Now

$$\frac{\partial w}{\partial t} = \frac{\partial}{\partial t}\left[\sin(x+ct) + \cos(2x+2ct)\right]$$

$$= \frac{\partial}{\partial t}((\sin(x+ct)) + \cancel{\cos(2x} \frac{\partial}{\partial t}(\cos(2x+2ct))$$

$$= \frac{\partial w}{\partial t} = c\cos(x+ct) - 2c\sin(2x+2ct$$

$$\boxed{\text{Now } \frac{\partial^2 w}{\partial t^2} = -c^2\sin(x+ct) - 4c^2\cos(2x+2ct}$$

②

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Now

$$\frac{\partial w}{\partial x} = \frac{\partial}{\partial x}\left[\sin(x+ct) + \cos(2x+2ct)\right]$$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - 2\sin(2x+2ct)$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial}{\partial x}\left[\cos(x+ct) - 2\sin(2x+2ct)\right]$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct) \Rightarrow (1)$$

$$= -c^2\sin(x+ct) - 4c^2\cos(2x+2ct) = c^2\left[-\sin(x+ct) - 4\cos(2x+2ct)\right]$$

$$= -c^2\cancel{\sin}(x+ct) - 4\cancel{c^2}\cos(2x+2ct) = -c^2\cancel{\sin}(x+ct) - 4c^2\cos\cancel{(2x+2ct)}$$

$0 = 0$ (satisfied)

ii) $w = \tan(2x + ct)$

Now $$\frac{\partial w}{\partial t} = c \sec^2(2x + ct)$$

And $$\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial t}\left(c \sec^2(2x + ct)\right)$$

$$= c^2 \cdot 2\sec^2(2x + ct)\tan(2x + ct)$$

Now $$\frac{\partial w}{\partial x} = 2\sec^2(2x + ct)$$

$$\frac{\partial^2 w}{\partial x^2} = 4\sec^2(2x + ct)\tan(2x + ct) = 70$$

$$4c^2\sec^2(2x + \cancel{ct})\tan(2x + ct) = 4\sec^2(2x + \cancel{ct})\tan(2x + ct)$$

$0 = 0$ (satisfied)

Q#02

Given Data is

$$F(x) = \begin{cases} x; & -\pi < x \le 0 \\ 2x; & 0 \le x \le \pi \end{cases}$$

We have to find the fouriers co-efficient, a_0, a_n, and ~~bn~~ b_n

Now

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x)\,dx = \frac{1}{\pi} \int_{-\pi}^{0} x\,dx + \frac{1}{\pi} \int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi} \left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi} \left|\frac{x^2}{2}\right|_{0}^{+\pi}$$

$$= \frac{1}{\pi} \left(0 - \frac{\pi^2}{2}\right) + \frac{2}{\pi} \left[\frac{\pi^2}{2} - 0\right]$$

$$\boxed{a_0 = -\frac{\pi}{2} + \pi = \frac{\pi}{2}} \longrightarrow (1)$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x) \cos nx \, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} (x \cos nx \, dx) + \frac{1}{\pi} \int_{0}^{\pi} (2x \cos nx) \, dx$$

$$= \frac{1}{\pi} \left[x \left(\frac{\sin nx}{n} \right) - \left(- \frac{\cos nx}{n^2} \right) \right]_{-\pi}^{0} + \frac{2}{\pi} \left[x \left(\frac{\sin nx}{n} \right) - \left(- \frac{\cos nx}{n^2} \right) \right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi} \left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2} \right] + \frac{2}{\pi} \left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2} \right]$$

$$a_n = \frac{1}{\pi} \left[\frac{1 - (-1)^n + 2(-1)^n - 2}{n^2} \right] = \frac{(-1)^n - 1}{\pi n^2}$$

So

$$a_n = \begin{cases} \frac{-2}{\pi n^2} ; & \text{if } n \text{ is add} \\ 0 ; & \text{if } n \text{ is even} \end{cases}$$

$\Rightarrow$ 2

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\sin nx\,dx + \frac{2}{\pi}\int_{0}^{\pi} x\sin nx\,dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0} +$$

$$\frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$\boxed{b_n = \frac{1}{\pi}\left[-\frac{\pi\cos n\pi}{n}\right] + \frac{2}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right] = \frac{-3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}}$$

③

So the required fourier Series is :

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos nx + b_n\sin nx\right)$$

$$= \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2x-1)x}{(2x-1)^2} + 3\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}$$

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Q#3

$$y'' - 4y' + 13y = 8\sin 3x \longrightarrow ①$$

$y(0) = 1$

$y'(0) = 2$

Sol: ~~Equation~~ Associated Homogenous Eq of ① is

$$y'' - 4y' + 13y = 0 \longrightarrow ②$$

Change ② ~~and~~ into Auxiliary equation

put $y = m$ in ②

$$m^2 - 4m + 3 = 0$$

use Quadratic formula

$a = 1$, $b = -4$, $c = 13$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 52}}{2}$$

$$= \frac{4 \pm \sqrt{-36}}{2}$$

$$= \frac{4 \pm \sqrt{36i}}{2}$$

$$= \frac{4 \pm 6i}{2}$$

$$\boxed{= 2 \pm 3i}$$

$$\boxed{\begin{aligned} m_1 &= 2+3i \\ m_2 &= 2-3i \end{aligned}}$$

$$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) \longrightarrow (A)$$

$$y_p = A\cos 3x + B\sin 3x \longrightarrow (*)$$

Diff w.r.t x

$$y'_p = -3A \sin 3x + 3B\cos 3x$$

Again Diff w.r.t x

$$y''_p = -9A \cos 3x - 9B\sin 3x$$

put in (1)

$$\Rightarrow (-9A\cos 3x - 9B\sin 3x) - 4(-3A\sin 3x + 3B\cos 3x) + 13(A\cos 3x + B\sin 3x) = 8\sin 3x$$

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$= -9A\cos 3x - 12B\cos 3x + 13A\cos 3x - 9B\sin 3x$

$+ 12A\sin 3x + 13B\sin 3x = 8\sin 3x$

$\Rightarrow (-9A - 12B + 13A)\cos 3x + (-9B + 12A + 13B)\sin 3x$
$= 8\sin 3x$

$\Rightarrow (4A - 12B)\cos 3x + (4B + 12A)\sin 3x = 8\sin 3x$

Comparing coefficient

$\sin 3x \Rightarrow \quad 4B + 12A = 8 \rightarrow (a)$

$\cos 3x \Rightarrow \quad 4A - 12B = 0 \Rightarrow (b)$

$4A = 12B$

$\Rightarrow \boxed{A = 3B \rightarrow b}$

Put b in ea

$4B + 12(3B) = 8 \Rightarrow 4B + 36B = 8$

$40B = 8$

Now 40 divid on Both sides

$$\frac{40B}{40} = \frac{8}{40} \Rightarrow \boxed{B = \frac{1}{5}} \rightarrow (e)$$

put (e) ine eq (b)

$$\Rightarrow A = \frac{3}{5} \rightarrow (d)$$

Put (e) and (d) in eq (*)

$$y_p = \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \longrightarrow (B)$$

The General Solution is

$$y = y_c + y_p$$

$$y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow (C)$$

Now need to find the value of

C_1 and C_2 for this

Put $x=0$, and $y=1$ in C

$$1 = e^{2(0)}(c_1 \cos 3(0) + c_2 \sin 3(0)) + \frac{3}{5}\cos 3(0)$$
$$+ \frac{1}{5}\sin 3(0)$$

$$1 = (c_1(1) + c_2(0)) + \frac{3}{5}(1) + \frac{1}{5}(0)$$

$$1 = C_1 + \frac{3}{5}$$

$$\boxed{C_1 = 1 - \frac{3}{5}}$$

$$\boxed{C_1 = \frac{2}{5}} \Rightarrow **$$

Diff equation C w.r.t x

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$y' = C_1 \left(2e^{2x}\cos 3x - 3e^{2x}\sin 3x\right) + \cancel{(C_2)}2e$

ε

$C_2 \cancel{2e} e^{2x}\sin 3x + 3e^{2x}\cos 3x - \frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x$

$\rightarrow$ (D)

Put $y' = 2$, $x = 0$ in eq (D)

$y' = C_1\left(2e^{2x}\cos 3x - 3e^{2x}\sin 3x\right) + C_2\left(2e^{2x}\sin 3x\right.$

$\left. + 3e^{2x}\cos 3x\right) - \frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x$

Put $y' = 2$, $x = 0$

$2 = C_1\left(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0)\right) + C_2\left(2e^{2(0)}\right.$

$\left. + 3e^{2(0)}\cos 3(0)\right) - \frac{6}{8}\sin 3(0) + \frac{3}{5}\cos 3(0)$

$= 2 = C_1(2) + C_2(3) - 0 + \frac{3}{5}$

$$2 = 2c_1 + 3c_2 + \frac{3}{5}$$

put $c_1 = 2/5$

$$2 = 2 \cdot \frac{2}{5} + 3c_2 + \frac{3}{5}$$

$$2 = \frac{4}{5} + 3c_2 + \frac{3}{5}$$

$$2 = \frac{4}{5} + \frac{3}{5} + 3c_2$$

$$2 = \frac{7}{5} + 3c_2$$

$$2 - \frac{7}{5} = 3c_2$$

$$3c_2 = \frac{\cancel{5-7}\ 10-7}{5}$$

$$3c_2 = \frac{3}{5}$$

No $3 \div$ on both sides

$$c_2 = \frac{3}{5} \div 3$$

$$\boxed{c_2 = \frac{3}{15}} \longrightarrow ***$$

eq (**) and eq (***) in (C)

$$y = e^{2x}\left(\frac{2}{5}\cos 3x + \frac{3}{15}\sin 3x\right) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x$$

$\cancel{y = \frac{2}{5}(}$

$\cancel{y = \frac{2}{5}e}$

$$\boxed{y = \frac{2}{5}e^{2x}\cos 3x + \frac{3}{15}e^{2x}\sin 3x + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x}$$

$\Rightarrow$

Required Answer

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Q# 4

Solution:: It is all already in symbolic form

$(D^2 - DD')z = \cos x \cos 2y \longrightarrow (a)$

Put A.E $= D^2 - DD' = 0$

As we know that

$\frac{D}{D'} = m$ i-e $D = m$, $D' = 1$

$\Rightarrow m^2 - m = 0$

$m = 0, 1$

There fore

$C.F = f_1(y) + f_2(y+x)$

from eq ⓐ

$$P.I = \frac{1}{D^2 - DD'} \cos x \cos 2y$$

$$= \frac{1}{2} \cdot \frac{1}{D^2 - DD'} 2 \cos x \cos 2y$$

As

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$C.F = f_1(y-x) + x f_2(y-x)$

$$P.I = \frac{1}{D^2 - 2DD' + D'^2} [2(y-x) + \sin(x-y)]$$

$$= \frac{1}{(D+D')^2} [2(y-x) + \sin(x-y)]$$

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By General Method

$m = -1 \; ; \; y - x = c$

$$= \frac{1}{D+D'} \int [2c + \sin(-c)]\, dx$$

$$= \frac{1}{D+D'} [2cx - (\sin c)x]$$

Replacing c by $y-x$

$$= \frac{1}{D+D'} [2x(y-x) - x \sin(y-x)]$$

Again put $y - x = c$

$$= \int (2x c - x \sin c)\, dx \Rightarrow cx^2 - \frac{x^2}{2} \sin c$$

$= \int (2xc - x\sin c)\,dx \Rightarrow cx^2 - \frac{x^2}{2}\sin c$

Replacing c by $y-x$

$= x^2(y-x) - \frac{x^2}{2}\sin(y-x) = x^2 y$

$= x^2 y - x^3 - \frac{x^2}{2}\sin(x-y)$

Here the required Solution is

$Z = C.F + P.I = f_1(y-x) + x f_2(y-x) + x^2 y - x^3 + \frac{1}{2} x^2 \sin(x-y)$