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"MATHS PAPER"

Question: 1

Consider the given below matrix as the augmented matrix of a linear system. Explain in your own words the next elementary row operation that should be performed in order to solve this system. Where ID3 is the 3^{rd} digit in your ID and ID_last is the last digit of your ID in inverse e.g: if your ID is 12345 then -ID_last = -5.

$$\begin{bmatrix} 1 & ID3 & 3 & 0 & 5 \\ 0 & 1 & -ID\_last & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & ID3 \end{bmatrix}$$

Solution:

My ID is 14672

Given Matrix be A

$$1x + 6y + 3z + 0t = 5$$
$$0x + 1y - 2z + 0t = 7$$
$$0x + 0y + 1z + 0t = -6$$
$$0x + 0y + 0z + 1t = 6$$

$$\begin{bmatrix} 1 & 6 & 3 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ -6 \\ 6 \end{bmatrix}$$

Let the augmented matrix be:

$$A = \left(\begin{array}{cccc|c} 1 & 6 & 3 & 0 & 5 \\ 0 & 1 & -2 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right)$$

$R_1 \to R_1 - 6R_2$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 15 & 0 & -37 \\ 0 & 1 & -2 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

$R_2 \to R_2 + 2R_3$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 15 & 0 & -37 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

$R_1 \to R_1 - 15R_3$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 53 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 6 \end{array}\right]$$

Hence:

So we get the values of x, y, z and t.

$x = 53$

$y = 5$

$z = -6$

$t = 6$

(a)

Q-2 Find the elementary row operation that transform the first matrix into second and reverse row operation that transforms the second matrix into first.

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Solution:

Let first matrix be A.

Let second matrix be B.

Elementary Row Operation:

$$A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - 2R_2$$

$$A = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Reverse Row Operation:

$$B = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$$R_3 \rightarrow 2R_2 + R_3$$

$$B = \begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

Hence Showed.

Q-2.(b) Given below are some matrices. Find whether these are in the forms written in front of them or not. Explain in your own words for each of the selection in detail.

a)- $\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$ is in echelon form.

Solution:

Let $A = \begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$

Yes, matrix A is in echelon form because of its definition, as echelon form of a matrix states that "If a column contains a leading entry then all entries below that leading entry are zero".
In matrix A, it satisfies the definition of echelon form of a matrix, so it is in echelon form.

(b). $\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in echelon form.

Solution:

Let $B = \begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Yes, matrix B is in echelon form because of its definition, which states that "If a column contains a leading entry then all entries below that leading entry are zero."

According to definition, matrix B's columns contains leading entries as 1 and below that all entries are zero. So matrix B is in echelon form.

(c). $\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form.

Solution:

Let $C = \begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

$C = \begin{bmatrix} 5/5 & 0/5 & 0/5 & 7/5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ $R_1/5$

$C = \begin{bmatrix} 1 & 0 & 0 & 7/5 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

Yes, matrix C is in reduced row echelon form, because reduced row echelon form states that " In reduced row echelon form the leading co-efficient must be 1, in each row is to the right of the leading co-efficient in the row above it". According to definition, matrix C satisfies the definition properties so it is in reduced row echelon form.

d). $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$ is in reduced row echelon form.

Solution:

Let $D = \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

No, matrix D is not in reduced row echelon form, because if a matrix is in reduced row echelon form then its rows (non-zero) contains its first entries as a number "1" which is known as leading 1 i-e: the first non-zero entry is 1. Also if there are any rows containing only zero entries then they are located in the botton part of the matrix but in matrix D the zero row is located in mid of matrix, so it is not in reduced row echelon form.

(b)

Q:3 Find an echelon form for the below matrix using row operations. Where ID2 is 2nd Digit in your ID. e.g if your ID is 12345, ID2 = 2, ID3 = 3. ID-first_last is the first and last digit of your ID. i-e: 15.

$$\begin{bmatrix} 1 & ID2 & 8 \\ 2 & 8 & -1 \\ -ID3 & 0 & 0 \\ 1 & -4 & ID\text{-}first\_last \end{bmatrix}$$

Solution:

My ID is 14672.

Let the given matrix be A.

$$A = \begin{bmatrix} 1 & 4 & 8 \\ 2 & 8 & -1 \\ -3 & 0 & 0 \\ 1 & -4 & 12 \end{bmatrix}$$

$R_2 \to 2R_1 - R_2$

$$A = \begin{bmatrix} 1 & 4 & 8 \\ 0 & 0 & 17 \\ -3 & 0 & 0 \\ 1 & -4 & 12 \end{bmatrix}$$

$R_3 \to 3R_1 + R_3$, $R_4 \to R_4 - R_1$

$$= \begin{bmatrix} 1 & 4 & 8 \\ 0 & 0 & 17 \\ 0 & 12 & 24 \\ 0 & -8 & 4 \end{bmatrix}$$

$R_2 \to R_4$

$$= \begin{bmatrix} 1 & 4 & 8 \\ 0 & -8 & 4 \\ 0 & 12 & 24 \\ 0 & 0 & 17 \end{bmatrix}$$

$R_2 / -4$

$$= \begin{bmatrix} 1 & 4 & 8 \\ 0 & 2 & -1 \\ 0 & 12 & 24 \\ 0 & 0 & 17 \end{bmatrix}$$

$R_3/2$

$$= \begin{bmatrix} 1 & 4 & 8 \\ 0 & 2 & -1 \\ 0 & 6 & 12 \\ 0 & 0 & 17 \end{bmatrix}$$

$R_3 \leftarrow R_3/6$

$$= \begin{bmatrix} 1 & 4 & 8 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 17 \end{bmatrix}$$

$R_3 \leftarrow 2R_2 + R_3$

$$= \begin{bmatrix} 1 & 4 & 8 \\ 0 & 2 & -1 \\ 0 & 5 & 0 \\ 0 & 0 & 17 \end{bmatrix}$$

$R_2 \leftarrow R_3$

$$= \begin{bmatrix} 1 & 4 & 8 \\ 0 & 5 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 17 \end{bmatrix}$$

$R_2/5$

$$= \begin{bmatrix} 1 & 4 & 8 \\ 0 & 1 & 0 \\ 0 & 2 & -1 \\ 0 & 0 & 17 \end{bmatrix}$$

$R_3 \leftarrow 2R_2 - R_3$

$$= \begin{bmatrix} 1 & 4 & 8 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 17 \end{bmatrix}$$

$R_4 \leftarrow 17R_3 - R_4$

$$= \begin{bmatrix} 1 & 4 & 8 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence, we get the echelon form of matrix A.

(a)

Q: 3 The row echelon form is used to solve the system of linear equations. What is the difference between the row echelon and reduced row echelon form? What is the practical use of reduced row echelon form? Give one example.

Answer:

Difference between row Echelon and reduced row Echelon form:

Row Echelon form	Reduced Row Echelon form
① Row Echelon form of a matrix is defined as: " the leading entry in row echelon form in each row (column) is the only non-zero entry in its row (column)".	① Reduced Row Echelon form is defined as: In reduced row echelon form the leftmost non-zero entry of a row is equal to 1. The leftmost non-zero entry of a row is the only non-zero entry in its column.

2. Echelon form of a matrix isn't unique which means there are infinite answers possible when we perform row reduction or elementary operation	Reduced Row Echelon form is unique, which means when we apply elementary row operation on a matrix it will produce the same answers, no matter how we perform the same row operations.
3. Each row containing a non-zero number has the number 1 appearing in the row's first non-zero column. Such entry will be known as "leading entry/one".	In reduced row echelon form, the leftmost non-zero entry of a row is equal to 1. The leftmost non-zero entry of a row is the only non-zero entry in its column.
4. The entries only below the first leading non-zero entry that must be zero, not necessary for above one's	The entries above and below the first 1 in each row must all be 0.

"Example"

(5). $\begin{bmatrix} 1 & 6 & 2 & -8 \\ 0 & 1 & 14 & 5 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ | $\begin{bmatrix} 1 & 0 & -2 & 0 & 6 \\ 0 & 1 & 7 & 0 & 1 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$

(*) Practical Use of Reduced Row Echelon form :

(1). This type of matrix is used to solve system of linear equations.

(2). Reduced Row Echelon form used in balancing chemical equations.

(3). Such matrix is used to solve computer operations.

(*) Example of Reduced Row Echelon form :

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$