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Subject: Applied Calculus

Semester: 1st final

Q1 Given that Coordinate.

$P = (4,1,3)$

$Q = (1,2,4)$

Sol:-

We know that

$OP = 4i + 1j + 3k$

$OQ = 1i + 2j + 4k$

using distance formula.

$$|\overline{PQ}| = \sqrt{(z_2-z_1)^2+(y_2-y_1)^2+(x_2-x_1)^2}$$

$$|\overrightarrow{PQ}| = \sqrt{(4-3)^2+(2-1)^2+(1-4)^2}$$

$$\overrightarrow{PQ} = \sqrt{(1)^2+(1)^2+(-3)^2}$$

$$\overline{PQ} = \sqrt{1+1+9}$$

$$\overline{PQ} = \sqrt{11} \quad \text{—(1)} \quad \text{Ans (1)}$$

M the point which divided PQ in ratio 1:3 then by ratio theorem

position $m = \overline{OM}$

$$M = \overline{OM}$$

$$= \frac{3(4i+1j+3k)+1(i+2j+4k)}{1+3}$$

$$= \frac{12i+3j+9k+i+2j+4k}{4}$$

$$\boxed{= \frac{13i+5j+13k}{4}}$$

Solution (2)

Q#2

$$\int \frac{4x^3+10x+4}{2x^2+x}$$

Solution:

$$\int \frac{4x^3+10x+4}{2x^2+x}$$

$$= \int \frac{2(2x^3+5x+2)}{x(2x+1)} dx$$

$$= 2\int \frac{2x^3+5x+2}{x(2x+1)} dx$$

perform polynomial long division

$$\int \left(\frac{11x+4}{2x(2x+1)} + \frac{2x+1}{2} \right) dx$$

$$= \frac{1}{2}\int \frac{11x+4}{x(2x+1} dx + \int x dx - \frac{1}{2}\int 1 dx$$

Now solving

$$\int \frac{11x+4}{x(2x+1)} dx$$

$$\int \frac{2(2x^3+5x+2)}{x(2x+1)}\,dx$$

$$=\frac{3\ln(2x+1)}{2}+4\ln(x)+x^2-x+c$$

$$=\frac{3\ln(2x+1)}{2}+4\ln(x)+(x-1)x+c$$

$= \ln(x)$

plug in solved integral.

$= 3\int \frac{1}{2x+1}\,dx + 4\int \frac{1}{x}\,dx$

$= \frac{3\ln(2x+1)}{2} + 4\ln(x)$

now

$\int x\,dx$

Apply power Rule.

$= \int x^n dx = \frac{x^{n+1}}{n+1}$ with $n=1$

$= \frac{x^2}{2}$

$= \int 1\,dx$

$= x$ plug solved integral.

$= \frac{1}{2}\int \frac{11x+4}{x(2x+1)}\,dx \int x\,dx - \frac{1}{2}\int 1\,dx$

$= \frac{3\ln(2x+1)}{4} + 2\ln(x) + \frac{x^2}{2} - \frac{x}{2}$

$= 2\int \frac{2x^3+5x+2}{x(2x+1)}\,dx$

$= \frac{3\ln(2x+1)}{2} + 4\ln(x) + x^2 - x$

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by partial fraction

$= \int \left(\frac{3}{2x+1} + \frac{4}{x} \right) dx$

$= 3 \int \frac{1}{2x+1} dx + 4 \int \frac{1}{x} dx$

$= \int \frac{1}{2x+1} dx$

Substitute $\Rightarrow$ $u = 2x+1 - \frac{du}{dx}$

$dx = \frac{1}{2} dx$

$= \frac{1}{2} \int \frac{1}{2} dx$

now solving

$\int \frac{1}{u} du$

$= \ln(u)$ $= \frac{1}{2} \int \frac{1}{u} du$

$= \frac{\ln(u)}{2}$

now $u = 2x+1$

$= \frac{\ln(2x+1)}{2}$

solving by x

$\int \frac{1}{x} dx$

Q#3 | part # (i)

$$\int_0^2 x^2 e^x dx$$

using integration by parts

Solution

$$\int_0^2 x^2 e^x dx = x^2 \int e^x dx - \int \frac{dx^2}{dx} \int e^x dx \Big|_0^2$$

$$= x^2 e^x - 2x e^x \Big|_0^2$$

$$= x^2 e^x \Big|_0^2 - 2x e^x \Big|_0^2$$

putt the value

$$= ((2)^2 (2.71)^2 - 2((2)(2.71)^2 - (e))$$

$$= (4)(7.34) - 1 - (4 \times 7.34) - 1)$$

$= e^0 = 1$

$$= \cancel{29.36} - 1 - \cancel{29.36} - 1$$

$$\boxed{= -2}$$

Ans

Q3 part H)

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

Sol

let $\sqrt{x} = y$ As $x - 2$

$x = y^2$ Then $y = \sqrt{2}$

$x - 1$

Diff: $y = 1$

$1 = 2y \Rightarrow dx = 2y\,dy$

$$= \int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int_1^{\sqrt{2}} \frac{\sin y}{y}\cdot 2y\,dy$$

$$= 2\int_1^{\sqrt{2}} \sin y\,dy = -2\cos y \Big|_1^{\sqrt{2}}$$

$$= -2\left[\cos\sqrt{2} - \cos 1\right)$$

$$= -2(0.9996 - 0.99984)$$

$$= -2(0.00024) =$$

$$\boxed{= 1.0008 \text{ Ans}}$$

Q#4 $u(x,y,z) = \frac{1}{x^2+y^2+z^2}$

Solved by Laplace three dimensional.

Sol

$u(x,y,z) = \frac{1}{x^2+y^2+z^2}$

Laplace equation

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{\partial^2 u}{dz^2} = 0 \quad \text{— (A)}$$

$$= u(x,y,z) = (x^2+y^2+z^2)^{-\frac{1}{2}}$$

$$\frac{\partial u}{dx} = -\frac{1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}}(2x)$$

$$\frac{\partial u}{dx} = -x(x^2+y^2+z^2)^{-\frac{3}{2}}$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[x\left(\frac{-3}{2}\right)(x^2+y^2+z^2)^{-\frac{5}{2}}(2x) + (x^2+y^2+z^2)^{-\frac{3}{2}}\right]$$

$$\boxed{\frac{d^2u}{dx^2} = 3x^2(x^2+y^2+z^2)^{-\frac{5}{2}} - (x^2+y^2+z^2)^{\frac{3}{2}}} \quad \text{— (1)}$$

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Now $\frac{du}{dy} = ?$

$$\frac{du}{dy} = -\frac{1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}} \cdot (2y)$$

$$\frac{d^2u}{dx^2} = -y(x^2+y^2+z^2)^{-\frac{3}{2}}$$

$$\frac{d^2u}{dx^2} = -\left\{y\left(-\frac{3}{2}\right)(x^2+y^2+z^2)^{-\frac{5}{2}}(2y) + (x^2+y^2+z^2)^{-\frac{3}{2}}\right.$$

$$\boxed{\frac{d^2u}{dx^2} = 3y^2(x^2+y^2+z^2)^{-\frac{5}{2}} - (x^2+y^2+z^2)^{-\frac{3}{2}}} \rightarrow (2)$$

$$\frac{du}{dz} = -\frac{1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}} \cdot (2z)$$

$$\frac{du}{dz} = -z(x^2+y^2+z^2)^{-\frac{3}{2}}$$

$$\boxed{\frac{d^2u}{dz^2} = 3z^2(x^2+y^2+z^2)^{-\frac{5}{2}} - (x^2+y^2+z^2)^{-\frac{3}{2}}} \rightarrow (3)$$

putting eq 1,2 & 3 in eq (A)

(P.T.O)

$$\Rightarrow 3x^2(x^2+y^2+z^2)^{-5/2}-(x^2+y^2+z^2)^{-3/2}+3y^2(x^2+y^2+z^2)^{-5/2}+(x^2+y^2+z^2)$$

$$+3z^2(x^2+y^2+z^2)^{-5/2}+(x^2+y^2+z^2)^{-3/2}$$

$$=(x^2+y^2+z^2)^{-5/2}\left[3x^2-(x^2+y^2+z^2)+3y^2-(x^2+y^2+z^2)+3z^2-(x^2+y^2+z^2)\right]$$

$$=(x^2+y^2+z^2)^{-5/2}\left[\cancel{3x^2}-\cancel{x^2}-\cancel{y^2}-z^2+3y^2-\cancel{x^2}-\cancel{y^2}-z^2+3x-\cancel{x^2}-\cancel{y^2}\right.$$

$$=(x^2+y^2+z^2)^{-5/2}(0)=0$$

$=0$

So the given $u(x,y,z)$ is laplace equation solution