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SECTION : "A"

SUBJECT : Diff equation.

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Application of Partial differential equation

Many Engineering Problems are governed by different type of partial differential equation and some of the more important types as:

*** Poisson's equation** ~~for ∇a~~

$$\partial_2 Q \, \partial x_2 + \partial_2 Q \cdot \partial y_2 = f(x,y)$$

*** Tricomi equation:**

$$y \frac{\partial^u U}{\partial x^2} + \frac{\partial^u U}{\partial y^2} = 0 \begin{cases} y > 0 \\ y < 0 \end{cases}$$

*** Laplace equation:** (or variants)

$$\partial_2 Q \, \partial x 2 + \partial_2 Q \partial y^2 = \nabla_2 Q = 0$$

*** Helmholtz equation:**

$$\partial_2 Q \, \partial x^2 + \partial_2 Q \, y_2 + C_2 Q = 0$$

*** plate bending:-**

$$\nabla_2 \; \nabla_2 w = \nabla 4 w = q D$$

* Wave equation:

$$\frac{\partial^2 U}{\partial t^2} - C^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 U}{\partial y^2}\right) = 0$$

* Fourier equation:

$$\partial T \partial t = a(\partial_2 T \partial x_2)$$

* Separable differential equation:

For equation which can be expressed in separable from as shown below, the solution can obtained easily as:

$$\frac{dy}{dx} = f(x,y) \quad \frac{dy}{f(y)} = f(x)\,dx \quad \int \frac{dy}{f(y)}$$

$$= \int f(x)\,dx + c$$

$$M(x,y)\,dx + N(x,y)\,dy = 0 \quad M(x)\,dx = -$$

$$= -N(y)\,dy$$

then $\int M(x)\,dx = -\int N(y)\,dy + C$

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Example:

$$\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)} \quad \text{Subject to } y(0) = -1$$

A separable function. The problem can be solved as

$$2(y-1)\,dy = (3x^2+4x+2)\,dx$$

$$y^2 - 2y = x^3 + 2x^2 + 2x + c$$

Example:

$$\frac{dy}{dx} = 3x^3 + (y^2+1) = \frac{dy}{y^2+1} = x^3\,dx$$

$$\int \frac{dy}{y^2+1} = \int x^3\,dx + c \Rightarrow \tan^{-1} y = \frac{1}{4x}\,x^4 + c$$

$$\Rightarrow y = \tan\left(\frac{1}{4}x^4 + c\right)$$

Example:

$$(x+1)\frac{dy}{dx} - n.y = e^x(x+1)^{n+1}$$

This equition is now expended as

$$\frac{dy}{dx} = P(x)\,y + Q(x)$$

$$\frac{dy}{dx} = \frac{n}{x+1}\,y + \underbrace{e^x(x+1)^n}_{Q(x)}$$

(4)

For $x \neq -1$

Solving the homogenous part of the ODE

$$\frac{dy}{dx} = \frac{n}{x+1} y \text{ then } \frac{dy}{y} = \frac{n\,dx}{x+1}$$

$$\ln(y) = n \ln|x+1| + C_1$$

$$y = c(x+1)^n$$

For solution $y = c(x)(x+1)^n$ where $c(x)$ is the variation of parameters substitute it to the ODE.

$$\frac{dc(x)}{dx}(x+1)^n + nc(x)(x+1)^{n-1} = nc(x)$$

$$(x+1)^{n-1} = nc(x)(x+1)^{n-1} + e^x (x+1)^n$$

$$\frac{dy}{dx} = \frac{n}{x+1} y + e^x (x+1)^n$$

Comparisons givens $\frac{dc(x)}{dx} = e^x$

Integration of this equation gives

$$c(x) = e^x + \vec{C}$$

General solution is hence given by $y = (x+1)^n (e^x + c)$

⇒ The bernoulli equation is an important equation type which can be solved in similar way by variation of parameters. Consider the following form of equation.

$$\frac{dy}{dx} = P(x)\, y + Q(x)\, y^n$$

• **Step 1st:** put $z = y^{1-n}$

• **Step 2nd:** Then $\frac{dz}{dx} = (1-n)\, y^{-n} \frac{dy}{dx}$

$$\frac{dz}{dx} = (1-n)\, P(x) z + (1-n) Q(x)$$

The non linear ODE Now become linear ODE. It can be solved by formula.

• **Step 3rd:**

$n = -1$, z, y^2. Inverting z to get y.

$$\frac{dy}{dx} = \frac{y}{2x} + \frac{x^2}{2y}$$

$$\frac{dz}{dx} = \frac{1}{x} z + x^2$$

$$z = e^{\int \frac{1}{x} dx} \left(\int x^2 e^{-\int \frac{1}{x} dx} + c \right)$$

$$= cx + \frac{1}{2} x^2$$

Back substitution of $z = y^2$ of $z = y^2$

$$y^2 = cx + \frac{1}{2} x^3$$

Homogenous equation:

For equation of the following types, where all the coefficient are constant it can be evaluated according different.

⇒ Two Diamensional heat Conduction.

$$a^2 (U_{xx} + U_{yy}) u_t$$

Two diamensional seepage problem

$$(K_x U_{xx} + K_y U_{yy}) = 0$$