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Q#1 Consider the following vectors R^3.

$$V_1 = \begin{bmatrix} ID1 \\ ID2 \\ ID3 \end{bmatrix}, \quad V_2 = \begin{bmatrix} ID2 \\ ID3 \\ ID4 \end{bmatrix}, \quad V_3 = \begin{bmatrix} ID3 \\ ID4 \\ ID5 \end{bmatrix}$$

Solution:-

$$a_1V_1 + a_2V_2 + a_3V_3 = 0$$

$$a_1(6, 8, 4) + a_2(8, 4, 9) + a_3(4, 9, 1) = 0$$

$$(6a_1 + 8a_2 + 4a_3,\ 8a_1 + 4a_2 + 9a_3,\ 4a_1 + 9a_2 + a_3) = 0$$

$$6a_1 + 8a_2 + 4a_3 = 0$$

$$8a_1 + 4a_2 + 9a_3 = 0$$

$$4a_1 + 9a_2 + a_3 = 0$$

Let $A = \begin{bmatrix} 6 & 8 & 4 \\ 8 & 4 & 9 \\ 4 & 9 & 1 \end{bmatrix}$

$$|A| = 6\begin{vmatrix} 4 & 9 \\ 9 & 1 \end{vmatrix} - 8\begin{vmatrix} 8 & 9 \\ 4 & 1 \end{vmatrix} + 4\begin{vmatrix} 8 & 4 \\ 4 & 9 \end{vmatrix}$$

$$= 6(4-81) - 8(8-36) + 4(72-16)$$

$$= 6(-77) - 8(-28) + 4(56)$$

$$= -462 + 224 + 224$$

$$|A| \neq -14$$

The system is linearly independent.

Q#2
$$\begin{bmatrix} 450 & 400 \\ 250 & 350 \\ 150 & 150 \end{bmatrix} \begin{bmatrix} 1000 \\ 500 \end{bmatrix}$$

solution:-

(a)
$$= \begin{bmatrix} 450 \times 1000 + 400 \times 500 \\ 250 \times 1000 + 350 \times 500 \\ 150 \times 1000 + 150 \times 500 \end{bmatrix}$$

$$= \begin{bmatrix} 450000 + 200000 \\ 250000 + 175000 \\ 150000 + 75000 \end{bmatrix}$$

$$= \begin{bmatrix} 650000 \\ 425000 \\ 225000 \end{bmatrix}$$

Now Total of material = 650000

Total cost of labour = 425000

Total cost of over head = 225000

(b) linear Transformation

let $U(F)$ and $V(F)$ are vector spaces: A mapping F such that $f: U \rightarrow V$ is a linear transformation of U in to V

if (i) $f(x+y) = f(x) + f(y)$ } $x, y \in V$

(ii) $f(ax) = a f(x)$ } $a \in F$

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Properties of L-T $T: U \to F$

(i) $f(0) = 0'$, when 0 is identify

(ii) $f(-\alpha) = -f(\alpha)\ \forall\ \alpha \in U$

(iii) $f(\alpha - \beta) = f(\alpha) - f(\beta)\ \forall \alpha, \beta \in U$

examples:

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \to \begin{pmatrix} x-y \\ x+y \\ 2x \end{pmatrix}$$

$$T\left(\begin{bmatrix} 6 \\ 8 \end{bmatrix}\right) \to \begin{bmatrix} 6-8 \\ 6+8 \\ 2(6) \end{bmatrix} = \begin{bmatrix} -2 \\ 14 \\ 12 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 6 \\ 8 \end{bmatrix}\right) \to \begin{bmatrix} -2 \\ 14 \\ 12 \end{bmatrix} \qquad T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \to \begin{pmatrix} x-4 \\ x-4 \\ y \end{pmatrix}$$

e.g

$$T(U+V) = T(U) + T(V)$$

$$U = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad V = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$T\left(\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}\right) \quad T\left(\begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \end{bmatrix}\right)$$

$$= \begin{pmatrix} a_1 + b_1 - (a_2 + b_2) \\ a_1 + b_1 + (a_2 + b_2) \\ 2(a_1 + b_1) \end{pmatrix} = \begin{bmatrix} a_1 + b_1 - a_2 - b_2 \\ a_1 + b_1 + a_2 + b_2 \\ 2a_1 + 2b_1 \end{bmatrix}$$

example:

$$T(U+V) = T(U) + T(V)$$

$$U = \begin{bmatrix} 6 \\ 8 \end{bmatrix}, \quad V = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 6 \\ 8 \end{bmatrix} + \begin{bmatrix} 4 \\ 9 \end{bmatrix}\right) = T\begin{bmatrix} 6+4 \\ 8+9 \end{bmatrix}$$

$$= T\begin{pmatrix} 10 \\ 17 \end{pmatrix} = T\begin{pmatrix} 10-17 \\ 10+17 \\ 2(10) \end{pmatrix}$$

$$= T\begin{pmatrix} -7 \\ 27 \\ 20 \end{pmatrix}$$

2nd conditions:

$$T(CU) = CT(U)$$

let $U = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ $= T\left[C\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right] = T\begin{bmatrix} C \cdot a_1 \\ C \cdot a_2 \end{bmatrix}$

$$= \begin{bmatrix} ca_1 - ca_2 \\ ca_1 + ca_2 \\ 2ca_1 \end{bmatrix}$$

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Q#4 Determinants Let $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is a 2x2 matrix.

$$A = \begin{pmatrix} ID1 & ID1 & ID2 \\ ID2 & ID3 & ID2 \\ ID4 & ID1 & ID5 \end{pmatrix}$$

Solution:-

Part a The following matrix have inverse exist

$$M_1 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \quad M_2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Part b

$$M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Part c

$$M_1 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad M_3 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Part d

$$A = \begin{bmatrix} 6 & 6 & 6 \\ 8 & 4 & 9 \\ 9 & 6 & 0 \end{bmatrix}$$

$$|A| = 6 \begin{vmatrix} 4 & 8 \\ 6 & 0 \end{vmatrix} - 6 \begin{vmatrix} 8 & 8 \\ 9 & 0 \end{vmatrix} + 6 \begin{vmatrix} 8 & 4 \\ 9 & 6 \end{vmatrix}$$

$$= 6(0-48) - 6(0-72) + 6(48-36)$$

$$= -288 + 432 + 72 = \underline{216} \text{ Ans}$$

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Q3 ~~(a)~~ Part (a)

Let V be a set where elements are called vector and F be a field, where element are called scaler. The set V is called vector space or linear space or linear vector space are field F if $V(F)$ satisfy the following condition.

(i) $(V, +)$ is abelian group w.r.t internal composition +.

(ii) $V(F)$ is closed under scaler multiplication w.r.t "."

i.e $\forall a \in F, \forall \alpha \in V \Rightarrow a.\alpha = \alpha.a \in V$

(iii) $(V, +, \cdot)$ satisfy ① $a\alpha \in V, \forall a \in F \; \& \; \forall \alpha \in V$

(i) $a.(\alpha+\beta) = a\alpha + a\beta \; \forall a \in F \; \& \; \alpha, \beta \in V$

(ii) $(a+b)\alpha = a\alpha + b\alpha \; \forall a, b \in F \; \alpha \in V$

(iii) $\left.\begin{array}{l} a(b\alpha) = (ab)\alpha \\ 1 \cdot \alpha = \alpha \end{array}\right\} \begin{array}{l} \forall 1, a, b \in F \\ \alpha \in V. \end{array}$

(a) . is a vector space because it's satisfy all the condition of a vector space.

(b) It is not a vector space because its not satisfy the closer property of a vector space.