Assignment No 01

Submitted To Sir Engr Fawad Khan

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Subject Advanced Mechanics of Materials.

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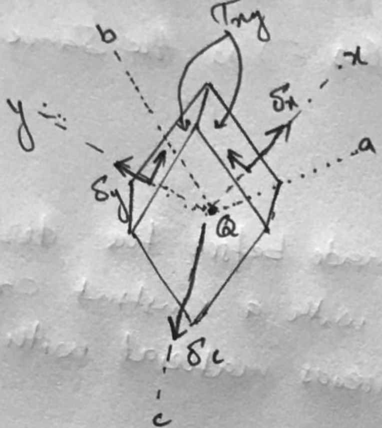
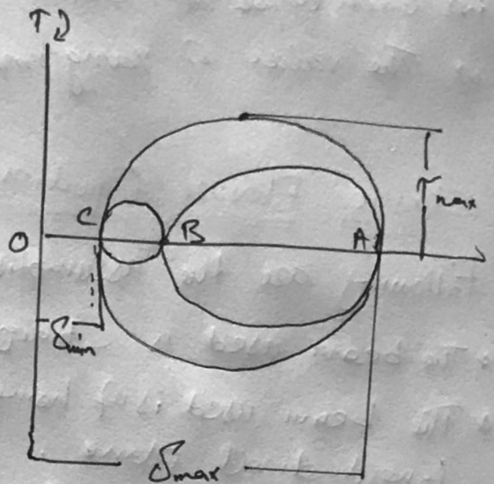
1) Application of Mohr's Circle to the three-Dimensional Analysis of stress.





* Transformation of stress for an element rotated around a Principle axis may be represented by Mohr's Circle
* Point A, B, & C represent the principle stresses on the principle planes (shearing Stress is zero
* The three Circles represent the normal & shearing stresses for rotation around each principle axis
* Radius of the Largest circle yields the max shearing stress

$$\tau_{max} = \tfrac{1}{2}\,|\delta_{max} - \delta_{min}|.$$

3) Simple Bending and Pure Bending

* When some external force act on a beam, the shear force & bending moments are set up at all the section of beam.
* Due to Shear force & Bending Moment, the beam undergoes deformation. The material of beam offer resistance to deformation.
* When a beam length is Subjected to a Constant amount of bending moment & a zero shear force. the stresses set up across the cross-section of the beam, due to bending is known as "Pure Bending Stress".

4) Assumptions in the Theory of PURE BENDING:

i) The material of the beam is Perfectly homogeneous & isotropic

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ii) The beam is stressed within its elastic limit & Hook's law (stress is proposanal to strain) is valid.

iii) The transverse Section, which were plane before bending remain plan after bending.

iv) Each layer of beam is free to expand or Contract, independently

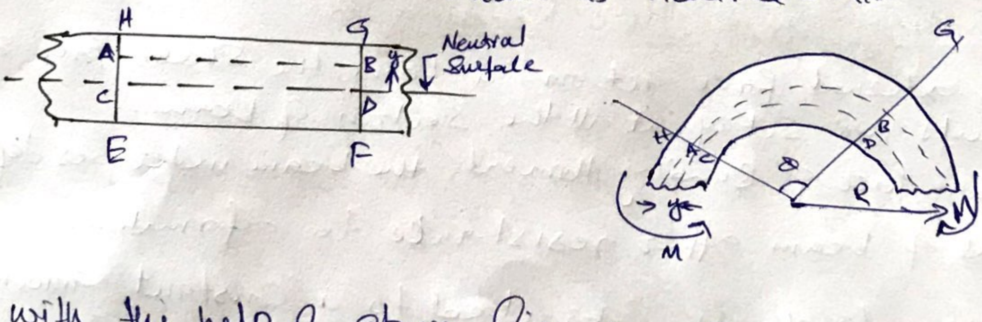
v) The value of Young's modulus (E) is the Same in tension & Compression

5) Classic Flexure Equation.

Following are the assumption made before the derivation of a bending equation

* The beam used is straight with a Constant cross-Section
* The beam used is of homogeneous material with a symmetrical longitudinal plane.
* The plan of Symmetry has all the resultant of applied loads.
* The primary Cause of failure is buckling.
* E remains Same for tension & Compression.
* Cross-Section remains the Same before & after bending.

Consider an unstressed beam, which is Subjected to a Constant bending moment such that the beam bends up to radius R. The top fibres are Subjected to tension whereas the bottom fibres are Subjected to Compression the locus of Point with zero Stress is known as neutral axis.



with the help of above fig the following are the Step involve in the derivation of flexure eq.

$$\text{Strain in fibre } AB = \frac{A'B' - AB}{AB}$$

$$= \frac{(R+y)\theta - R\theta}{R\theta} = \frac{R\theta + y\theta - R\theta}{R\theta} = y/R \quad \frac{\sigma}{E} = y/R$$

Where E is Young's Modulus of Elasticity.

or $\sigma/y = E/R$

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$\sigma = E/R\, y$ (eq.1)

$F = \sigma\,\delta A = E/R\, y\, \delta A$ (Force acting on the strip with area dA)

$Fy = E/R\, y^2 \delta A$ (Momentum about Neutral axis).

$M = \Sigma\, E/R\, y^2$ (total momentum for entire cross-sectional area)

$\delta A = E/R \Sigma y^2 \delta A$ $\Sigma y^2 \delta A$ is known as second moment of area & is represented as I.

$\therefore M = E/R\, I$ (eq 2)

From Eq 1 & 2

$\sigma/y = M/I = E/R$

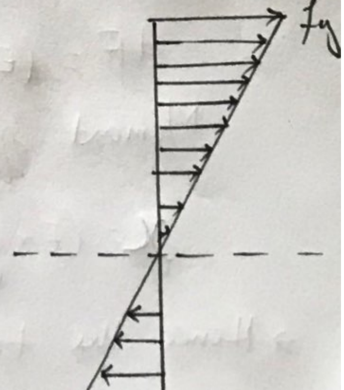
Therefore the above is the flexure theory Equation

6) __Section Modulus:__

Section modulus is a geometric property of the cross section used for designing beam & flexural members. it does not represent anything physically

To defin Section Modulus it may be defined as the ratio of total moment resisted by the Section to the stress in the extreme fibre which is equal to yield stress.

In elastic bending, the stress distribution in the beam is as follow



Elastic linear Stress distribution.

Fig for Question No 6

7) Application of Bending Equation in any Object.

When you try to break a wood with your foot the breaking occurs due to bending moment. You push the wood from middle point & apply a force perpendicular to allignment of the wood.

Slowly applying the force initially the wood bends. when it bends, wood is subjected to bending moment due to its inner forces which happen to be compression (closer to your foot), Tension (further to your foots) in this case. The forces are aligned with the longitudinal axis of the wood. They create the moment in the following way.

Force x radius = Bending Moment.

when the bending moment on the part subjected to tension exceeds the stress of the material; breaking occurs.

8) Moment of Resistance.

→ Due to the tensile & Compressive stresses, forces are exerted on the layers of a beam subjected to simple bending.

→ These forces will have moment about the neutral axis. The total moment of these forces about the neutral axis is known as moment of resistance of that section.

→ we have seen that force on layer of cross sectional area dA at a distance y from the neutral axis

$dF = (E \times y \times dA)R$

Moment of force dF about the neutral axis = dF_x

$= (E \times y \times dA)R \times y = E/R \times (y^2 dA)$.

→ Hence the total moment of force about the neutral axis = Bending moment Applied $= \int E/R \times (y^2 dA) = E/R \times I_{xx}$; I_{xx} is the moment of area about the neutral axis/centroidal axis.

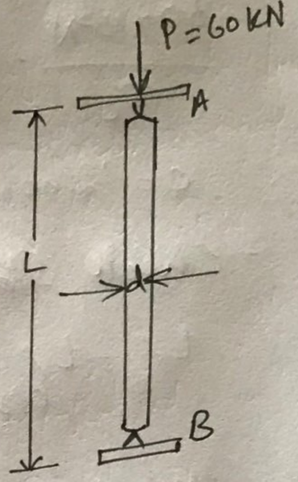
Hence $M/I_{xx} = E/R = \sigma_b/y$: σ_b is also known as flexural stress (F_b)

→ Hence $M/I_{xx} = E/R = F_b/y$

→ The above equation is known as bending equation.

9) Design of Column under Centric load

Using the aluminum alloy 2014-T6, determine the smallest diameter rod which can be used to support the centric load $P = 60\,kN$ if a) $L = 750\,mm$ b) $L = 300\,mm$



Solution

→ With the diameter unknown, the slenderness ratio can not be evaluated. Must make an assumption on which slenderness ratio regime to utilize.

→ Calculate required diameter for assumed slenderness ratio regime.

→ Evaluate slenderness ratio and verify initial assumption.

Repeat if necessary

→ For $L = 750\,mm$, assume $L/r > 55$

c = cylinder radius

r = radius of gyration

- Determine cylinder radium

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{\pi c^4/4}{\pi c^2}} = c/2$$

$$\sigma_{all} = P/A = \frac{372 \times 10^3\,MPa}{(L/r)^2}$$

$$\frac{60 \times 10^3\,N}{\pi c^2} = \frac{372 \times 10^3\,MPa}{\left(\frac{0.750\,m}{c/2}\right)^2} \qquad c = 18.44\,mm$$

- Check slenderness ratio assumption.

$$L/r = L/c/2 = \frac{750\,mm}{(18.44\,mm)} = 81.3 > 55$$

assumption was correct.

$$\boxed{d = 2c = 36.9\,mm}$$

- For $L = 300mm$, assume $L/r < 55$
- Determin cylinder radius

$$\sigma_{all} = P/A = \left[212 - 1.585\left(\frac{L}{r}\right)\right] MPa$$

$$\frac{60\times10^3 N}{\pi c^2} = \left[212 - 1.585\left(\frac{0.3m}{c/2}\right)\right]\times10^6 Pa$$

$c = 12.00mm$

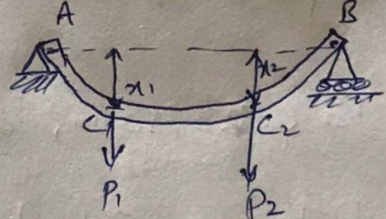
- Check Slenderness ratio assumption.

$$L/r = L/_{c/2} = \frac{300mm}{(12.00mm)} = 50 < 55$$

assumption was Correct.

$$\boxed{d = 2c = 24.0mm}$$

10) Deflection by Castigliano's Theorem.

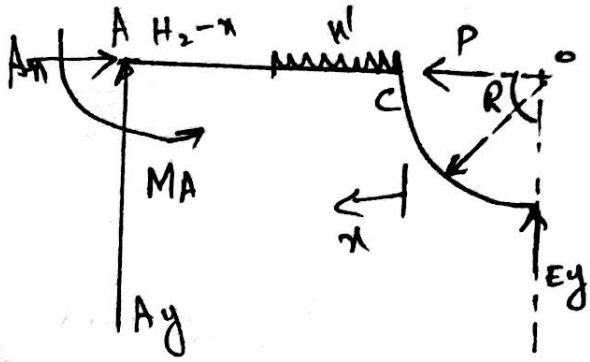


- Application of Castigliano's theorem is simplified if the differentiation with respect to the load P_j is performed before the integration or summation to obtain the strain energy U.
  - In the Case of beam.

$$U = \int_0^L \frac{M^2}{2EI} dx \qquad x_j = \frac{\partial U}{\partial P_j} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial P_j} dx$$

Example: The given beam Consist of Straight beam AC where A is fixed. & Curved beam CE load P is horizontal. Draw free body diagram & moment equation for AC & CE part, express reduntand at E by using Castigliano's theorem. for deflection.

Sol 1. ⓐ FBD



Ay and Ey are vertical reaction

A x-horizontal & M_A moment of A

ⓑ In EC,

$$M_{EC} = E_y = R.\cos\theta \,.\ 0 \le \theta \le \pi/2$$

at E,

In AC,

$$M_{CA} = E_y \times (R+x) - \frac{wx^2}{2} \quad \text{for } x \le L/2$$

$$M_{CA} = E_y(R+x) - \frac{wL}{2} \times (x - L/4) \text{ for } L/2 \le x \le L$$

ⓒ Considering Ey as redundant, Strain energy stored in beam

$$U = U_{EC} + U_{CA} = \int_E^A \frac{\left(E_y(R+x) - \frac{wL}{2}(x - L/4)\right)^2}{2EI} dx + \int_0^{L/2} \frac{\left(E_y(R+x) - \frac{wx^2}{2}\right)^2}{2EI} dx$$

$$+ \int_{\pi/2}^{0} \frac{E_y^2 \cdot R^2 \cos\theta}{2EI} \cdot R d\theta = \int_E^A \frac{M^2}{2EI} ds$$

:- Note : only Bending Energy Considered.

once we find U after integration use the Compatibility eq to find support/redundant at E

Def at E, $\delta_E = \frac{dU}{dE_0}$; But $\delta_E = 0$; so

Put $\frac{\partial U}{\partial E_y} = 0$ to find Ey