Quiz No # 1

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Subject :- Applied Calculus

Q) Find

$$\int_0^1 \frac{4t^3 - 2t^2 + 3t - 1}{2t^2 + 1}\, dt$$

* Since degree of Numerator > Degree of Denominator so we divide Divided by Divisor

$$\begin{array}{r|l} & 2t - 1 \\ \hline 2t^2+1 & 4t^3 - 2t^2 + 3t - 1 \\ & \cancel{\pm 4t^3} \qquad \pm 2t \\ \hline & -\cancel{2t^2} + t - 1 \\ & \mp \cancel{2t^2} \qquad -1 \\ \hline & t \end{array}$$

→ Remender [R(x)]

$$\left[Q(x) + \frac{R(x)}{D(x)}\right]$$

$$\frac{4t^3 - 2t^2 + 3t - 1}{2t^2 + 1} = 2t - 1 + \frac{t}{2t^2 + 1}$$

$$y = \int_0^1 \frac{4t^3 - 2t^2 + 3t - 1}{2t^2 + 1}\, dt$$

$$y = \int_0^1 \left(2t - 1 + \frac{t}{2t^2+1}\right) dt$$

$$y = \int_0^1 2t\, dt - \int_0^1 1\, dt + \int_0^1 \frac{t}{2t^2+1}\, dt$$

$$y = 2\int_0^1 t^1 dt - \int_0^1 1\, dt + P \quad \text{— eq } 1$$

$$P = \int \frac{t}{2t^2+1}\, dt \quad \text{— eq } 2$$

Using Substitution Method

let

$$2t^2 + 1 = x \quad \text{— (i)}$$

$$d(2t^2 + 1) = d(x)$$

$$2\, d(t^2) + d(1) = dx$$

$$2 \cdot 2t\, dt + 0 = dx$$

$$4t\,dt = dx$$

$$t\,dt = \frac{dx}{4} \longrightarrow (ii)$$

Putting $2t^2 + 1 = x$ & $t\,dt = \frac{dx}{4}$

in eq (2)

R.W

$$\frac{d}{dx}(\ln x) = 1/x$$

$$\ln x = 1/x \; \frac{dx}{d}$$

$$\ln x = \frac{1}{d}\,\frac{1}{x}\,dx$$

$$\ln x = \int \frac{1}{x}\,dx$$

$$P = \int \frac{dx/4}{x}$$

$$P = \frac{1}{4} \int \frac{1\,dx}{x}$$

$$P = \frac{1}{4} \ln x + c$$

Put $x = 2t^2 + 1$

$$P = \frac{1}{4} \ln|2t^2+1| + c$$

$$P = \int \frac{t}{2t^2+1}\,dt = \frac{1}{4} \ln|2t+1) + c$$

$$P = \int_0^1 \frac{t}{2t^2+1}\,dt = \frac{1}{4} \ln|2t^2+1|$$

$$y = 2\int_0^1 t\,dt - \int_0^1 1\,dt + \int \frac{t}{2t^2+1}\,dt \rightarrow eq\ 1$$

$$y = 2\left|\frac{t^{1+1}}{1+1}\right|_0^1 - |t|_0^1 + \frac{1}{4}\left|\ln|2t^2+1|\right|_0^1$$

$$y = \cancel{2}\left|\frac{t^2}{\cancel{2}}\right|_0^1 - [(1)-(0)] + \frac{1}{4}\{\ln[2(1)^2+1] -$$

$$\ln[2(0)^2+1]\}$$

$$y = [(1)^2 - (0)^2] - [1-0] + \frac{1}{4}\{\ln[2+1] - \ln[0+1]\}$$

$$y = [1-0] - [1] + \frac{1}{4}[\ln 3 - \ln 1]$$

$$y = [1] - [1] + \frac{1}{4}[\ln 3 - 0]$$

$$y = \cancel{1} - \cancel{1} + \frac{1}{4}\ln 3.$$

$$y = \frac{1}{4}\ln 3 \quad Ans.$$

Q Find

$$\int_2^3 t \sin t^2 \, dt$$

Sol:-

let $y = \int_2^3 t \sin t^2 \, dt$ — eq ①

$$\frac{d}{dt}(\cos t^2) = -\sin t^2 \cdot \frac{d}{dt}(t^2)$$

$$\frac{d}{dt}(\cos t^2) = -\sin t^2 \cdot 2 t^{2-1} \frac{dt}{dt}$$

$$\frac{d}{dt}(\cos t^2) = -2t \cdot \sin t^2 \frac{dt}{d}$$

$$\frac{\cos t^2}{-2} = \frac{1}{d} t \sin t^2 \cdot dt$$

$$-\frac{\cos t^2}{2} = \int t \sin t^2 \, dt$$

From eq ①

$$y = \int_2^3 t \sin t^2 \, dt$$

0.00 (6) - 0.

- 0.

$$y = \left| -\frac{\cos t^2}{2} \right|_2^3$$

$$y = -\frac{1}{2} \left| \cos t^2 \right|_2^3$$

$$y = -\frac{1}{2} \left[\cos 3^2 - \cos 2^2 \right]$$

$$y = -\frac{1}{2} \left[\cos 9 - \cos 4 \right]$$

$$y = 0.0049$$