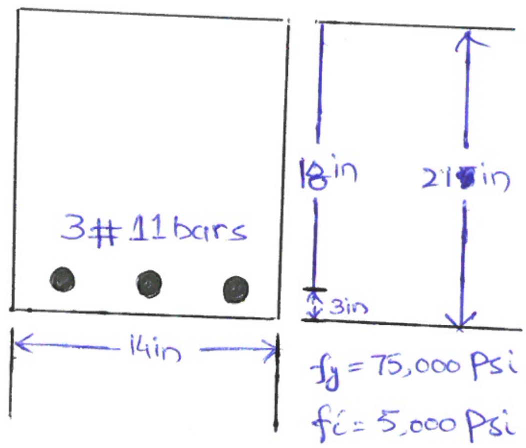
Midterm Paper

Submitted by: I.D #15345 Module: 4th Semester

Submitted to: Engr. Fawad Ahmad

Subject : RCD

Q1 (A) Given Data :



$f_y = 75{,}000$ Psi
$f'_c = 5{,}000$ Psi

Required : Values of i) ϵ_t ii) ϕ iii) ϕM_n

Also discuss strength analysis. Does the reinforcement is done according to design standard or not. Defend your design analysis.

Solution : i) $\epsilon_t = ?$

$$\epsilon_t = \frac{d-c}{c}(0.003) \longrightarrow (*)$$

$$a = \frac{A_s f_y}{0.85 f'_c b}$$

From Table A.2
∴ $A_s = 1.56 \times 3 = 4.68 \text{ in}^2$
or From Table A.4
$A_s = 4.68 \text{in}^2$

$$a = \frac{4.68 \times 75}{0.85 \times 5 \times 14}$$

$$a = 5.899 \text{ in}$$

$$c = \frac{a}{\beta} = \frac{5.899\text{in}}{0.85} = 6.940\text{in}$$

$$\text{Eq } (*) \Rightarrow \epsilon_t = \frac{18 - 6.940}{6.94}(0.003) \Rightarrow \boxed{\epsilon_t = 0.00478}$$

$\epsilon_t = 0.00478$

$\epsilon_t > 0.004$

$\epsilon_t < 0.005$

Hence Beam is in transition zone -

ii) $\phi = ?$

$$\phi = 0.65 + (\epsilon_t - 0.002)\frac{250}{3}$$

$$\phi = 0.65 + (0.00478 - 0.002)\frac{250}{3}$$

$$\boxed{\phi = 0.881}$$

iii) $\phi M_n = ?$

$$M_n = A_s * f_y\,(d - a/2)$$

$$M_n = 4.68 * 75\left(18 - \frac{5.899}{2}\right)$$

$$M_n = 5282.72 \text{ in-K.}$$

$$M_n = 5282.72 \cancel{\text{in}}\text{-K} * \frac{1\text{ft}}{12\cancel{\text{in}}}$$

$$M_n = 440.227 \text{ ft-K}$$

Now,

$$\phi M_n = 0.881 * (440.227)$$

$$\boxed{\phi M_n = 387.84 \text{ ft-K}}$$

Checking Solution :

$$\rho_s = \frac{A_s}{bd} = \frac{4.68}{14 * 18} = 0.01857$$

$$\rho_{min} = \frac{3\sqrt{f_c'}}{f_y} \geq \frac{200}{f_y}$$

$$= \frac{3\sqrt{5000}}{75000} \geq \frac{200}{75000}$$

$$\rho_{min} = 0.00282 \geq 0.00266$$

so $\boxed{\rho_{min} = 0.00282}$

Now,

$$\rho_{max} = 0.85\,\beta_1 * \frac{f_c'}{f_y} * \frac{\epsilon_u}{\epsilon_u + 0.004}$$

For ρ_{max}, $\epsilon_u = 0.003$.

$$\rho_{max} = 0.85 * 0.85 * \frac{5}{75} * \frac{0.003}{0.003 + 0.004}$$

$$\boxed{\rho_{max} = 0.0206}$$

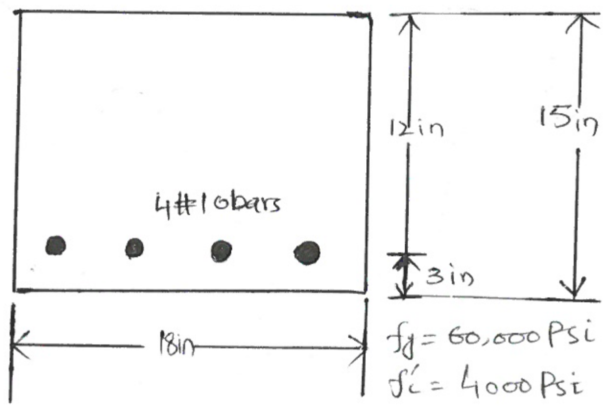
So,

$$\rho_s = 0.01857 > \rho_{min} = 0.00282.$$

$$< \rho_{max} = 0.0206$$

$\rho_{max} > \rho_s > \rho_{min}$ $\Longrightarrow$ The design reinforcement is $\boxed{\text{OK}}$

Q1 **Given Data:**



Required: The values of i) ϵ_t, ii) ϕ, iii) ϕM_n

Solution: 1) $\epsilon_t = ?$

$\boxed{\because A_s = 5.06}$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{5.06 * 60}{0.85 * 4 * 18} = 4.96in$$

$$c = \frac{a}{\beta} = \frac{4.96in}{0.85} = 5.835\ in$$

$$\epsilon_t = \frac{(d-c)}{c} * (0.003)$$

$$\epsilon_t = \frac{(12 - 5.835)}{5.835} * (0.003) = 0.00316$$

$\epsilon_t = 0.00316 < 0.004$

Section is not ductile and may not be used as per ACI Section 10.3.5

2)

$$\phi = 0.65(\epsilon_t - 0.002)\frac{250}{3}$$

$$\phi = 0.65(0.00316 - 0.002)\frac{250}{3}$$

$$\boxed{\phi = 0.746}$$

3. $\phi M_n = ?$

$$M_n = A_s f_y (d - a/2)$$

$$M_n = 5.06 * 60 \left(12 - \frac{4.96}{2}\right)$$

$$M_n = 2890.27 \text{ in-K}$$

$$M_n = \frac{2890.27}{12} = 120.428 \text{ ft-K}$$

Now,

$$\phi M_n = 0.746 * 120.428$$

$$\boxed{\phi M_n = 89.11 \text{ ft-K}}$$

Q.No. 1 (b): Design Doubly reinforced beam

Given Data:

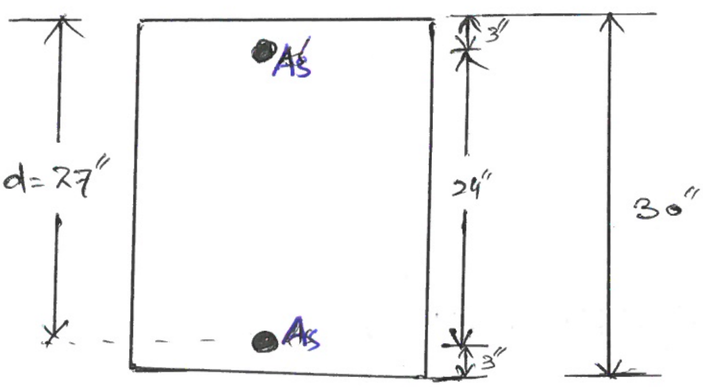
$M_D = 153$ ft-K

$M_L = 410$ ft-K

$f_c' = 4000$ Psi

$f_y = 60,000$ Psi

Diagram:



Solution:

① **Factored moment**

$$M_u = 1.2M_D + 1.6M_L$$

$$= 1.2(153) + 1.6(410)$$

$$\boxed{M_u = 839.6 \simeq 840 \text{ ft-K}}$$

② Nominal moment $(M_n) = ?$

$$M_n = \frac{M_u}{\phi} = \frac{840}{0.90} \qquad \because \phi = 0.90$$

$$\boxed{M_n = 933.33 \text{ ft-K}}$$

Assuming max. possible tensile steel with no compression steel and computing beams nominal strength moment.

ρ_{max} (From Appendix A. table A.7)

$$\rho_{max} = 0.0181$$

$$A_{s_1} = \rho_{max}\, bd$$

$$= 0.0181 * 14 * 27$$

$$= 6.842 \text{ in}^2$$

For $\rho_{max} = 0.0181$, $\frac{M_u}{\phi b d^2} = 912$ psi

$$M_{u_1} = 912 * \phi b d^2$$

$$= 912 * 0.9 * 14 * (27)^2$$

$$= \frac{8377084.8}{12 \text{ in}} \text{ in-lb}$$

$$= 698090 \text{ ft-lb}$$

$$\boxed{M_{u_1} = 698 \text{ ft-k}}$$

$$\boxed{M_{n_1} = \frac{M_{u_1}}{\phi} = \frac{698}{0.9} = 775.55 \text{ ft-k}}$$

$$M_{n_2} = M_n - M_{n_1} = 933.33 - 698 = 235.33 \text{ ft-k}$$

$$\boxed{M_{n_2} = 235.33 \text{ ft-k}}$$

③ Theoritical A_s' required:

$$A_s' = \frac{M_{n_2}}{f_y(d-d')} = \frac{235.33 \times 12}{60(27-3)}$$

$$A_s' = 1.96 \simeq 2\ in^2$$

Try 2#9 (2 in²)

$$A_s' f_s' = A_{s_2} f_y$$

$$A_{s_2} = \frac{A_s f_y}{f_s'} = \frac{2 \times \cancel{60}}{\cancel{60}} = 2\ in^2$$

$$A_s = A_{s_1} + A_{s_2}$$

$$A_s = 6.842 + 2$$

$$\boxed{A_s = 8.842\ in^2}$$

Try 8#10 ($A_s = 10.12\ in^2$)

Note: The actual value of A_s' is exactly the same as the theoritical value.

The actual value of A_s is however is higher than the theoritical value by $10.12 - 96 = 0.52\ in^2$.

If new bar selection for A_s' is made where by the actual value of A_s' exceeds the theoritical value by about this much ($0.52\ in^2$), the design will be adequate.

Select 3#8 ($A_s' = 2.36\ in^2$) & repeat the previous steps.

Now,

Assuming $f_s' = f_y$

① $c = \dfrac{(A_s - A_s')f_y}{0.85 f_c' b \beta_1}$

$c = \dfrac{(10.12 - 2.36) * 60}{0.85 * 4 \times 14 * 0.85} = 11.5 \text{ in}$

② $\epsilon_s' = \dfrac{(c - d')}{c} * (0.003)$

$\epsilon_s' = \dfrac{(11.5 - 3)}{11.5} * (0.003) = 0.00217 > \epsilon_y.$

③ $\epsilon_t = \dfrac{(d - c)}{c} * (0.003)$

$= \dfrac{(27 - 11.5)}{11.5} * (0.003)$

$\epsilon_t = 0.00404 < 0.005$

so $\phi \neq 0.9$

$\phi = 0.65 + (\epsilon_t - 0.002) * \dfrac{250}{3}$

$\phi = 0.65 + (0.00404 - 0.002) * \dfrac{250}{3}$

$\boxed{\phi = 0.82}$

$A_{s_2} = \dfrac{A_s' f_s'}{f_y} = \dfrac{2.36 * \cancel{60}}{\cancel{60}} = 2.36 \text{ in}^2$

$A_{s_1} = A_s - A_{s_2} = 10.12 - 2.36 = 7.76 \text{ in}^2$

$M_{n_1} = A_{s_1} f_y (d - a/2) = 7.76 * 60\left(27 - \dfrac{0.85 * 10.74}{2}\right)$

$\boxed{M_{n_1} = \dfrac{10912.5 \text{ in-K}}{12} = 909.2 \text{ ft-K}}$

$$M_{n_2} = A_s f_y (d - d')$$

$$= 2.36 * 60 * (27 - 3)$$

$$M_{n_2} = 3398 \text{ in-K}$$

$$= \frac{3398}{12 \text{ in}} \cancel{\text{in}}\text{-K}$$

$$\boxed{M_{n_2} = 283 \text{ ft-K}}$$

$$M_n = M_{n_1} + M_{n_2}$$

$$M_n = 909 + 283 = 1192 \text{ ft-K}$$

$$\boxed{M_n = 1192 \text{ ft-K}}$$

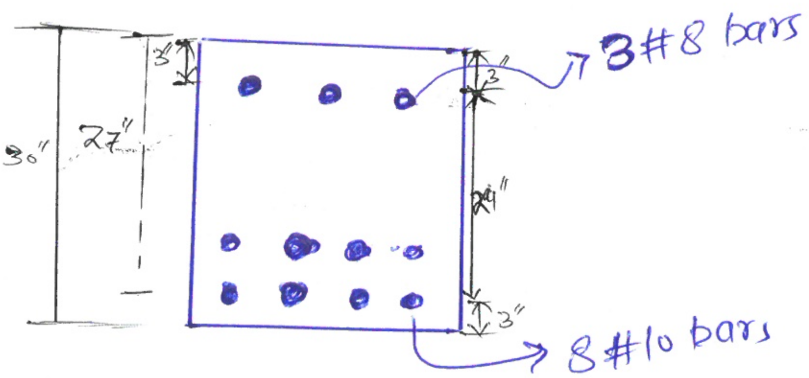
$$\phi M_n = 0.82 * 1192$$

$$\boxed{\phi M_n = 977 \text{ ft-K}} > M_u$$

$\boxed{\text{OK}}$

$A_s' = 2.36 \text{ in}^2$ (3#8 bars)

$A_s = 10.12 \text{ in}^2$ (8#10 bars)



Q No. 2

Solution :

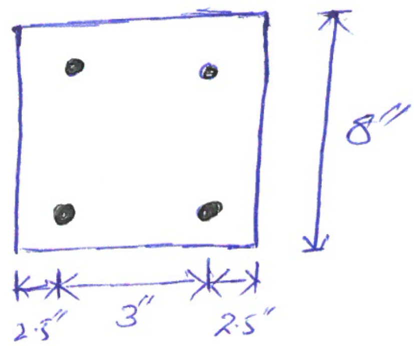
Given DATA : $P_u = 153K$, $M_u = 15\ ft\text{-}K$, $f'_c = 4000psi$, $f_y = 60{,}000psi$

Assume the column will have an average Compressive stress = about $0.6f'_c = 0.6 \times 4000 = 2400\ psi = 2.4Ksi$

$$A_g = \frac{P_u}{\text{Avg. compressive stress}}$$

$$A_g = \frac{153\ K}{2.4\ ksi} = 63.75\ in^2$$

Try $8in \times 8in$ Column ($A_g = 64in^2$) with bar arrangment



$$e = M_u / P_u$$

$$e = \frac{15\ ft\text{-}K}{153\text{-}K}$$

$$\boxed{e = 1.17\ in}$$

$$\boxed{P_n = P_u/\phi = \frac{153K}{0.65} = 235.38\ K}$$

$$K_n = \frac{P_n}{f'_c\ A_g}$$

$$K_n = \frac{235.38K}{(14Ksi)(8'' \times 8'')} = 0.919$$

$$\boxed{K_n = 0.919}$$

$$R_n = \frac{P_n e}{f'_c \cdot A_g \cdot h} = \frac{(235.38\,k)(1.17\,in)}{(14\,ksi)(8'' \times 8'')(8'')}$$

$$\boxed{R_n = 0.1344}$$

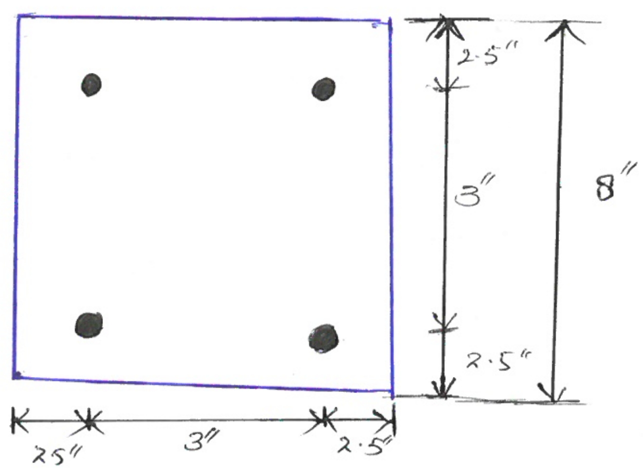
$$\boxed{\gamma = \frac{3''}{8''} = 0.375}$$

Interpolating b/w values given in Graphs "6" and "7" of appendix A.

$$A_s = (0.0123) * (8'' * 8'')$$

$$A_s = 0.78\,in^2$$

use $4\#4 = 0.78\,in^2$



Q No 3

Given Data :

$P_D = 153K$

$P_L = 160K$

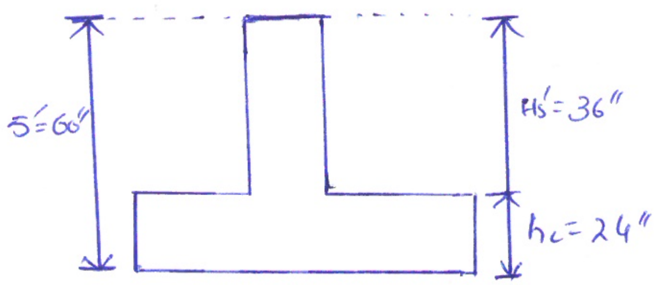
$\gamma_s = 100\ lb/ft^3$

$f_y = 60{,}000$

$f_c = 3000psi$

$q_a = 1534\ Psf$

Column is reinforced by #8 bars



Assume data :

Unit weight of concrete $= \gamma_c = 150\ lb/ft^3$

$h_c = 24''$

$d = 19.5''$

$H_s = 36''$

Solution :

Step #01 : Effective soil pressure "q_e"

$$q_e = q_a - (h_c * \gamma_c) - (H_s' * \gamma_s)$$

$$= 1534 - (\frac{24}{12} * 150) - (\frac{36}{12} * 100)$$

$$q_e = 934\ Psf$$

$$\boxed{q_e = 0.934\ Ksf}$$

Step # 02 : Area of footing.

$$\text{Area of footing} = \frac{P_D + P_L}{0.934}$$

$$= 335\ ft^2$$

Use $\boxed{18.5' \times 18.5' = 342\ ft^2}$ → Area of footing

Step # 03 :- ultimate bearing capacity. (q_u)

$$q_u = \frac{1.2P_D + 1.6P_L}{\text{Area of footing}}$$

$$q_u = \frac{(1.2 * 153) + (1.6 * 160)}{342}$$

$$\boxed{q_u = 1.28\ ksf}$$

Step # 04 :-

Depth require for two way or punching shear

The "d" required for two way is,

The largest value obtained from the following expression

(i) $$d = \frac{V_{u_2}}{\phi 4 \sqrt{f_c'}\ b_o} \longrightarrow (*)$$

(ii) $$d = \frac{V_{u_2}}{\phi \left(\frac{\alpha_s d}{b_o} + 2\right) \sqrt{f_c'}\ b_o} \longrightarrow (\#)$$

b_o = Perimeter around the punching area

$b_o = 4(a + d)$

$b_o = 4(16+19.5)$

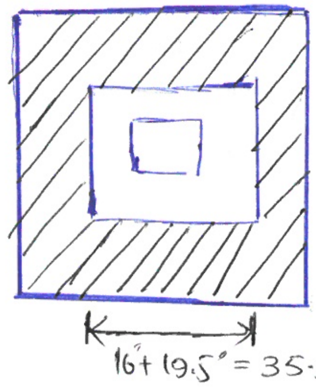
$\boxed{b_o = 142 \text{ in}}$

$V_{u_2} = \{A - (a+d)\} * q_u$

$V_{u_2} = \{335 - \frac{(16+19.5)}{12}\} * (1.28)$

$V_{u_2} = 433.973 K$

$\boxed{V_{u_2} = 433973 \text{ lb}}$



Two way shear

i) Eq (*) $\Rightarrow$ $d = \frac{433973}{0.75 * 4\sqrt{3000} * 142} = 18.59'' < 19.5'' \Rightarrow \boxed{OK}$

ii) Eq (#) $\Rightarrow$ $d = \frac{433973}{0.75 \times \left(\frac{40 * 19.5}{142} + 2\right)\sqrt{3000} * 142}$

$\because \alpha_s = 40$ for column, where perimeter is four sided (square column)

$d = 9.928'' < 19.5'' \Rightarrow \boxed{OK}$

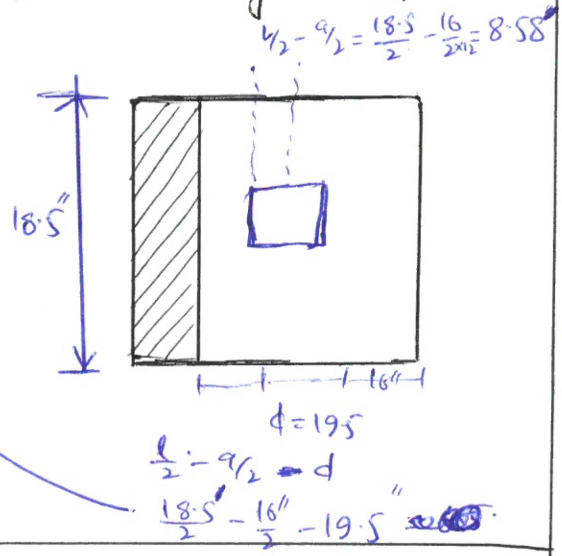
So both value of d are less than the assumed value of 19.5" so punching shear is $\boxed{OK}$

Step # 05 : Depth required for one way shear.

$= 9.25 - \frac{8}{12} - \frac{19.5}{12}$

$= 9.25' - 0.667' - 1.625'$

$= 6.958'$



$V_{u_1} = (185 * 6.958) * q_u$

$V_{u_1} = (18.5 * 6.958) * 1.28k$

$V_{u_1} = 164.576\ k$

$\boxed{V_{u_1} = 164576\ lb}$

$d = \dfrac{V_{u_1}}{\phi 2 \sqrt{f_c'}\ b_w}$

$= \dfrac{164576}{0.75 * 2\sqrt{3000} * (18.5 * 12)}$

$d = 9.01'' < 19.5'' \Rightarrow \boxed{ok}$

Use $h = 24''$ in total depth

Moment:

$M_u = 8.58 * 18.5 * 1.28 * \dfrac{8.58}{2}$

$\boxed{M_u = 871\ ft\text{-}k}$

$\dfrac{M_u}{\phi b d^2} = \dfrac{871 * 1000 * 12}{0.9 * (18.5 * 12)(19.5)^2} = 137.5\ Psi$

Use Appendix A. Table (A-12)

$\dfrac{M_u}{\phi b d^2} = 139.9$

$\rho = 0.0024$

$< \rho_{min}$ for flexure.

Then use greater of

(1) $\frac{153}{60000} = 0.00255$

(2) $\frac{3\sqrt{3000}}{60000} = 0.00273$ so, $\rho = 0.00273$

Area of Steel:

$A_s = \rho bd = 0.00273 * (18.5*12) * (19.5)$

$\boxed{A_s = 11.81 in^2}$

Use Table A-4

8#11 bars in both direction

(As selected = $12.5 in^2$)

Development length:

$\psi_t = \psi_e = \psi_s = \lambda = 1$

$\frac{l_d}{d_b} = \frac{3}{40} \frac{f_y}{\lambda\sqrt{f_c}} \frac{\psi_t \psi_e \psi_s}{c_b/d_b} \longrightarrow (1)$

If $\frac{c_b}{d_b} > 2.5$ then use 2.5

c_b = side cover = 3.5"

d_b = dia of bar = $\frac{8}{8} = 1"$

$c_b/d_b = \frac{3.5}{1} = 3.5$ so use 2.5

using equation (1)

$\frac{l_b}{d_b} = \frac{3}{40} * \frac{60000}{1\sqrt{3000}} * \frac{1*1*1}{2.5} = 32.86$

$$\frac{l_b}{d_b} * \frac{A_{s\,required}}{A_{s\,selected}} = 32.86 * \frac{11.81}{12.5} = 31.04$$

$$\Rightarrow \frac{l_b}{d_b} = 31.04$$

$$\Rightarrow l_b = 31.04 * l_b$$

$$\Rightarrow l_b = 31.04 * 1$$

$$\Rightarrow l_b = 31.04 \simeq 31''$$

$$\boxed{l_b = 31''} \Rightarrow$$

So Development length is 31''

The End