Usama Khurshid

7983

Section B

Differential Equation.

Q.no 1:

Solve the following objective type questions.

1)- Order of AB is m x n.

2)- Rank of matrix.

3)- a = 8

4)- $-2i^2 \times i^2 = -3(+1) = 3$

5)- Scalar matrix.

6)- $dy/dx + 2xy$: $\log y = x - x^2 + C$ / $y = e^{x-x^2} + C$

7)- its order is 1 and degree is 3.

8)- order = 2, degree is undefined.

9)- It's a Homogenius equation.

10)- $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

Scanned by TapScanner

Q no 3:

(iii)-

$$\begin{bmatrix} 3 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 3 \end{bmatrix}.$$

Solution:

$$|A - \lambda I| = 0$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} 3 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = 0$$

Taking Determinant.

$$\begin{bmatrix} 3 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}$$

Scanned by TapScanner

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

expanding by column 1.

$$\rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$\rightarrow \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} = 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1\begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}.$$

$$= (3-\lambda)\left[((3-\lambda)(2-\lambda) - (-1)(-1)) + 1((-1)(2-\lambda) - (-1)(-1)) - 1((-1)(-1) - (-1)(3-\lambda))\right]$$

$$= (3-\lambda)(6 - 3\lambda - 2\lambda + \lambda^2 - 1) + (-2 + \lambda - 1) - (1 + 3 - \lambda)$$

$$= (3-\lambda)(\lambda^2 - 5\lambda + 5) + (3+\lambda) - (4-\lambda)$$

$$= 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda.$$

$$= -\lambda^3 + 8\lambda^2 - 18\lambda + 8.$$

Scanned by TapScanner

$$\rightarrow 1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

expanding by column 1.

$$-1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$= -1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$$

$$= \lambda^2+5\lambda-5-3+\lambda.$$

$$= -\lambda^2+6\lambda-8$$

$$\rightarrow -1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-\left[(-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$= -\left[-1(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right]$$

$$= (3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

Scanned by TapScanner

Now:

$$=(-2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+17\lambda^2-8\lambda-\lambda^2+6\lambda$$
$$-8-\lambda^2+16\lambda-8).$$

$$=\lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2-36\lambda-8\lambda+6\lambda+6\lambda+16-16$$

$$=\lambda^4+10\lambda^3+32\lambda^2-32\lambda=0$$

we get:

$$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$$

$$(\lambda=0)$$

$$\lambda-2=0\ ,\ \lambda=2$$

$$\lambda^2-8\lambda+16=0$$

Factorizing.

$$\lambda^2-4\lambda-4\lambda+16=0$$

$$\lambda(\lambda-4)\ -4(\lambda-4)=0$$

$$\Rightarrow(\lambda-4)(\lambda-4)=0.$$

Scanned by TapScanner

Qno3:

The rate of change in the form of differential equation.

Given:

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$x = 2 \quad , \quad y = 6$$

Solution:

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$(x^2+3y^2)dx = 2xy\,dy.$$

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^2}{2xy} + \frac{3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right]$$

$$v = y/x \quad , \quad y = vx.$$

As:

$$dy = v\,dx + x\,dv$$

Scanned by TapScanner

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = 1/g\left[\frac{x}{xv} + 3\frac{vx}{x}\right]$$

$$v + \frac{xdv}{dx} = \frac{1}{g}\left[\frac{1}{v} + 3v\right]$$

$$gv + gx\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$gx\frac{dv}{dx} = \frac{1}{v} + 3v - gv$$

$$gx\frac{dv}{dx} = \frac{1}{v} + v$$

$$gx\frac{dv}{dx} = \frac{1 + v^2}{v}$$

If we multiply b/s by dx/dv

we will get:

Scanned by TapScanner

$$\delta x \delta v = \frac{1+v^2}{v} \, dx$$

now xing both l s by $\frac{v}{x(1+v^2)}$

$$\frac{v}{1+v^2} \, dv = \frac{1}{8} \, \delta x.$$

Integration.

$$\int \frac{gv}{1+v^2} \, \delta v = \int \frac{1}{x} \, \delta x + C$$

$$\ln|1+v^2| = \ln x + \ln C$$

Taking "e".

$$e^{\ln|1+v^2|} = e^{\ln|x+c|}$$

$$1+v^2 = xc$$

put $v = y/x$.

$$1+(y/x)^2 = xc$$

$$\frac{x^2+y^2}{x^2} - xc$$

Scanned by TapScanner

$x^2 + y^2 = x^3 c$

As given put $x = 2$, $y = 6$

$(4) + (36) = 8c$

$c = \frac{40}{8}$, $c = 5$

$x^2 + y^2 = 5x^2$

$y^2 = 5x^2 - x^2$

$y^2 = x^2(5 - x)$ .1

$\sqrt{y^2} = \sqrt{x^2}\sqrt{5-x}$

$y = x\sqrt{5-x}$

$y = \pm x\sqrt{5-x}$.

Scanned by TapScanner

Q no 9:

(i) -

Express the Determinant.

$$\begin{bmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{bmatrix}.$$

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$a(b^2c^2 - c^2b^3) - b(a^2c^3 - c^2a^3) + c(a^2b^3 - b^2a^3)$$

$$= ab^2c^2 - ac^2b^3 - ba^2c^3 + bc^2a^3 + ca^2b^3 - cb^2a^3$$

Answer.