Date:

Question 1 (a):-

Estimate $\int \theta \sqrt[4]{1-\theta^2}\, d\theta$

Sol:-

$$\int \theta \sqrt[4]{1-\theta^2}\, d\theta$$

Appling u-substitude $u = 1-\theta^2$

$$\int \sqrt[4]{1-\theta^2}$$

$$= \int = \sqrt[4]{\frac{u}{2}}\, du$$

take constant out

$$= \frac{1}{2} \cdot \int u^{1/4}\, du$$

$$= -\frac{1}{2} \frac{u^{1/4}+1}{1/4+1}$$

$$= \frac{-1}{2} \frac{(1-\theta^2)^{1/4+1}}{1/4+1}$$

$$= \frac{2}{5} (-\theta^2+1)^{5/4}$$

$$= -\frac{2}{5} (-\theta^2+1)^{5/4} + C$$

Answer

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Question 1(b):-

Estimate $\int_0^1 x^3 (1+x^4)^3 dx$ using substitution methode.

Sol:-

$$\int_0^1 x^3(1+x^4)^3 dx$$

$$x^3(1+x^4)^3 : x^3 + 3x^7 + 3x^{11} + x^{15}$$

$$\int_0^1 x^3 + 3x^7 + 3x^{11} + x^{15} dx$$

$$\int_0^1 x^3 dx + \int_0^1 3x^7 dx + \int_0^1 3x^{11} dx + \int_0^1 x^{15} dx$$

$$\left[\frac{x^{3+1}}{3+1}\right]_0^1 + 3\left[\frac{x^{7+1}}{7+1}\right]_0^1 + 3\left[\frac{x^{11+1}}{11+1}\right]_0^1 + \left[\frac{x^{15+1}}{15+1}\right]_0^1$$

$$\left[\frac{x^4}{4}\right]_0^1 + 3\left[\frac{x^8}{8}\right]_0^1 + 3\left[\frac{x^{12}}{12}\right]_0^1 + \left[\frac{x^{16}}{16}\right]_0^1$$

$$\left(\frac{1}{4}\right) + 3\left(\frac{1}{8}\right) + 3\left(\frac{1}{12}\right) + \left(\frac{1}{16}\right)$$

$$\frac{1}{4} + \frac{3}{8} + \frac{3}{12} + \frac{1}{16}$$

$$\frac{4+6+4+1}{16}$$

$$\Rightarrow \frac{15}{10} \quad \underline{\text{Answer}}.$$

Date:

Question 2 :-

The region between the curve $y=\sqrt{x}$, $0 \le x \le 4$, ... Find the volume of solid

Sol :-

Given that

$$y=\sqrt{x}$$

$$0 \le x \le 4 \qquad a \le x \le b$$

As

$$V=\int_a^b \pi y^2 \, dx$$

$$V=\int_0^4 \pi (\sqrt{x})^2 \, dx$$

$$V=\pi \int_0^4 x \, dx \qquad \pi \cdot \frac{x^2}{2} \Big|_0^4$$

$$V=\frac{\pi}{2}\left[(4)^2-0\right]$$

$$V=8\pi$$

Answer

Question 3 :-

If $A = 2i - 4j + \sqrt{5}k$ & $B = -2i + 4j - \sqrt{5}k$

i) the scalar component of B in the direction of A.

ii) the vector $proj_A B$.

Sol :-

$$B \cdot A = (-2i + 4j - \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$$

$$B \cdot A = 4i - 16j - 5k$$

$$\boxed{B \cdot A = -25}$$

$$A \cdot A = (2i - 4j + \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$$

$$= 4 + 16 + 5$$

$$\boxed{A \cdot A = 25}$$

$$proj_A B = \left(\frac{B \cdot A}{A \cdot A}\right) A.$$

$$= \left(\frac{-25}{25}\right)(2i - 4j + \sqrt{5}k)$$

$$= -1(2i - 4j + \sqrt{5}k)$$

$$\boxed{= -2i + 4j - \sqrt{5}k}$$ Answer.

Date:

Question 4 :-

Sol :-

Fubini Theorem

$= \iint_R f(x,y)\,dA$

$= \int_0^2 \int_{-1}^1 f(x,y)\,dA$

$f(x,y) = 1 - 6x^2 y$

$= \int_0^2 \int_{-1}^1 (1 - 6x^2 y)\,dy\,dx$

$= \int_0^2 \left(\int_{-1}^1 dy - 6x^2 \int_{-1}^1 y\,dy \right) dx$

$= \int_0^2 \left(y\Big|_{-1}^1 - 6\frac{x^2 y^2}{2}\Big|_{-1}^1 \right) dx$

$= \int_0^2 (2 - 6x^2)\,dx$

$= 2\int_0^2 dx - 6\int_0^2 x^2\,dx$

$= 2x\Big|_0^2 - 6\frac{x^3}{3}\Big|_0^2$

$= 4 - 2(8)$

$= 4 - 16$

$\boxed{= -12}$ Answer.

Date:

Question 5(a):-

Find the minima maxima of curve $y = -2x^3+6x^2-3$.

Sol:-

$y = -2x^3+6x^2-3$

1st find $f'(x)$ & $f''(x)$

$$f'(x) = \frac{dy}{dx} = \frac{d}{dx}(-2x^3+6x^2-3)$$

$$= -2\frac{d}{dx}x^3 + 6\frac{d}{dx}x^2 - \frac{d}{dx}3$$

$$= -2(3x^2) + 6(2x) - 0$$

$$= -6x^2 + 12x - 0$$

$$f'(x) = -6x^2 + 12x$$

Now,

$$f''(x) = \frac{d}{dx}(-6x^2+12x)$$

$$f''(x) = -12x + 12$$

for maxima, minima

$f'(x) = 0$

$-6x^2+12x = 0$

$6(-x^2+2x) = 0$

$-x^2+2x = 0$

$x(-x+2) = 0$

$-x+2 = 0 \Rightarrow \boxed{x=2}$

MCM Haroon Notes

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put $x=2$ in $f''(x)$

$f''(x) = (-12x+12)$ put $x=2$

$f''(2) = (-12(2)+12)$

$f''(2) = -24+12$

$f''(2) = -12$

So $f''(x) < 0$

so the function

$y = -2x^3+6x^2-3$ is minima

at $x=2$.

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Question 5 (b):-

Change the spherical coordinate equation for the sphere $x^2+y^2+(z-1)^2=1$.

Sol:-

$$x^2+y^2+(z-1)^2=1$$

$$(\rho \sin\phi \cos\phi)^2+(\rho \sin\theta \sin\theta)^2+(\rho\cos\theta-1)^2=1$$

$$\rho^2\sin^2\theta \cos^2\theta+\rho^2\sin^2\theta \sin^2\theta+\rho^2\cos^2\theta+1-2\rho\cos\theta=1$$

$$\rho^2 \sin^2\theta(\cos^2\theta+\sin^2\theta)+\rho^2\cos^2\theta+1-2\rho\cos\theta=1$$

$$\rho^2(\sin^2\theta)+\rho^2\cos^2\theta+2\rho\cos\theta=1-1$$

$$\rho^2(\sin^2\theta+\cos^2\theta)-2\rho\cos\theta=0$$

$$\rho^2=2\rho\cos\theta=2\cos\theta$$

Answer.