Department of Electrical Engineering
Assignment
Date:13/04/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:	**Pir Meher Ali Shah**	**Total Marks:**	30

Student Details

Name: **YASIR AHMAD** **Student ID:** **13788**

Q1.	(a)	Consider the following analog signal $x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$ i. Determine the minimum sampling rate required to avoid aliasing. ii. Suppose that the signal is sampled at the rate $F_s = 100\text{Hz}$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal. iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?	Marks 5 CLO 1
	(b)	Consider a discrete time signal which is given by $x(n) = \begin{cases} 0.5n, & n \geq 0 \\ 0, & n < 0 \end{cases}$ This is signal is sampled at the rate $F_s = 200\text{Hz}$. i. Draw the sampled signal. ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i . iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.	Marks 5 CLO 1
Q2.	(a)	Determine the response of the system to the following input signal with given impulse response $x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$, $h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$	Marks 5 CLO 2

<table>
<tr>
<td></td>
<td>(b)</td>
<td>Compute the convolution y(n) of the following signal

$$x(n)=\begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & elsewhere \end{cases}$$

$$h(n)=\begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & elsewhere \end{cases}$$
</td>
<td>Marks 5
CLO 2</td>
</tr>
<tr>
<td>Q3.</td>
<td></td>
<td>Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC).

i)

$$X(n)\begin{cases} (\frac{1}{4}^{n}) & , n \ge 0 \\ (\frac{1}{3})^{-n} & , n < 0 \end{cases}$$

ii)

$$X(n)=\begin{cases} (\frac{1}{2})^{n} - 3^{n} & , \quad n \ge 0 \\ 0 & , \quad elsewhere \end{cases}$$
</td>
<td>Marks 10
CLO 2</td>
</tr>
</table>

Q1(a) Consider The following Analog Signal:

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

(i) Determine the minimum Sampling rate required to Avoid aliasing.

$F_s \geq 2F_{max}$

$F_2 = \frac{\omega}{2\pi}$

$F_1 = \frac{100\pi}{2\pi}$

$F_1 = 50\,Hz$

$F_2 = \frac{200\pi}{2\pi}$

$F_2 = 100\,Hz$

F_2 is max (greater Then F_1)

$F_s \geq 2 \times 100\,Hz$. Sample frequency to Avoid aliasing.

(ii)

$F_s = 100\,Hz$

F_1 becomes

$F_1' = \frac{F_1}{F_s} = \frac{180}{100} = 0.5\,Hz$

F_2 becomes.

$F_2 = \frac{F_2}{100} = \frac{100}{100} = 1\,Hz$

So that

$\omega_1' = 2\pi f_1'$

$\omega_1' = 2\pi \times 0.5$

$\omega_1' = \pi$

$\omega_2' = 2\pi F_2'$

$\omega_2' = 2\pi \times 1$

$\omega_2' = 2\pi$

$x[n] = 3\cos 100\pi n + 4\sin 200\pi n$

The sampling are

$x[n] = 3\cos \pi n + 4\sin 2\pi n$

The Effect of Sampling rate on the newly generated discrete Time signal is that There will be no Aliasing phenomena means.
There will not present unwanted Component is the re construction of the signal.
The reconstruct original signal.

$\omega_1 = 100\pi \quad F_2 = 200\pi$

$F_1 = \frac{\cancel{100}\cancel{\pi}}{\cancel{2}\cancel{\pi}} \quad F_2 = \cancel{100}\pi$

$\boxed{F_1 = 50}$

Page 2

(iii) what is the Analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?

Ans: Folding frequency of the sampled signal is.

$$\text{Folding frequency} = F_s/2 = \frac{100}{2}$$

$$= 50\,Hz$$

We have frequency of the original signal

$$F_1 = 50\,Hz, \quad F_2 = 100\,Hz.$$

Both The frequency are either equal are greater The folding frequency.

Hence for ideal interpolation we can contrust the original signal.

$$X_a(t) = 3\cos 100\pi t + 4\sin 200\pi t.$$

The original signal is Constructed because we use Sampling frequency at Nyqusist rate.

we can also reconstruct The signal for Sampling frequency above The Nyquist rate.

Q1b Consider a discrete time signal which is given by:

$$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

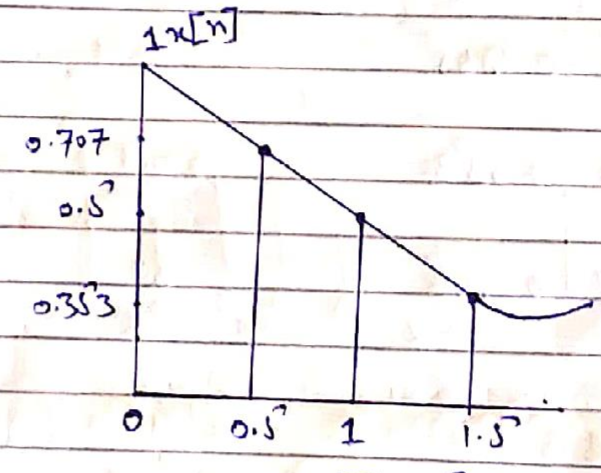
$F_s = 2\,Hz$

$F_s = 1/T$

$T = \frac{1}{F_s} = \frac{1}{2} = 0.5\,sec$

(i) Draw The simpled signal.

x(n)	0.5^n
0	1
0.5	0.7071
1	0.5
1.5	0.353



T = 0.5 sec.

(ii) quantization level.

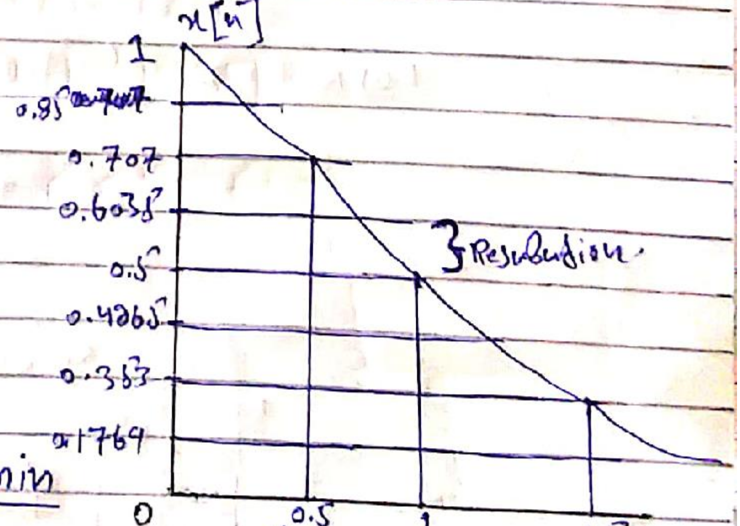
$L = 2^n$

$n = bits = 3$

$L = 2^3 = 8$ levels

$$\text{Resolution} = \frac{X_{max} - X_{min}}{L}$$

$$= \frac{1-0}{8} = 0.125$$



Scanned with CamScanner

iii

	Discreate Time signal	Truncation	Rounding	error.
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	-0.1
2	0.707	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4268	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1768	0.1	0.2	-0.1

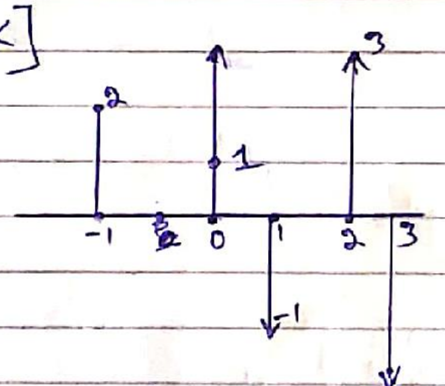
Q2 (a) Determine the response of the system to the following input signal with given ~~Example~~ impulse response.

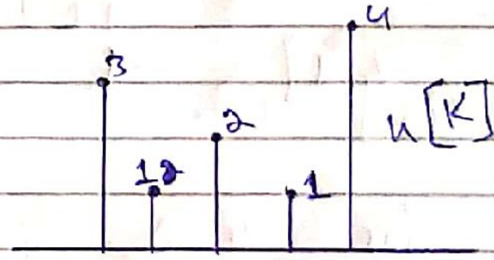
$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}, \quad h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

Solution.

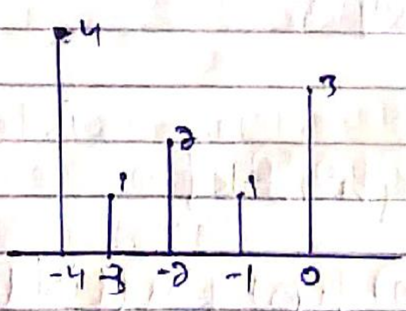
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$x[k]$





now we find the folded signal $h[-k]$



for

$n = 0$

$$y(0) = \sum_{k=-1}^{0} x[-1]\,h[-1] + x(0)\,h(0)$$

$$= 2 \times 1 + 1 \times 3$$

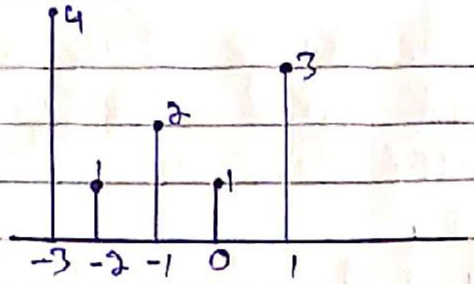
$$= 2 + 3$$

$$y(0) = 5$$

Page 6

for $n=1$

$h[1-k]$



$$y[1] = \sum_{k=-1}^{1} x[k]h[1-k]$$

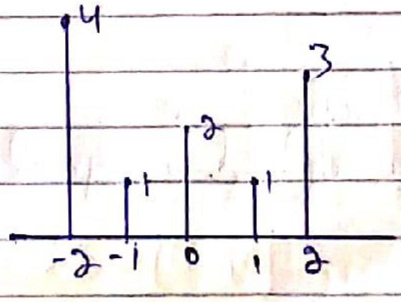
$$= x(-1)h(-1) + x(0)h(0) + x(1)h(1)$$

$$= (2)(2) + (1)(1) + 3(-2)$$

$$= 4 + 1 - 6$$

$$y[1] = -1$$

for $n=2$



By Putting in formula.

$$y[2] = \sum_{k=-1}^{2} x[n]h[2-k]$$

$$= x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2)$$

$$= 2(1) + (1)(2) + (-2)(1) + 3(3)$$

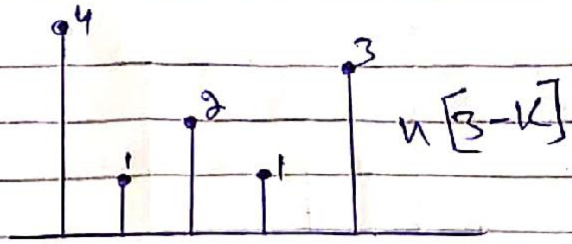
$$= 2 + 2 - 2 + 9$$

$$= 11$$

$$y[2] = 11$$

Page 7

for $n=3$



$$y[3] = \sum_{k=-1}^{3} x[k]\, h[3-k]$$

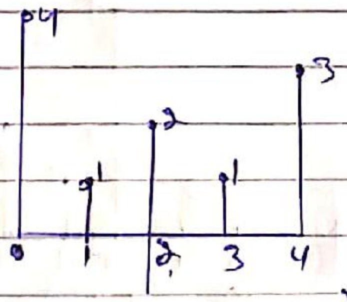
$$= x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(1)h(2) + x(3)h(3)$$

$$= 2\times 4 + (1)(1) + (-2)(2) + (3)(1) + (-4)(3)$$

$$= 8+1-4+3-12$$

$$y[3] = -4$$

for $n=4$



$$y[4] = \sum_{k=0}^{3} x(n)\, h(4-k)$$

$$= x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$= (1)(4) + (-2)(1) + (3)(2) + (-4)(1)$$

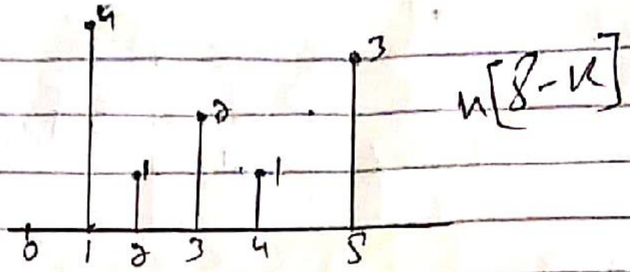
$$= 4-2+6-4$$

$$= 8-4$$

$$= 4$$

$$y[4] = 4.$$

for $n=8$



So that

$$y[8] = \sum_{k=1}^{3} x[k]\, h[8-k]$$

$= x(1)\, h(1) + x(2)\, h(2) + 3(3)\, h(3) + 0 + 0$

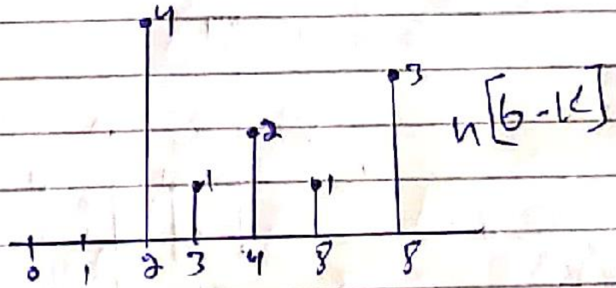
$= (-2)(4) + (3)(1) + (-4)(2)$

$= -8 + 3 - 8$

$= -13$

$y[8] = -13.$

For $n=6$



$$y[6] = \sum_{k=2}^{3} x(k)\, h[6-k]$$

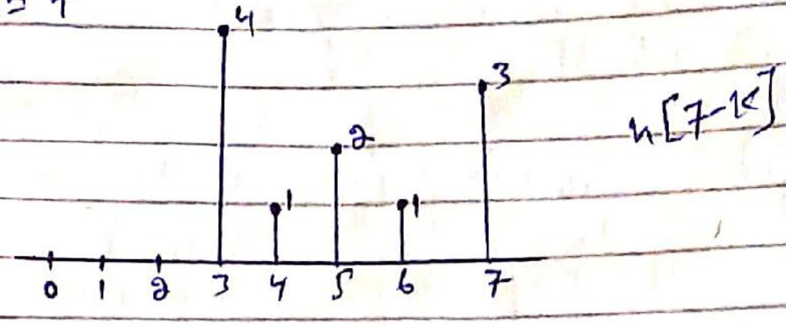
$= x(2)\, h(2) + x(3)\, h(3) + 0 + 0$

$= (3)(4) + (1)(-4)$

$= 12 - 4$

$y[6] = 8$

for $n=7$



So that

$$y[7] = x(3)\,h(3) + 0 + 0$$

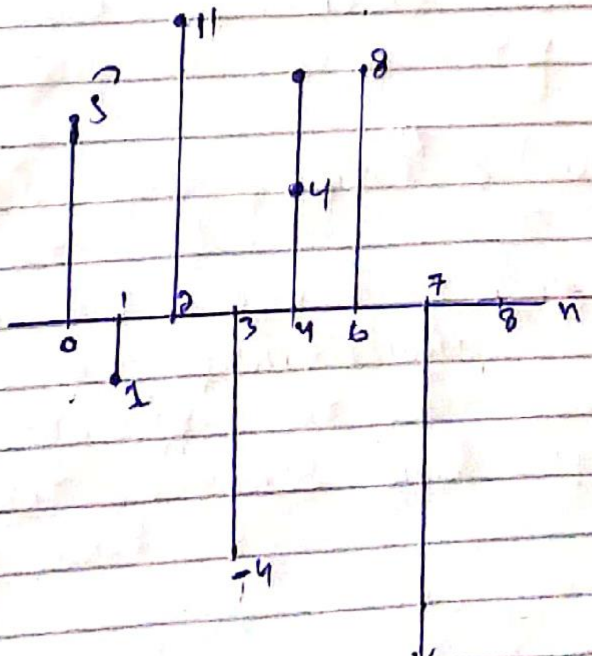
$$y[7] = (4)(-4)$$

$$y[7] = -16$$

As for $n=8$ There is overlap

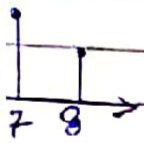
so $\{\underset{\uparrow}{3}, -1, 11, -4, 4, -13, 8, -16\}$

Response.



Q2

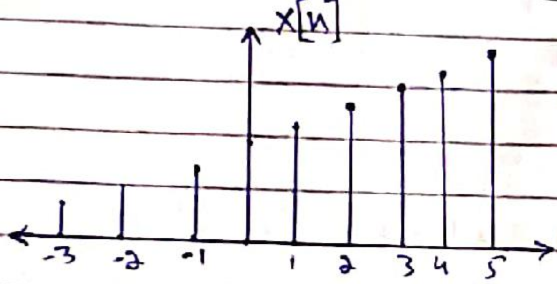
(b) Compute the convolution $y(n)$ of the following signal.

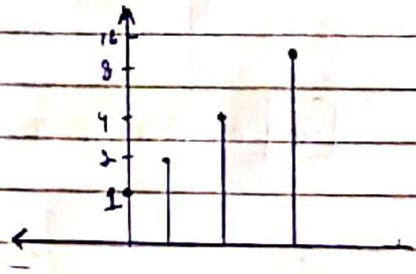


$$x(n) = \begin{cases} a^{n+1}, & -3 \le n \le 5 \\ 0, & \text{elsewhere.} \end{cases}$$

$$h(n) = \begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & \text{else where} \end{cases}$$

Solution:

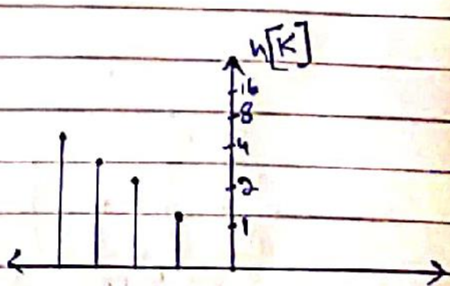




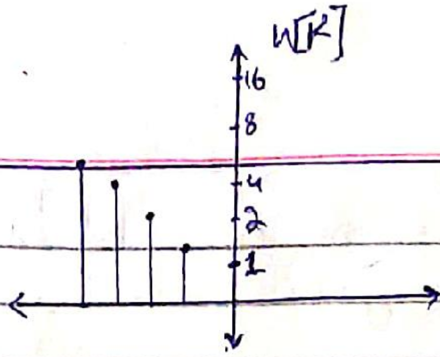
$$x[n] = \{a^{-2}, a^{-1}, 1, \underset{\uparrow}{a}, a^{2}, a^{3}, a^{4}, a^{5}, a^{6}\}$$

$$h[n] = \{\underset{\uparrow}{1}, 2, 4, 8, 16\}$$

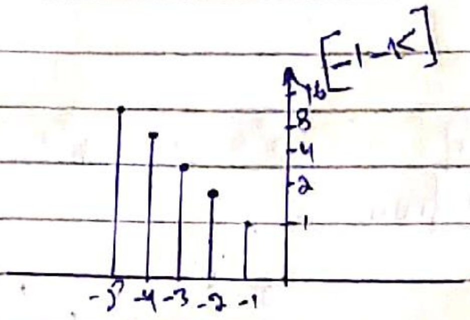
$$y[n_0] = \sum_{K=-\alpha}^{\alpha} x[K]\, h[n_0 - K]$$



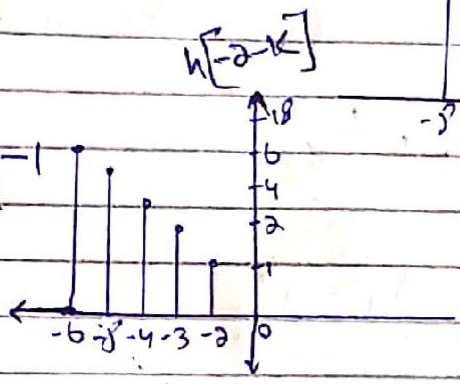
Scanned with CamScanner



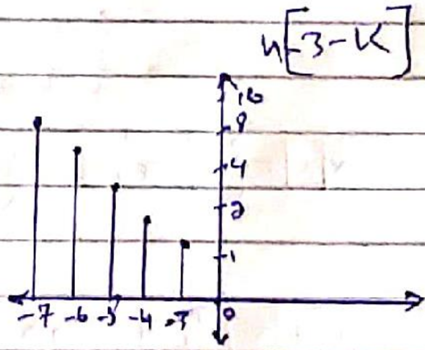
$y[0] = a-2+4a^{-1}+8a^{-2}$



$y[-1] = 1+2a^{-1}+4a^{-2}$

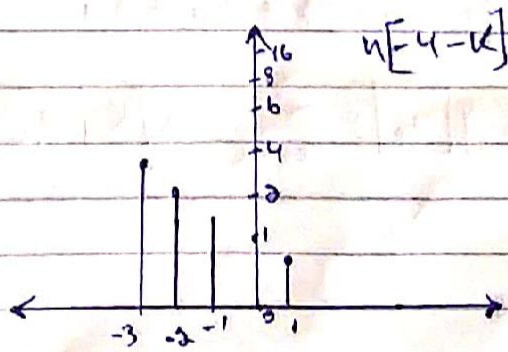


$y[-2] = 2a^{-2}+a^{-1}$

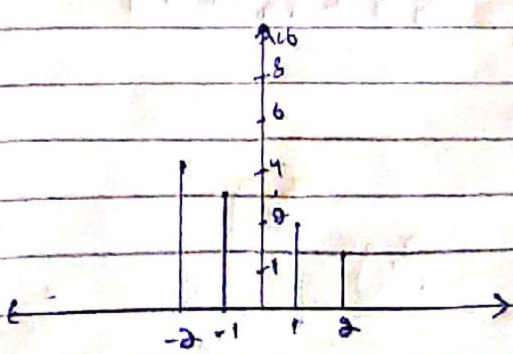


$y[-3] = a^{-2}$

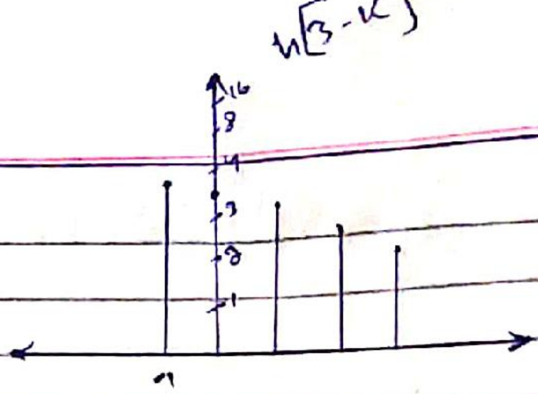
$y[1] = a^{2}+2a+4+8a^{-1}+16a^{-2}$



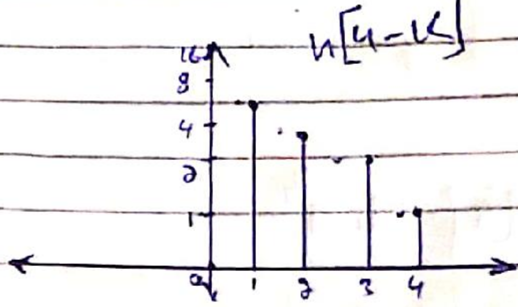
$y[2] = a^{3}+2a^{2}+4a+8+16a^{-1}$



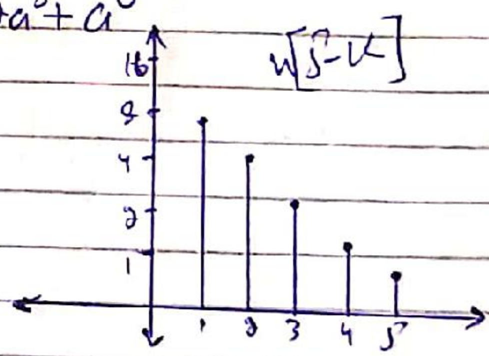
$y[3] = a^4 + 2a^3 + 4a^2 + 8a + 16$



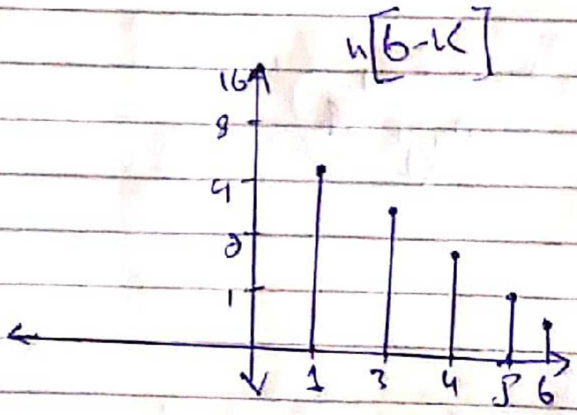
$y[4] = a^5 + 2a^4 + 4a^3 + 8a^2 + 16a$



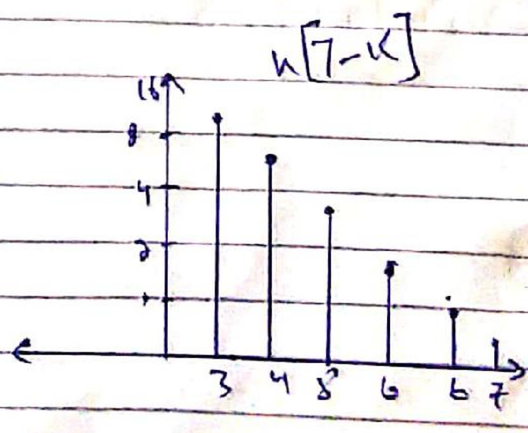
$y[5] = 16a^2 + 8a^3 + 4a^4 + 2a^5 + a^6$



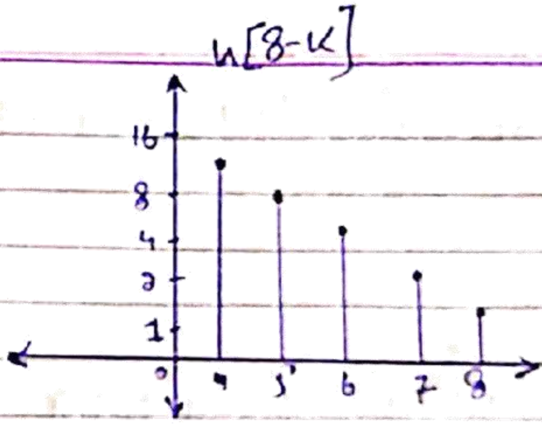
$y[6] = 16a^3 + 8a^4 + 4a^5 + 2a^6$



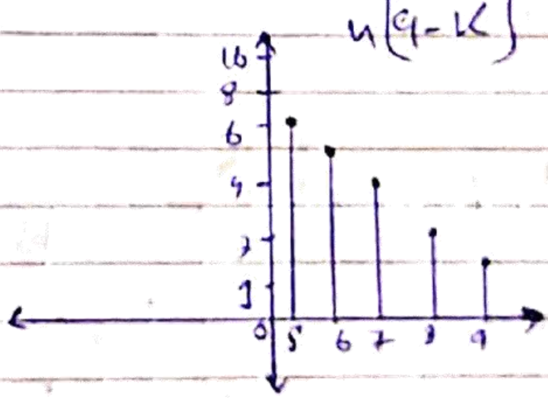
$y[7] = 16a^4 + 8a^5 + 4a^6$



$y[8] = 16a^5 + 8a^6$



$y[9] = 16a^6$



Page 14

Q9
(b) Con
Sign

Solution

Q3 Determine the z-transform of the following signal and also sketch Region of Convergence (ROC).

(i) $x(n) = \begin{cases} (\frac{1}{4})^n, & n \geq 0 \\ (\frac{1}{3})^{-n}, & n < 0 \end{cases}$

Solution: $x(n) = \begin{cases} (\frac{1}{4})^n & n \geq 0 \\ (\frac{1}{3})^{-n} & n < 0 \end{cases}$

writing in The form of z-Transform.

$$x(z) = \sum_{n=0}^{\infty} (\tfrac{1}{4})^n z^{-n} + \sum_{n=-\infty}^{0} (\tfrac{1}{3})^n z^n - 1$$

using geometric series.

$$= \frac{1}{1 - \frac{1}{4} z^{-1}} + \sum_{n=0}^{\infty} (\tfrac{1}{3})^n z^n - 1$$

$$= \frac{1}{1 - \frac{1}{4} z^{-1}} + \frac{1}{1 - \frac{1}{3}} - 1$$

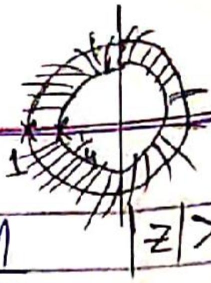
$$= \frac{1 - \frac{1}{\frac{1}{4} z^{-1}} + 1 - \frac{1}{1} - 1}{(1 - \frac{1}{4} z^{-1})(1 - \frac{1}{3} z)} - 1$$

$$= \frac{1 - \frac{1}{3} z + 1 - \frac{1}{4} z^{-1} - (1 - \frac{1}{4} z^{-1})(1 - \frac{1}{3} z)}{(1 - \frac{1}{4} z^{-1})(1 - \frac{1}{3} z)}$$

Page 13

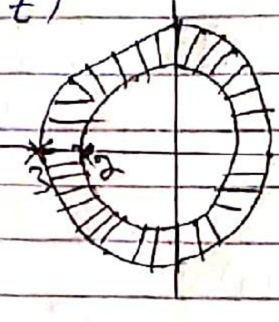
$$= \frac{1 - \frac{1}{3} z + 1 - \frac{1}{4} z^{-1} (1 + \frac{1}{3} z - \frac{1}{4} z^{-1} + \frac{1}{12} z^{-1} + \cancel{z}}{(1 - \frac{1}{4} z^{-1})(1 - \frac{1}{3} z)}$$

Scanned with CamScanner



$|z| > \frac{1}{4}$

$$= \frac{1 - \frac{1}{3} z + 1 - \frac{1}{4} z^{3} - 1 + \frac{1}{3} + \frac{1}{4} z^{-3} + \frac{1}{12}}{(1 - \frac{1}{4} z^{-1})(1 - \frac{1}{3} z)}$$



ROC

$|z| < 3$.

$$= \frac{1 + \frac{1}{12}}{(1 - \frac{1}{4} z^{-1})(1 - \frac{1}{3} z)}$$

$$= \frac{\frac{13}{12}}{(1 - \frac{1}{4} z^{-1})(1 - \frac{1}{3} z)}$$ Hence The ROC is $\frac{1}{4} < |z| < 3$

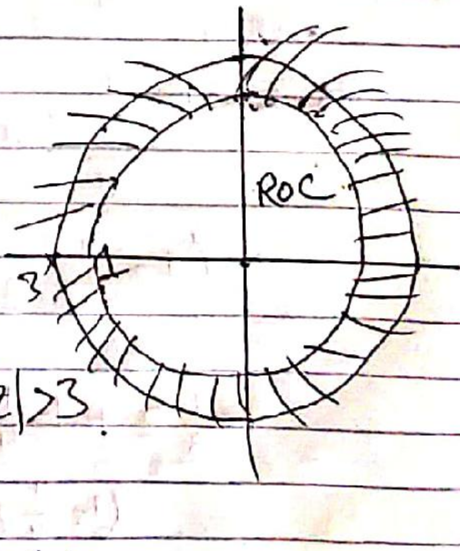
Q3 ii $x(n) \begin{cases} (\frac{1}{2})^n - 3^n, & n \geq 0 \\ 0, & \text{else where} \end{cases}$

Solution. $x(n) = \begin{cases} (\frac{1}{2})^n - 3^n, & n \geq 0 \\ 0, & \text{else where} \end{cases}$

in the form of Z-Transform

$$X(z) = \sum_{n=0}^{\infty} (\tfrac{1}{2})^{n} z^{-n} - \sum_{n=0}^{\infty} 3^n z^{-n}$$

using geometric series to simply



$$= \frac{1}{1 - \frac{1}{2} z^{-1}} - \frac{1}{1 - 3z^{-1}}$$

$$= \frac{1 - 3z^{-1} - 1 + \frac{1}{2} z^{-1}}{(1 - \frac{1}{2} z^{-1})(1 - 3z^{-1})}$$ $|z| > 3$.

$$= \frac{-\frac{5}{2} z^{-1}}{(1 - \frac{1}{2} z^{-1})(1 - 3 z^{-1})}$$ The ROC is $|z| > 3$.