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ASSIGNMENT # 01

SUBJECT # DIFFERENTIAL EQUATION

I.D # 13131

SUBMITTED To # SIR LATIF JAN

Q) Solve & graph the solution. Show details of your work.

12) $x^2y'' - 4xy' + 6y = 0$, $y(1) = 0.4$, $y'(1) = 0$

SOLUTION:-

$y = x^m$, $y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$

into the given ODE. This gives:

$x^2m(m-1)x^{m-2} - 4x\,mx^{m-1} + 6x^m = 0$

$x^2m(m-1)x^m \cdot x^{-2} - 4x\,mx^m x^{-1} + 6x^m = 0$

we can see that x^m is common factor. it gives:

$m(m-1) - 4m + 6 = 0 \longleftrightarrow m^2 - 5m + 6 = 0 \quad (*)$

So $y = x^m$ is a solution of the ODE if m is a root of the equation $(*)$

let find the root of equation $(*)$

$m^2 = 5m + 6 = 0 \longleftrightarrow m_{1/2} = \dfrac{5 \pm \sqrt{(-5)^2 + 4 \cdot 6}}{2}$

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$$\Longleftrightarrow m_{1/2} = \frac{5 \pm 1}{2}$$

So it has the distinct real roots:

$$m_1 = 3 \wedge m_2 = 2$$

real different roots m_1 & m_2 provide two real solution.

$$y_1 = x^{m_1} = x^3 \wedge y_2 = x^{m_2} = x^2$$

this quotient is not constant, so the solution y_1 & y_2 are linearly independent & consitute a basic of solution for given ODE, for all x for which.

$$y_1 . y_2 \in R.$$

So general solution is

$$y = C_1 y_1 + C_2 y_2$$

$$= C_1 x^3 + C_2 x^2$$

$$y' = 3C_1 x^2 + 2C_2 x$$

Now, all we need to is determine C_1 & C_2 from IVP

$$\begin{cases} 0.4 = y(1) = C_1 . 1^3 + C_2 . 1^2 \\ 0 = y'(1) = 3C_1 . 1^2 + 2C_2 . 1 \end{cases} \Longrightarrow \begin{cases} 0.4 = C_1 + C_2 \\ 0 = 3C_1 + 2C_2 \end{cases}$$

$$\Longrightarrow \begin{cases} 0.4 - C_2 = C_1 \\ 0 = 3(0.4 - C_2) + 2C_2 \end{cases}$$

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$$\Rightarrow \begin{cases} 0.4 - C_2 = C_1 \\ 0 = 1.2 - C_2 \end{cases}$$

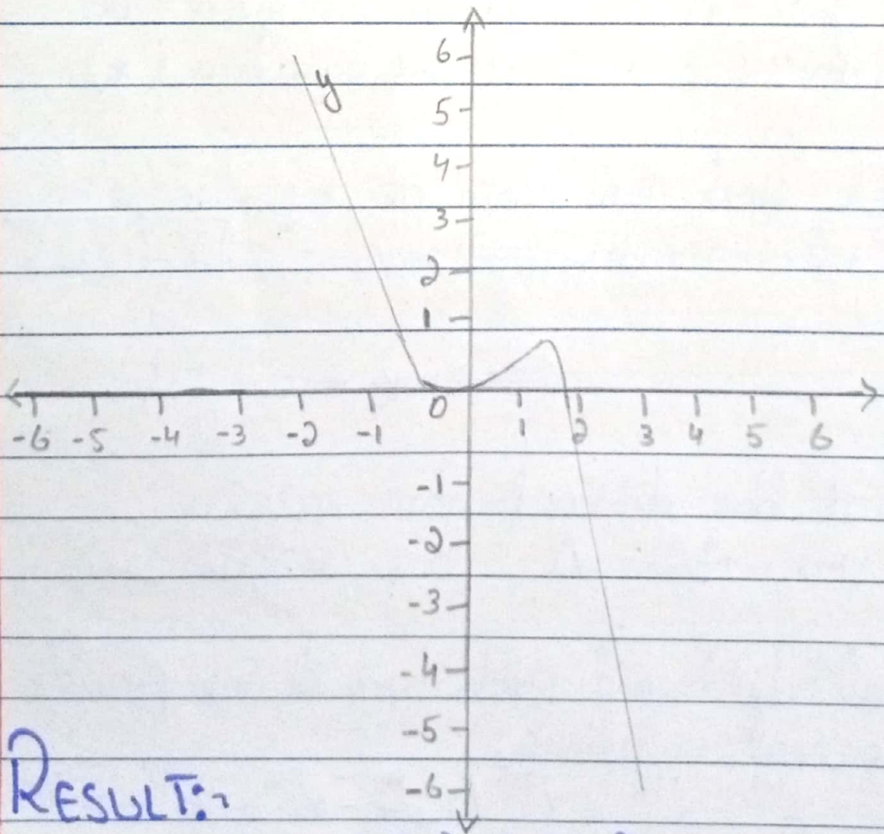
$$\Rightarrow \begin{cases} 0.4 - C_2 = C_1 \\ 1.2 = C_2 \end{cases}$$

$$\Rightarrow \begin{cases} 0.4 - 1.2 = C_1 \\ 1.2 = C_2 \end{cases}$$

$$\Rightarrow \begin{cases} -0.8 = C_1 \\ 1.2 = C_2 \end{cases}$$

the particular solution of IVP is:

$$y = -0.8x^3 + 1.2x^2$$



RESULT:

$$y = -0.8x^3 + 1.2x^2$$

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13) $x^2y'' + 3xy' + 0.75y = 0, y(1) = 1, y'(1) = -1.5$

SOLUTION:-

$y = x^m, y' = mx^{m-1}, y'' = m(m-1)x^{m-2}$

into given ODE. this gives:

$x^2m(m-1)x^{m-2} + 3xmx^{m-1} + 0.75x^m = 0$

$\cancel{x^2}m(m-1)x^m \cdot \cancel{x^{-2}} + 3xmx^m\cancel{x^{-1}} + 0.75x^m = 0$

we can see that x^m is Common factor it gives:

$m(m-1) + 3m + 0.75 = 0 \Longleftrightarrow m^2 + 2m + 0.75 = 0 \; (*)$

So $y = x^m$ is a solution of given ODE if m is a root of the equation $(*)$

Let find the root of equation $(*)$.

$m^2 + 2m + 0.75 = 0 \Longleftrightarrow m_{1/2} = \dfrac{-2 \pm \sqrt{2^2 - 4 \cdot 0.75}}{2}$

$\Longleftrightarrow m_{1/2} = \dfrac{-2 \pm 1}{2}$

So it has distinct real roots:

$m_1 = -1/2 \wedge m_2 = -3/2$

Real different roots m_1 & m_2 provide two real solutions:

$y_1 = x^{m_1} = x^{-1/2} = x^{-0.5} \wedge y_2 = x^{m_2} = x^{-3/2} = x^{-1.5}$

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This quotient is not constant, so the solutions y_1 & y_2 are linearly independent & constitute a basics of solution for given ODE, for all x for which $y_1, y_2 \in R$.

So general solution is.

$$y = C_1 y_1 + C_2 y_2$$

$$= C_1 x^{-0.5} + C_2 x^{-1.5}$$

$$\Rightarrow y' = 0.5 C_1 x^{-1.5} - 1.5 C_2 x^{-2.5}$$

Now all we need to do is determine C_1 & C_2 from IVP:

$$\begin{cases} 1 = y(1) = C_1 \cdot 1^{-0.5} + C_2 \cdot 1^{-1.5} \\ -1.5 = y'(1) = -0.5 C_1 \cdot 1^{-1.5} - 1.5 C_2 \cdot 1^{-2.5} \end{cases}$$

$$\Rightarrow \begin{cases} 1 = C_1 + C_2 \\ -1.5 = -0.5 C_2 - 1.5 C_2 / (\div -0.5) \end{cases}$$

$$\Rightarrow \begin{cases} 1 = C_1 + C_2 \\ 3 = C_1 + 3C_2 \end{cases}$$

$$\Rightarrow \begin{cases} 1 - C_2 = C_1 \\ 3 = 1 - C_2 + 3C_2 \end{cases}$$

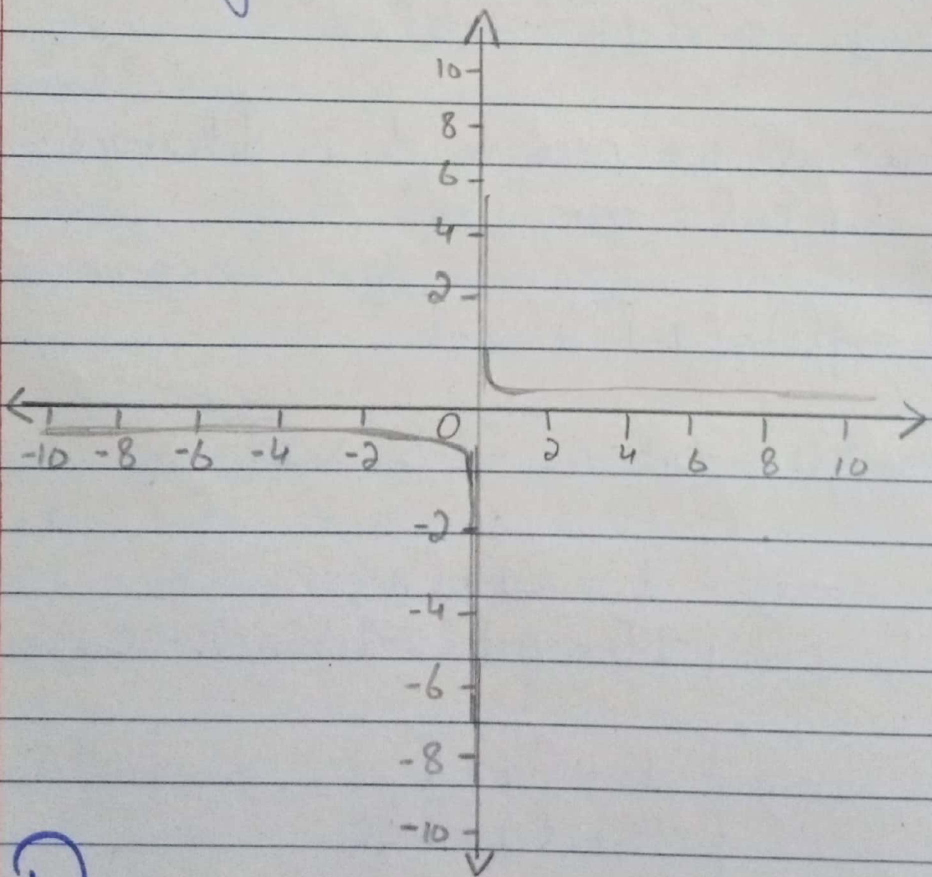
$$\Longrightarrow \begin{cases} 1 - C_2 = C_1 \\ 2 = 2C_2 \quad /(\div 2) \end{cases}$$

$$\Longrightarrow \begin{cases} 1 - C_2 = C_1 \\ 1 = C_2 \end{cases}$$

$$\Longrightarrow \begin{cases} 0 = C_1 \\ 1 = C_2 \end{cases}$$

The particular solution of IVP is:

$$y = x^{-1.5}$$



RESULT:-

$$y = x^{-1.5}$$

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14) $x^2 y'' + x y' + 9y = 0$, $y(1) = 0$, $y'(1) = 2.5$

SOLUTION:

$y = x^m$, $y'' = m(m-1)x^{m-2}$

into given ODE gives:

$x^2 m(m-1)x^{m-2} + m x^m + 9x^m = 0$

$x^2 m(m-1)x^m \cdot x^{-2} + m x^m + 9x^m = 0$

we can see that x^m is a common factor it gives:

$m(m-1) + m + 9 = 0 \Longleftrightarrow m^2 - m + m + 9 = 0 \Longleftrightarrow m^2 + 9 = 0 \; (*)$

So $y = x^m$ is a solution of the given ODE if m is a root of the equation $(*)$

Lets find the root of the equation $(*)$

$m^2 + 9 = 0 \Longleftrightarrow m^2 - (3i)^2 = 0 \Longleftrightarrow (m-3i)(m+3i) = 0$

So it has complex conjugate roots:

$m_1 = 3i \quad \wedge \quad m_2 = -3i$

Now we use the fact that $x = e^{\ln x}$:

$x^{m_1} = x^{3i} = (e^{\ln x})^{3i} = e^{3i\ln x}$

$x^{m_2} = x^{-3i} = (e^{\ln x})^{-3i} = e^{-3i\ln x}$

Recall that

$e^z = e^{a+ib} = e^a(\cos b + i \sin b)$, $z \in \mathbb{C}$

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So we have

$$e^{3i\ln x} = e^0(\cos(3\ln x)+i\sin(3\ln x)) = \cos(3\ln x)+i\sin(3\ln x)$$

&

$$e^{-3i\ln x} = e^0(\cos(3\ln x)-i\sin(3\ln x)) = \cos(3\ln x)-i\sin(3\ln x)$$

This gives:

$$x^{m_1} = \cos(3\ln x)+i\sin(3\ln x)$$

$$x^{m_2} = \cos(3\ln x)-i\sin(3\ln x)$$

Adding this two formulae gives:

$$x^{m_1}+x^{m_2} = \cos(3\ln x)\cancel{+i\sin(3\ln x)}+\cos(3\ln x)\cancel{-i\sin(3\ln x)}$$

$$= 2\cos(3\ln x)$$

Now divide by 2

$$\frac{x^{m_1}+x^{m_2}}{2} = \frac{\cancel{2}\cos(3\ln x)}{\cancel{2}} = \cos(3\ln x)$$

Next, subtract the second formulae from the first & divide it by 2i after that.

$$x^{m_1} = x^{m_2} = \cancel{\cos(3\ln x)}+i\sin(3\ln x)\cancel{-\cos(3\ln x)}+i\sin(3\ln x)$$

$$= 2i\sin(3\ln x)$$

Divid it by 2i:

$$\frac{x^{m_1}-x^{m_2}}{2i} = \frac{\cancel{2i}\sin(3\ln x)}{\cancel{2i}} = \sin 3(\ln x)$$

By the super position principal, $\cos(3\ln x)$ & $\sin(3\ln x)$ are the solution of the Euler Cauchy equation.

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This quotient is not constant, so solution
$y_1 = \cos(3\ln x) \wedge y_2 = \sin(3\ln x)$
are linearly independent & form a basic solutions.

So, the general solution is:

$$y = C_1 y_1 + C_2 y_2$$

$$= C_1 \cos(3\ln x) + C_2 \sin(3\ln x)$$

$$\Rightarrow y' = -C_1 \sin(3\ln x)^{\text{chain rule}} \cdot (3\ln x)' + C_2 \cos(3\ln x)^{\text{chain rule}} \cdot (3\ln x)'$$

$$= \frac{-3C_1}{x} \sin(3\ln x) + \frac{3C_2}{x} \cos(3\ln x)$$

Now all we need to do is determine C_1 & C_2 from IVP:

$$\begin{cases} 0 = y(1) = C_1 \cos(3\ln 1) + C_2 \sin(3\ln 1) \\ 2.5 = y'(1) = -3C_1 \sin(3\ln 1) + 3C_2 \cos(3\ln 1) \end{cases} \Rightarrow \begin{cases} 0 = C_1\cos(0) + C_2 \sin(0) \\ 2.5 = -3\sin 1(0) + 3C_2 \cos(0) \end{cases}$$

$$\Rightarrow \begin{cases} 0 = C_1 \\ 5/2 = 3C_2 \quad | :3 \end{cases}$$

$$\Rightarrow \begin{cases} 0 = C_1 \\ 5/6 = C_2 \end{cases}$$

The particular solution of IVP is:
$$y = 5/6 \sin(3\ln x)$$

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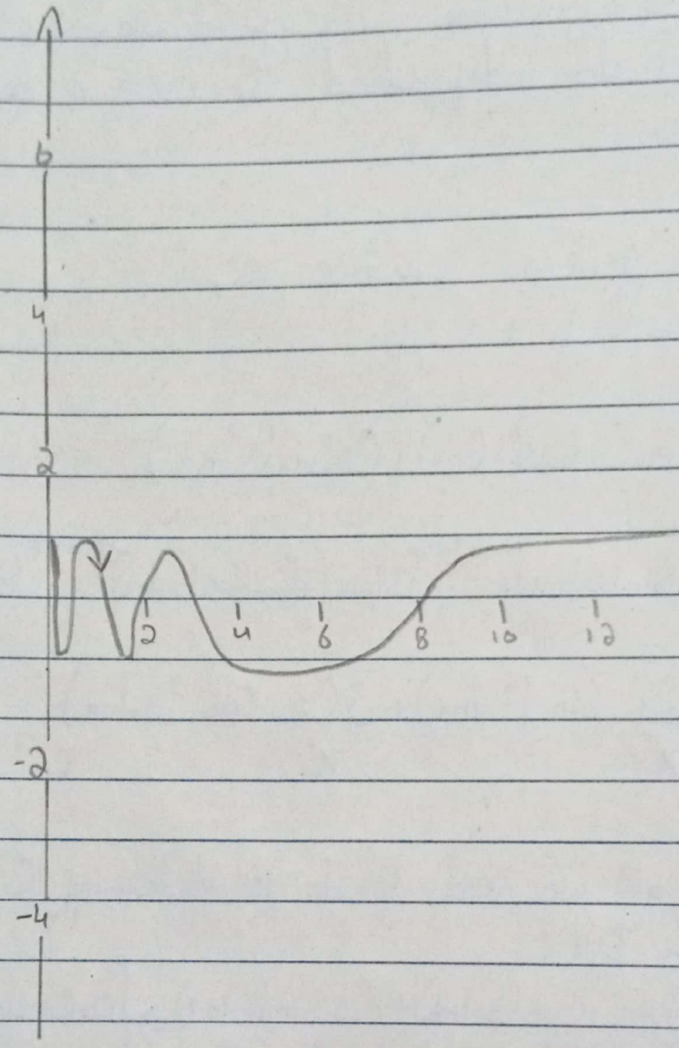

RESULT:

$$y = \frac{5}{6}\sin(3\ln x)$$

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Q15) $x^2y'' + 3xy' + y = 0$, $y(1) = 3.6$, $y'(1) = 0.4$

SOLUTION:

$$y = x^m, \quad y'' = m(m-1)x^{m-2}$$

into the given ODE, this gives

$$x^2 m(m-1)x^{m-2} + 3mx^m + x^m = 0$$

$$x^2 m(m-1)x^m \cdot x^{-2} + 3mx^m + x^m = 0$$

we can see that x^m is common factor, it gives:

$$m(m-1) + 3m + 1 = 0 \Longleftrightarrow m^2 - m + 3m + 1 = 0 \Longleftrightarrow m^2 + 2m + 1 = 0 \quad (\star)$$

So, $y = x^m$ is a solution of given ODE is m is a root of equation $(\star)$

let's find the root of the equation $(\star)$

$$m^2 + 2m + 1 = 0 \Longleftrightarrow (m+1)^2 = 0$$

So it has the real double root:

$$m = 1$$

Real double root m provides a real solution

$$y_1 = x^m = x^{-1} = 1/x$$

we can find a second linearly independent solution y_2 using the method of reduction order.

First write given ODE in standard form:

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$$y'' + \frac{3}{x} \cdot y' + \frac{1}{x^2} \cdot y = 0$$

Now, we can see that:

$P(x) = 3 \cdot \frac{1}{x} \implies \int P\,dx = 3 \ln|x|$

Put $y_2 = U y_1$

where $U = \int u\,dx \wedge u = \frac{1}{y_1^2} e^{-\int P dx}$

let find u:

$e^{-\int P dx} = e^{-3 \ln|x|} = \left(e^{\ln|x|}\right)^{-3} = x^{-3}$

$\implies u = x^{-3} \cdot \frac{1}{x^2} = x^{-3+2} = x^{-1} = \frac{1}{x}$

By integration, we have:

$U = \int \frac{dx}{x} = \ln|x|$

So $y_2 = U y_1 = y_1 \ln x = \frac{1}{x} \cdot \ln x$

Since their quotient is not constant, y_1 & y_2 are linearly independent & constitute a basic of solution for all x for which.

$y_1 \cdot y_2 \in \mathbb{R}$.

General Solution is:

$y = C_1 y_1 + C_2 y_2$

$= C_1 \cdot \frac{1}{x} + C_2 \cdot \frac{1}{x} \cdot \ln x$

$= \frac{1}{x} \cdot (C_1 + C_2 \ln x)$

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Product Rule

$$\Rightarrow y' = (x^{-1})'(C_1 + C_2 \ln x) + x^{-1}(C_1 + C_2 \ln x)'$$
$$= -x^{-2}(C_1 + C_2 \ln x) + 1/x \; C_2 \cdot 1/x$$
$$= 1/x^2 (-C_1 - C_2 \ln x + C_2)$$

Now determine C_1 & C_2 from IVP:

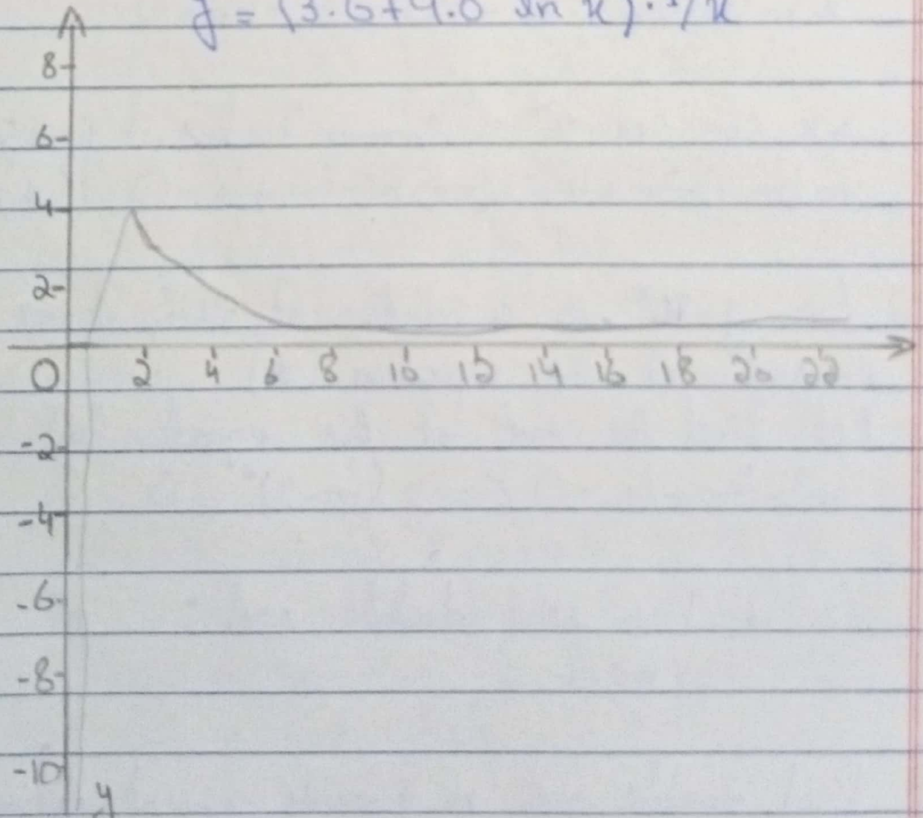
$$\begin{cases} 3.6 = y(1) = 1/1 (C_1 + C_2 \ln 1) \\ 0.4 = y'(1) = 1/1^2 (-C_1 - C_2 \ln 1 + C_2) \end{cases} \Rightarrow \begin{cases} 3.6 = C_1 \\ 0.4 = -C_1 + C_2 \end{cases}$$

$$\Rightarrow \begin{cases} 3.6 = C_1 \\ 0.4 = -3.6 + C_2 \end{cases}$$

$$\Rightarrow \begin{cases} 3.6 = C_1 \\ 4.0 = C_2 \end{cases}$$

the particular solution of the IVP is:

$$y = (3.6 + 4.0 \ln x) \cdot 1/x$$



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16) $(x^2D^2 - 3xD + 4I)y = 0, y(1) = -\pi, y'(1) = 2\pi$

SOLUTION:-

First, we need to apply the given operator to the given function:

$x^2D^2 + 3xDy + 4Iy = x^2D(Dy) - 3xDy + 4y$
$= x^2y'' - 3xy' + 4y$

let solve the equation:

$x^2y'' - 3xy' + 4y = 0$

Lets subtitute:

$y = x^m, y' = mx^{m-1}, y'' = m(m-1)x^{m-2}$

into given ODE gives:

$x^2m(m-1)x^{m-2} - 3xmx^{m-1} + 4x^m = 0$

$x^2m(m-1)x^m \cdot x^{-2} - 3xmx^m x^{-1} + 4x^m = 0$

we can see x^m is common factor, it gives:

$m(m-1) - 3m + 4 = 0 \Leftrightarrow m^2 - 4m + 4 = 0$ (*)

So, $y = x^m$ is a solution of ODE if m is root of the equation (*)

Let find the root of the equation (*)

$m^2 - 4m + 4 = 0 \Leftrightarrow (m-2)^2 = 0$

So, it has real double root:

$m = 2$

Real double root m provide a real solution:

$y_1 = x^m = x^2$

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we can find second linearly independent solution y_2 using method of reduction order.

First write ODE in standard form:

$$y'' - 3/x \cdot y' + 4/x^2 \cdot y = 0$$

Now we can see that:

$$P(x) = -3 \cdot 1/x \Rightarrow \int P\,dx = -3 \ln|x|$$

Put:

$$y_2 = u y_1$$

where

$$u = \int U\,dx \quad \wedge \quad U = \frac{1}{y_1^2} e^{-\int P dx}$$

Let find U:

$$e^{-\int P dx} = e^{3 \ln|x|} = (e^{\ln|x|})^3 = x^3$$

$$\Rightarrow U = x^3 \cdot \frac{1}{(x^2)^2} = x^{3-4} = x^{-1} = 1/x$$

By integration we have:

$$u = \int \frac{dx}{x} = \ln|x|$$

So

$$y_2 = u y_1 = y_1 \ln x = x^2 \ln x$$

Quotient is not constant, y_1 & y_2 are linearly independent & constitute a basic of solution for the given ODE, for all x for which $y_1, y_2 \in R$.

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So, general Solution is

$$y = C_1 y_1 + C_2 y_2$$
$$= C_1 x^2 + C^2 \ln x$$
$$= x^2 (C_1 + C_2 \ln x)$$

PRODUCT RULE

$$\Rightarrow y' = (x^2)' (C_1 + C_2 \ln x) + x^2 (C_1 + C_2 \ln x)'$$

$$= 2x (C_1 + C_2 \ln x) + C_2 x^2 \cdot 1/x$$

$$= 2C_1 x + 2C_2 x \ln x + C_2 x$$

$$= 2C_1 x + C_2 x (2 \ln x + 1)$$

Now, we need to determine C_1 & C_2 from IVP:

$$\begin{cases} -\pi = y(1) = 1^2 (C_1 + C_2 \ln 1) \\ 2\pi = y'(1) = 2C_1 + C_2 (2 \ln 1 + 1) \end{cases} \Rightarrow \begin{cases} -\pi = C_1 \\ 2\pi = 2C_1 + C_2 \end{cases}$$

$$\Rightarrow \begin{cases} -\pi = C_1 \\ 4\pi = C_2 \end{cases}$$

Particular Solution for IVP is:

$$y = x^2 (-\pi + 4\pi \ln x)$$

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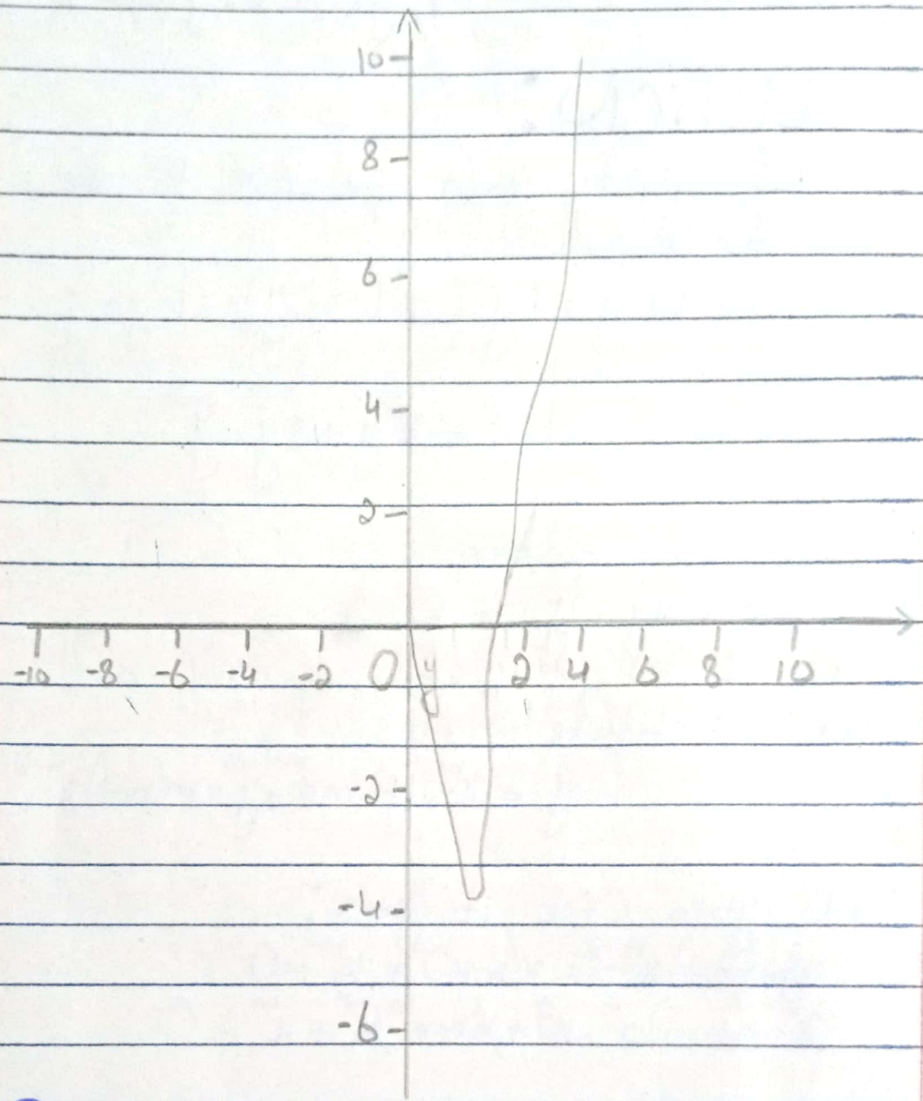

RESULT:-

$$y = x^2 \pi (4 \ln x - 1)$$

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17) $(x^2D^2 + xD + 1)y = 0$, $y(1) = 1$, $y'(1) = 1$

SOLUTION:

First apply given operator to the give function:

$$x^2D^2y + xDy + Iy = x^2D(Dy) + xDy + y$$

$$= x^2y'' + xy' + y$$

Let solve equation

$$x^2y'' + xy' + y = 0$$

let substitute

$$y = x^m, \; y' = mx^{m-1}, \; y'' = m(m-1)x^{m-2}$$

into given ODE it gives:

$$x^2m(m-1)x^{m-2} + x\,mx^{m-1} + x^m = 0$$

$$\cancel{x^2}m(m-1)x^m \cdot \cancel{x^{-2}} + \cancel{x}mx^m\cancel{x^{-1}} + x^m = 0$$

we can see that x^m is common factor it gives:

$$m(m-1) + m + 1 = 0 \iff m^2 - m + m + 1 = 0 \iff m^2 + 1 = 0 \quad (*)$$

So, $y = x^m$ is a solution of given ODE if m is a root of the equation $(*)$

let find the root of the equation $(*)$

$$m^2 + 1 = 0 \iff m^2 - i^2 = 0 \iff (m-i)(m+i) = 0$$

So it has complex conjugate roots:

$$m_1 = i \quad \wedge \quad m_2 = -i$$

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Now, we use the fact that $x = e^{\ln x}$:

$$x^{m_1} = x^{i} = (e^{\ln x})^{i} = e^{i\ln x}$$

$$x^{m_2} = x^{-i} = (e^{\ln x}) = e^{-i\ln x}$$

Recall that

$$e^{z} = e^{a+bi} = e^{a}(\cos b + i\sin b). \; z \in \mathbb{C}$$

So we have

$$e^{i\ln x} = e^{0}(\cos(\ln x) + i\sin(\ln x)) = \cos(\ln x) + i\sin(\ln x)$$

&

$$e^{-3i\ln x} = e^{0}(\cos(\ln x) - i\sin(\ln x)) = \cos(\ln x) - i\sin(\ln x)$$

This gives:

$$x^{m_1} = \cos(\ln x) + i\sin(\ln x)$$

$$x^{m_2} = \cos(\ln x) - i\sin(\ln x)$$

Adding two formulae gives:

$$x^{m_1} + x^{m_2} = \cos(\ln x) + \cancel{i\sin(\ln x)} + \cos(\ln x) - \cancel{i\sin(\ln x)}$$

$$= 2\cos(\ln x)$$

Now divide it by 2

$$\frac{x^{m_1} + x^{m_2}}{2} = \frac{\cancel{2}\cos(\ln x)}{\cancel{2}} = \cos(\ln x)$$

Next subtract the second formulae from the first & divide it by $2i$ after that.

$$x^{m_1} - x^{m_2} = \cancel{\cos(\ln x)} + i\sin(\ln x) - \cancel{\cos(\ln x)} + i\sin(\ln x)$$

$$= 2i\sin(\ln x)$$

Divide it by $2i$:

$$\frac{x^{m_1} - x^{m_2}}{2i} = \frac{2i\sin(\ln x)}{2i} = \sin(\ln x)$$

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By the superposition principle, $\cos(\ln x)$ $\sin(\ln x)$ are the solution of Euler-Cauchy equation.

Their quotient is not constant, so the solution

$$y_1 = \cos(\ln x) \quad \wedge \quad y_2 = \sin(\ln x)$$

are linearly independent & form basic solution.

So general solution is.

$$y = C_1 y_1 + C_2 y_2 = C_1\cos(\ln x) + C_2 \sin(\ln x)$$

$$\Rightarrow y' = -C_1 \sin(\ln x)^{\text{chain rule}} \cdot (\ln x)' + C_2 \cos(\ln x)^{\text{chain rule}} \cdot (\ln x)'$$

$$= \frac{C_1}{x} \sin(\ln x) + \frac{C_2}{x} \cos(\ln x)$$

Now determine C_1 & C_2 from IVP:

$$\begin{cases} 1 = y(1) = C_1 \cos(\ln 1) + C_2 \sin(\ln 1) \\ 1 = y'(1) = -C_1 \sin(\ln 1) + 3C_2 \cos(\ln 1) \end{cases} \Rightarrow \begin{cases} 1 = C_1 \cos(0) + C_2 \sin(0) \\ 1 = -C_1 \sin(0) + C_2 \cos(0) \end{cases}$$

$$\Rightarrow \begin{cases} 1 = C_1 \\ 1 = C_2 \end{cases}$$

The solution of IVP is:

$$y = \sin(\ln x) + \cos(\ln x)$$

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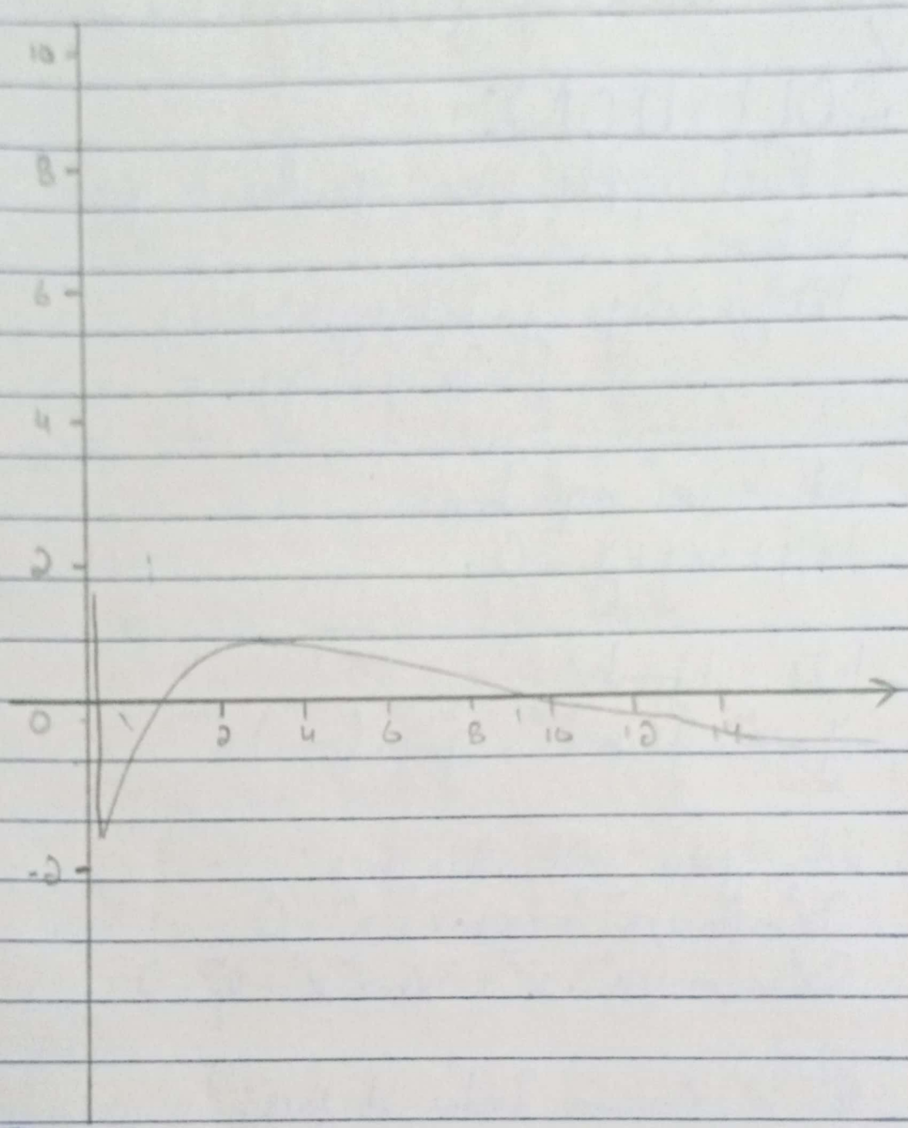

RESULT:-

$y = \sin(\ln x + \cos(\ln x)$

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18) $(9x^2D^2+3xD+I)y$, $y(1)=1$, $y'(1)=0$

SOLUTION:-

First apply given operator to given function:

$$9x^2D^2y+3xDy+Iy = 9x^2D(Dy)+3xDy+y$$
$$= 9x^2y''+3xy'+y$$

let solve equation:

$$9x^2y''+3xy'+y=0$$

lets subtitute:

$$y=x^m,\ y'=mx^{m-1},\ y''=m(m-1)x^{m-2}$$

into given ODE this gives:

$$9x^2m(m-1)x^{m-2}+3x\,mx^{m-1}+x^m=0$$
$$9\cancel{x^2}m(m-1)x^m\cdot\cancel{x^{-2}}+3\cancel{x}mx^m\cancel{x^{-1}}+x^m=0$$

x^m is Common factor it gives:

$$9m(m-1)+3m+1=0 \iff 9m^2-9m+3m+1=0 \iff 9m^2-6m+1=0 \quad (*)$$

So, $y=x^m$ is a solution of given ODE if m is a root of equation $(*)$

let find the roots of the equation $(*)$

$$m^2-4m+4=0 \iff (m-2)^2=0$$
$$9m^2-6m+1=0 \iff m_{1/2}=\frac{6\pm\sqrt{6^2-4\cdot 9}}{18}$$
$$\iff m_{1/2}=6/18$$
$$\iff m_{1/2}=1/3$$

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So it has real double root:

$$m = 1/3$$

Real double root m Provides a real Solution:

$$y_1 = x^m = x^{1/3}$$

we Can find a second linearly independent Solution y_2 using the method of reduction orders.

write ODE in standard form:

$$y'' + \frac{1}{3x} \cdot y' + \frac{1}{9x^2} \cdot y = 0$$

we Can see that

$$P(x) = 1/3 \cdot 1/x \Rightarrow \int P\,dx = 1/3 \ln|x|$$

Put:

$$y_2 = U y_1$$

where

$$U = \int u\,dx \quad \wedge \quad u = \frac{1}{y_1^2} e^{-\int P\,dx}$$

let find u:

$$e^{-\int P\,dx} = e^{-1/3 \ln|x|} = \left(e^{\ln x}\right)^{-1/3} = x^{-1/3}$$

$$\Rightarrow u = x^{-1/3} \cdot \frac{1}{(x^{1/3})^2} = x^{-1/3 - 2/3} = x^{-1} = 1/x$$

By integration we have

$$U = \int dx/x = \ln|x|$$

So

$$y_2 = U y_1 = y_1 \ln x = x^{1/3} \ln x$$

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Since their quotient is not constant, y_1 & y_2 are linearly independent & contitute a basic of solution for give ODE, for all x for which $y_1, y_2 \in R$.

So the general solution is:

$$y = C_1 y_1 + C_2 y_2$$
$$= C_1 x^{1/3} + x^{1/3} \ln x$$
$$= x^{1/3} (C_1 + C_2 \ln x$$

PRODUCT RULE

$$\Rightarrow y' = (x^{1/3})' (C_1 + C_2 \ln x) + x^{1/3} (C_1 + C_2 \ln x)'$$
$$= 1/3 \cdot x^{-2/3} (C_1 + C_2 \ln x) + x^{1/3} C_2 \cancel{x} \cdot \frac{1}{\cancel{x}}$$
$$= 1/3 \cdot x^{-2/3} (C_1 + C_2 \ln x) + x^{1/3} C_2$$

Now, all we need to do is determine C_1 & C_2 from IVP:

$$\begin{cases} 1 = y(1) = 1^{1/3} (C_1 + C_2 \ln 1) \\ 0 = y'(1) = 1/3 \cdot 1^{-1/3} (C_1 + C_2 \ln 1) + 1^{1/3} C_2 \end{cases} \Rightarrow \begin{cases} 1 = C_1 \\ 0 = C_1/3 + C_2 \end{cases}$$

$$\Rightarrow \begin{cases} 1 = C_1 \\ -1/3 = C_2 \end{cases}$$

The particular solution of the IVP is:

$$y = x^{1/3} (1 - 1/3 \ln x)$$

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