Iqra National University, Peshawar
Department of Electrical Engineering

FINAL – ASSIGNMENT SPRING2020
Date:26/6/2020

Course Code:	MTH 102	**Course Title:**	Calculus and analytic geometry
Prerequisite:		**Instructor:**	HIMAYATULLAH
Module:	3 **Program:** BEE	**Total Marks:**	50 **:**

Note: Attempt all questions.PLO: program learning outcome C:Cognitive

Q1.	a	. **Estimate** $\int \theta \sqrt[4]{1-\theta^2}\, d\theta$	Marks 7 PLO2 C2
	b	**Estimate** $\int_0^1 x^3(1+x^4)^3 dx$ using substitution method.	Marks 7 PLO2 C2
Q2	(a)	**Illustrate** the centre and radius of the sphere $x^2+y^2+z^2+3x-4z+1$.	Marks 5 PLO1 C3
	(b)	The region between the curve $y=\sqrt{x}$, $0 \le x \le 4$, and the x-axis is revolved about the x-axis to generate a solid. **Apply** the integration find the volume of solid.	Marks 4 PLO1 C3
Q3		If A= $2i-4j+\sqrt{5}k$, $and\ B=-2i+4j-\sqrt{5}k$ then **illustrate** the vector $proje_A\, B$	Marks 9 PLO1 C3
Q4		Find the area of the region between the graph and the x-axis Where $y=-x^2+5x-4$, $[0,\ 2]$.	Marks 9 PLO1 C3
Q5	(a)	**Estimate** the angle between $A=i-2j-2k\ and\ B=6i+3j+2k$	Marks 5 PLO1 C3
	(b)	**Change** into a spherical coordinate equation for the sphere $x^2+y^2+(z-1)^2=1$	Marks 4 PLO1 C3

			PLO2 C2

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Department: BE(E)

Subject: Calculus And Analytical Geometry

Teacher : Sir Himayat Ullah

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Q # 1

Part A:

Estimate $\int \theta \sqrt[4]{1-\theta^2}\, d\theta$.

Solutions.

$$\int \theta \sqrt{1-\theta^2}\, d\theta$$

Applying u-substitude: $u = 1-\theta^2$

$$\cancel{\int t \sqrt[4]{1-\theta^2}}$$

$$= \int -\frac{\sqrt[4]{u}}{2}\, du.$$

Take constant out.

$$= \frac{-1}{2} \int \sqrt[4]{u}\, du$$

Applying Radical rule.

$$= \frac{-1}{2} \cdot \int u^{1/4}\, du.$$

$$= \frac{-1}{2} \quad \frac{u^{1/4+1}}{1/4+1}$$

$$= \frac{-1}{2} \cdot \frac{(1-\theta^2)^{1/4+1}}{\frac{1}{4}+1}$$

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$$= \frac{-2}{5}\left(-\theta^2+1\right)^{5/4}$$

$$= \frac{-2}{5}\left(-\theta^2+1\right)^{5/4} + C$$

Ans.

Q# 1

Part B:

Estimate $\int_0^1 x^3(1+x^4)^3\,dx$

using substitution method.

Solution:

$$\int_0^1 x^3(1+x^4)^3\,dx.$$

$$x^3(1+x^4)^3 = x^3 + 3x^7 + 3x^{11} + x^{15}$$

$$\Rightarrow \int_0^1 x^3 + 3x^7 + 3x^{11} + x^{15}\,dx.$$

$$\Rightarrow \int_0^1 x^3\,dx + \int_0^1 3x^7\,dx + \int_0^1 3x^{11}\,dx$$

$$+ \int_0^1 x^{15}\,dx.$$

$$\Rightarrow \left[\frac{x^{3+1}}{3+1}\right]_0^1 + 3\left[\frac{x^{7+1}}{7+1}\right]_0^1 + 3\left[\frac{x^{11+1}}{11+1}\right]_0^1 + \left[\frac{x^{15+1}}{15+1}\right]_0^1$$

$$\Rightarrow \left[\frac{x^{4}}{4}\right]_0^1 + 3\left[\frac{x^{8}}{8}\right]_0^1 + 3\left[\frac{x^{12}}{12}\right]_0^1 + \left[\frac{x^{16}}{16}\right]_0^1$$

$$\Rightarrow \frac{1}{4} + 3\left(\frac{1}{8}\right) + 3\left(\frac{1}{12}\right) + \left(\frac{1}{16}\right)$$

$$\Rightarrow \frac{1}{4} + \frac{3}{8} + \frac{\cancel{3}\,1}{\cancel{12}\,4} + \frac{1}{16}$$

$$\Rightarrow \frac{1}{4} + \frac{3}{8} + \frac{1}{4} + \frac{1}{16}$$

$$\Rightarrow \frac{4+6+4+1}{16}$$

$$\Rightarrow \frac{15}{10} \quad \text{As.}$$

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Attempt all questions.

Q 2

Part A.

Illustrate the center & radius of sphere $x^2+y^2+z^2+3x-4z+1$

Solutions.

$$x^2+y^2+z^2+3x-4z+1=0.$$

$$(x^2+3x)+y^2+z^2-4z+1=0.$$

$$\left(x^2+3x+\left(\frac{3}{2}\right)^2\right)+(y-0)^2+\left(z^2-4z+\left(\frac{-4}{2}\right)^2\right.$$

$$=-1+\left(\frac{3}{2}\right)^2+\left(\frac{-4}{2}\right)^2$$

$$\left(x+\frac{3}{2}\right)^2+(y-0)^2+(z-2)^2=\frac{21}{4}$$

So (x_0, y_0, z_0) = center

$$=\left(\frac{-3}{2}, 0, 2\right)$$

and radius $a=\sqrt{\frac{21}{4}}$

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Q2

Part B:

The region between the curve $y = \sqrt{x}$, $0 \le x \le 4$, and the x-axis is revolved about the x-axis to generate a solid. Apply the integration find the volume of solid.

Solution:

Given That: $y = \sqrt{x}$

$0 \le x \le 4 \Rightarrow a \le x \le b$

As $V = \int_a^b \pi y^2 \, dx$

$V = \int_0^4 \pi (\sqrt{x})^2 \, dx.$

$V = \pi \int_0^4 x \, dx = \pi \cdot \frac{x^2}{2} \Big|_0^4$

$V = \frac{\pi}{2} \left[(4)^2 - 0 \right]$

$V = 8\pi$ Ans

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Q 3

If A is $= 2i - 4j + \sqrt{5}k$, and $B = -2i + 4j - \sqrt{5}k$ then illustrate the vector $proj_A B$.

Soloutions:-

$$B \cdot A = (-2i + 4j - \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k$$

$$B \cdot A = -4i - 16j - 5k$$

$$\boxed{B \cdot A = -25}$$

$$A \cdot A = (2i - 4j + 5k) \cdot (2i - 4j + \sqrt{5}k)$$

$$= 4 + 16 + 5$$

$$\boxed{= 25}$$

$$Proj_A B = \left(\frac{B \cdot A}{A \cdot A}\right) A$$

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$$= \left(\frac{-25}{25}\right)(2i - 4j + \sqrt{5}k)$$

$$= -1(2i - 4j + \sqrt{5}k)$$

$$\boxed{= -2i + 4j - \sqrt{5}k}$$

Q # 4.

Find the area of the region between the graph and the a-xis.

where $y = -x^2 + 5x - 4$, $[0, 2]$.

Solutions

$$y = f(x) = -x^2 + 5x - 4$$

$$[a, b] = [0, 2]$$

$$A = \int_a^b f(x)\, dx.$$

$$A = \int_0^2 (-x^2 + 5x - 4)\, dx$$

$$A = \left(\frac{-x^3}{3} + \frac{5x^2}{2} - 4x - 0 \right) \Big|_0^3$$

$$A = \frac{(3)^3}{3} + \frac{5(3)^2}{2} - 4(3) - 0.$$

$$A = \frac{-\cancel{27}^{\,9}}{\cancel{3}} + \frac{5(9)}{2} - 12$$

$$A = -9 + \frac{45}{2} - 12$$

$$A = 13.5 - 12$$

$\boxed{A = 1.5}$ Ans.

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Q 5

part A

Estimate the angle between ~~A = i - 2j~~

$A = i - 2j - 2k$ and $B = 6i + 3j + 2k$.

Solution:

$$A = i - 2j - 2k.$$

$$|A| = \sqrt{1+4+4} = \sqrt{9} = 3.$$

$$B = 6i + 3j + 2k.$$

$$|B| = \sqrt{36+9+4} = \sqrt{49} = 7$$

$$\theta = \cos^{-1}\left(\frac{A \cdot B}{|A||B|}\right).$$

$$\theta = \cos^{-1}\left(\frac{(i - 2j - 2k)\cdot(6i + 3j + 2k)}{3 \times 7}\right)$$

$$\theta = \cos^{-1}\left(\frac{(1)(6) + (-2)(3) + (-2)(2)}{21}\right)$$

$$\theta = \cos^{-1}\left(\frac{-4}{21}\right)$$

$$\theta = 100.98.$$

P.T.O.

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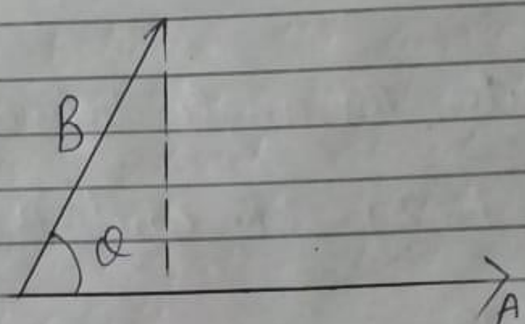
Vector Projection:



Projection of B on A is

$$\text{Proj}_A B = \left(\frac{B \cdot A}{A \cdot A} \right) A.$$

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Q5

Part B.

Change into a spherical coordinate equation for the sphere $x^2+y^2+(z-1)^2=1$

Solutions

$$x^2+y^2+(z-1)^2=1$$

$$(\rho \sin\phi \cos\theta)^2+(\rho \sin\phi \sin\theta)^2$$
$$+(\rho\cos\phi-1)^2=1$$

$$\rho^2\sin^2\phi\cos^2\theta+\rho^2\sin^2\phi\sin^2\theta$$
$$+\rho^2\cos^2\phi+1-2\rho\cos\phi=1$$

$$\rho^2\sin^2\phi(\cos^2\theta+\sin^2\theta)$$
$$+\rho^2\cos^2\phi+1-2\rho\cos\phi=1$$

$$\rho^2(\sin^2\phi)+\rho^2\cos^2\phi+2\cdot\rho\cos\phi=1-1$$

$$\rho^2(\sin^2\phi+\cos^2\phi)-2\rho\cos\phi=0$$
$$\rho^2=2\rho\cos\phi=2\cos\phi.$$