MIDTERM PAPER.

NAME:. HAMAD-UR-RAHMAN.

ID:. 7669

Subject:. Numerical Analysis

Teacher:. Maam Shomaila Mazhar.

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Numerical Analysis.

Q1):- Find a root of the equation

$x^3 + 3.993 \times 10^{-4} = 0.165x^2$

Use Newton Raphson method with $x_0 = 0.02$.

Solution:.

Rearranging the given Equation:

$x^3 - 0.165x^2 + 0.00039 = 0$

By formula, we know that,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f'(x) = 3x^2 - 0.33x = 0$.

$f(x_0) = 0.00033$

$f'(x_0) = -0.0054$

Put $n = 0$ in formula.

$$x_{0+1} = x_0 - \frac{f(x_0)}{f'(x_0)}.$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x)}.$$

Putting values.

$$x_1 = 0.02 - \frac{0.00033}{-0.0054}$$

$$\boxed{x_1 = 0.081}$$

For,

$f(x_1) = -0.00016$

$f'(x_1) = -0.0070$

Put $n = 1$

$$x_{1+1} = x_1 - \frac{f(x_1)}{f'(x_1)}.$$

$$x_2 = 0.081 - \frac{(-0.00016)}{-0.0070}$$

$$\boxed{x_2 = 0.058}$$

Now,

$$f(x_2) = 0.000030$$

$$f'(x_2) = -0.0090$$

Now,

Put $n = 2$

$$x_{2+1} = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

Putting values,

$$x_3 = 0.058 - \frac{0.000030}{(-0.0090)}$$

$$\boxed{x_3 = 0.061}$$

Q No: 4. Use the numbers, $x_0 = 2, x_1 = 2.75, x_2 = 4$. Find the Lagrange interpolation polynomial for $f(x) = 1/x$ at $x = 3$.

Solution:

$x_0 = 2$, $y_0 = 0.5$

$x_1 = 2.75$, $y_1 = 0.36$

$x_2 = 4$, $y_2 = 0.25$

We know that,

By formula of Lagrange Interpolation.

$$y = \frac{(x-x_1)(x-x_2)\cdots\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots\cdots(x_0-x_n)} y_0.$$

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x-x_1)} y_2$$

Now Putting values

$$y = \frac{(3-2.75)(3-4)}{(2-2.75)(2-4)}(0.5) + \frac{(3-2)(3-4)}{(2.75-2)(2.75-4}(0.36)$$

$$+ \frac{(3-2)(3-2.75)}{(4-2)(4-2.75)}(0.25)$$

By Calculating these all we get,

$$y = (-0.833) + 0.384 + 0.025)$$

$$\boxed{y = -0.424}$$

QNo:3. Complete the divided difference table for the given data & construct the interpolating polynomial that uses all this data.

x	$y=f(x)$	$f(,)$	$f(,,)$	$f(,,,)$	$f(,,,,)$
	$f(x_0)$	$f(x_0,x_1)$	$f(x_0,x_1,x_2)$	$f(x_0,x_1,x_2,x_3)$	$f(x_0,x_1,x_2,x_3,x_4)$
$1(x_0)$	0.7651977				
		-0.4837056			
$1.3(x_1)$	0.6200860		-0.10873		
		-0.54896		0.0658785	
$1.6(x_2)$	0.4554022		-0.0494433		-0.0028049
		-0.57812		0.0625125	
$1.9(x_3)$	0.2818186		0.006818		
		-0.57152			
$2.2(x_4)$	0.1103623				

First Divided Difference:-

i) $f(x_0,x_1) = \dfrac{f(x_1)-f(x_0)}{x_1-x_0}$

$f(x_0,x_1) = \dfrac{0.6200860-0.7651977}{1.3-1}$

$f(x_0,x_1) = -0.4837056$

2): $f(x_1, x_2) = \frac{f(x_2) - f(x_1)}{}$

$f(x_1, x_2) = \frac{0.4554022 - 0.6200860}{1.6 - 1.3.}$

$\boxed{f(x_1, x_2) = -0.548946}$

3): $f(x_2, x_3) = \frac{f(x_3) - f(x_2)}{x_3 - x_2.}$

$f(x_2, x_3) = \frac{0.2818186 - 0.4554022}{1.9 - 1.6}$

$\boxed{f(x_2, x_3) = -0.578612}$

4): $f(x_3, x_4) = \frac{f(x_4) - f(x_3)}{x_4 - x_3.}$

$f(x_3, x_4) = \frac{0.1103623 - 0.2818186}{2.2 - 1.9.}$

$\boxed{f(x_3, x_4) = 0.571521}$

Second Divided Difference:

$$f[x_0,x_1,x_2] = \frac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0}$$

$$f[x_0,x_1,x_2] = \frac{-0.548946-(-0.4837056)}{1.6-1}$$

$$\boxed{f[x_0,x_1,x_2] = -0.108734}$$

$$f[x_1,x_2,x_3] = \frac{f[x_2,x_3]-f[x_1,x_2]}{x_3-x_1}$$

$$f[x_1,x_2,x_3] = \frac{-0.578612-(-0.548946)}{1.9-1.3}$$

$$\boxed{f[x_1,x_2,x_3] = -0.0494433}$$

$$f[x_2,x_3,x_4] = \frac{f[x_3,x_4]-f[x_2,x_3]}{x_4-x_2}$$

$$f[x_2,x_3,x_4] = \frac{-0.571521-(-0.578612)}{2.2-1.6}$$

$$\boxed{f[x_2,x_3,x_4] = 0.006818}$$

Third Divided Difference:

$$f[x_0,x_1,x_2,x_3] = \frac{f[x_1,x_2,x_3] - f[x_0,x_1,x_2]}{x_3 - x_0}$$

$$f[x_0,x_1,x_2,x_3] = \frac{-0.049443 - (-0.108734)}{1.9 - 1}$$

$$\boxed{f[x_0,x_1,x_2,x_3] = 0.065878}$$

$$f(x_1,x_2,x_3,x_4) = \frac{f[x_2,x_3,x_4] - f(x_1,x_2,x_3)}{x_4 - x_1}$$

$$f(x_1,x_2,x_3,x_4) = \frac{0.006818 - (-0.049443)}{2.2 - 1.3}$$

$$\boxed{f(x_1,x_2,x_3,x_4) = 0.0625125}$$

⋮

Fourth Divided Difference:

$$f[x_0,x_1,x_2,x_3,x_4] = \frac{f(x_1,x_2,x_3,x_4) - f(x_0,x_1,x_2,x_3)}{2.2 - 1}$$

$$f(x_0,x_1,x_2,x_3,x_4) = \frac{0.0625125 - 0.065878}{2.2 - 1}$$

$$\boxed{f(x_0,x_1,x_2,x_3,x_4) = -0.0028049}$$

Using formula,

$$f(x) = f(x_0) + (x-x_0) f(x_0,x_1) + (x-x_0)(x-x_1) f(x_0,x_1,x_2)$$

$$+ (x-x_0)(x-x_1)(x-x_2) f(x_0,x_1,x_2,x_3) + (x-x_0)(x-x_1)(x-x_2)$$

$$(x-x_3) f(x_0,x_1,x_2,x_3,x_4).$$

Putting values,

$$f(x) = 0.7651977 + (x-1) - 0.4837056 + (x-1)(x-1.3)$$

$$(-0.108734) + (x-1)(x-1.3)(x-1.6)(0.0658785)$$

$$+ (x-1)(x-1.3)(x-1.6)(x-1.9)(-0.0028049)$$