Name = BILAL

ID = 16020

sub = Linear Algebra

BS-CS-2

25/6/2020

(1)

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(Q4A) Let

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Sol = M^{-1} adj M $\frac{1}{|M|}$

$|M| = a \times d - c \times b$

$|M| = ad - bc$

$$\text{adj}M = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$M^{-1} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{[ad - bc]}$$

$$\boxed{M^{-1} = [ad - bc] \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}$$

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B) Write down 2x2 bit matrices with determinant 1. Remember bits are either 0 or 1 and 1+1 = 0 in bits.

Let $D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$|D| = 1 \times 1 - 0 \times 0$

~~|D| = 1~~

$|D|\ 1 - 0$

$\boxed{|D| = 1}$

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Q4 D) compute det A for below 3x3 matrix.

Let $A = \begin{bmatrix} 3 & 1 & 2 \\ 5 & 6 & 7 \\ 1 & 2 & 5 \end{bmatrix}_{3\times 3}$

$|A| = 3\begin{bmatrix} 6 & 7 \\ 2 & 5 \end{bmatrix} - 1\begin{bmatrix} 5 & 7 \\ 1 & 5 \end{bmatrix} - 1\begin{bmatrix} 5 & 7 \\ 1 & 5 \end{bmatrix} + 2\begin{bmatrix} 5 & 6 \\ 1 & 2 \end{bmatrix}$

$|A| = 3[30-14] - 1[25-7] + 2[10-6]$

$|A| = 3[16] - 1[18] + 2[4]$

$|A| = 48 - 18 + 8$

$\boxed{|A| = 38}$

C) Writ down 2x2 bit matrices with determinant 0.

$F = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$

$|F| = 4\times 1 - 2\times 2$

$|F| = 4-4$

$\boxed{|F| = 0}$

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Q1 Consider the following vectors R^3

let $v_1 = \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix}$ $v_2 = \begin{bmatrix} 7 \\ 1 \\ 5 \end{bmatrix}$ $v_3 = \begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix}$

for independent

$a_1 v_1 + a_2 v_2 + a_3 v_3 = 0$

linearly independent

$$\left[\begin{array}{ccc|c} 1 & 7 & 6 & 0 \\ 6 & 1 & 9 & 0 \\ 3 & 5 & 3 & 0 \end{array}\right]$$

$R_2 - 6R_1$

$$\left[\begin{array}{ccc|c} 1 & 7 & 6 & 0 \\ 0 & -41 & -27 & 0 \\ 3 & 5 & 3 & 0 \end{array}\right]$$

$R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 7 & 6 & 0 \\ 0 & -41 & -27 & 0 \\ 0 & -16 & -15 & 0 \end{array}\right]$$

$\frac{1}{-16} R_3$ $$\left[\begin{array}{ccc|c} 1 & 7 & 6 & 0 \\ 0 & -41 & -27 & 0 \\ 0 & 1 & \frac{15}{16} & 0 \end{array}\right]$$

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$R_1 - 6R_3$

$$\left(\begin{array}{ccc|c} 1 & 1 & \frac{90}{16} & 0 \\ 0 & -4 & -27 & 0 \\ 0 & 1 & \frac{15}{16} & 0 \end{array}\right)$$

$$\left[\begin{array}{ccc|c} 1 & 1 & \frac{135}{4} & 0 \\ 0 & -41 & -27 & 0 \\ 0 & 1 & \frac{15}{16} & 0 \end{array}\right]$$

$R_3 - R_1$

$R_1 - R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 13.5 & 0 \\ 0 & -41 & -27 & 0 \\ 0 & 0 & \frac{15}{16} & 0 \end{array}\right]$$

$$a_1 + \frac{135}{4}a_3 = 0$$

$$\boxed{a_1 = -\frac{135}{4} a_3}$$

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(Q6A) $V = \{2\times2$ matrices with entries in $R\}$ usual matrix addition.

$V_2 = [x_2, y_2, z_2]$ then Show that ~~$v_1 + v_2$~~

$v_1 + v_2 = v_2 + v_1$

Sol:- Given $v_1 = [x_1, y_1, z_1]$ and $v_2 = [x_2, y_2, z_2]$

then show $v_1 + v_2 = v_2 + v_1$

L.H.S $v_1 + v_2 = [x_1, y_1, z_1] + [x_2, y_2, z_2]$

$= [x_1 + x_2, y_1 + y_2, z_1 + z_2]$

$= [x_2 + x_1, y_2 + y_1, z_2 + z_1]$

$v_2 + v_1$

RHS

$\boxed{\text{Hence } v_1 + v_2 = v_2 + v_1 \text{ proved}}$

~~6B~~

~~let $v_1 \cdot v_2$~~

(6B) $V_1 \cdot V_2 = V_2 \cdot V_1$

Proof let $v_1 \cdot v_2 = [x_1, y_1, z_1] \cdot [x_2, y_2, z_2] =$

$x_1 x_2 + y_1 y_2 + z_1 z_2$

$v_1 \cdot v_2 = x_2 x_1 + y_2 y_1 + z_1 z_2$

$\boxed{v_1 \cdot v_2 = v_2 \cdot v_1}$

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(Q2b) $T(u+v) = T(u) + T(v)$

$T(cu) = cT(u)$

(i) $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) \to \begin{bmatrix} x-y \\ x+y \\ 2+x \end{bmatrix}$

$= w\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x+y \\ y+2 \end{pmatrix}$

$= T\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} x+y \\ x+y \\ 2x \end{pmatrix}$

$T\begin{pmatrix} 4 \\ 15 \end{pmatrix} \to \begin{bmatrix} 4-15 \\ 4+15 \\ 2(4) \end{bmatrix} = \begin{vmatrix} 11 \\ 19 \\ 8 \end{vmatrix}$

$T(u+v) \quad T(u) + T(v)$

$u\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \qquad v = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

$T\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = T\begin{pmatrix} a_1 + b_1 \\ a_2\ b_2 \end{pmatrix}$

$\begin{pmatrix} a_1+b_1 & -(a_2+b_2 \\ a_1+b_1 & +(a_2+b_2) \\ 2(a+b_2) & \end{pmatrix}$

$\begin{matrix} a_1+b_1 & -a_2 & -b_2 \\ a_1+b_1 & +b_1 & +b_2 \\ 2a_1 & +2b_1 & \end{matrix}$

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$T(u) + T(v)$

$T\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + T\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

$= \begin{pmatrix} a_1 - a_2 \\ a_1 + a_2 \\ 2a_1 \end{pmatrix} + \begin{pmatrix} b_1 - b_2 \\ b_1 + b_2 \\ 2b_1 \end{pmatrix}$

$\begin{pmatrix} a_1 - a_2 + b_1 - b_2 \\ a_1 + a_2 + b_1 + b_2 \\ 2a_1 + 2b_1 \end{pmatrix} = T(u+v)$

$T(u+v) = T(u)$

$a\begin{pmatrix} x \\ y \end{pmatrix}$

$T(u+v) = T(u) + T(v)$

$T(cu) = cT(u)$

$u = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$

$T\left(c\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}\right) T\begin{pmatrix} c \cdot a_1 \\ c \cdot a_2 \end{pmatrix}.$

$\begin{pmatrix} ca_1 - ca_2 \\ ca_1 + ca_2 \\ 2ca \end{pmatrix}$

$c.T\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$

$c\begin{pmatrix} a_1 - a_2 \\ a_1 + a_2 \\ 2a \end{pmatrix} \rightarrow \begin{pmatrix} c(a_1 - a_2) \\ c(a_1 + a_2) \\ c(2a_1) \end{pmatrix} = \underline{c.T(u)}$

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Q3) A vector space over R is a nonempty set V of object called vectors on which are defined two operation called addition + and multiplication by scalars,

A1 closure of addition
for all $u, v \in V$, $u+v$ is defined and
$u+v \in V$.

A2 commutativity for addition
$u+v = v+u$ for all $u, v \in V$.

M1. closure for multiplication
$r \in R$ and each $u \in V$, $r \cdot u$ is defined and
$r \cdot u \in V$.

M2 Multiplication by 1
$1 \cdot u = u$ for all $u \in V$.

Vector space:- Suppose that V is set upon which we have defined two operation
(1) vector addition which combines two elements of V and is denoted by "+" ~~and~~

Ex The singleton set
$\left\{\begin{bmatrix}0\\0\\0\\0\end{bmatrix}\right\}$

is a vector space under the operation

$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix} + \begin{bmatrix}0\\0\\0\\0\end{bmatrix} = \begin{pmatrix}0\\0\\0\\0\end{pmatrix} \quad r \cdot \begin{pmatrix}0\\0\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\end{pmatrix}$$

that it inherits from R^4