Zia Ur Rehman. (1)

ID 11473. Mid term

Differential equation.

22/8/20

Q1 (a) $y' = (x+2)y^2$.

Sol: $y' = (x+2)y^2$

$$\frac{dy}{dx} = (x+2)y^2$$

$$\int \frac{1}{y^2}\,dy = \int (x+2)\,dx.$$

$$\int y^{-2}\,dy = \int (x+2)\,dx.$$

$$\frac{y^{-2+1}}{-1} = \frac{x^2}{2} + 2x + c.$$

$$\frac{y^{-1}}{-1} = \frac{x^2}{2} + 2x + C$$

xing both side by -1.

$$y^{-1} = -\left(\frac{x^2}{2} + 2x + c\right).$$

$$\boxed{y = -\left(\frac{1}{x^2/2 + 2x + c}\right)}$$

Q1 (b) Estimate the general solution of $y' = (y + 9x)^2$.

Sol: let $y' = (y+9x)^2 \rightarrow$ (i)

let $y + 9x = u$.

$$\frac{dy}{dx} + 9 = \frac{du}{dx}.$$

$$\frac{dy}{dx} = \frac{du}{dx} - 9.$$

So (i) become

$$\frac{du}{dx} - 9 = u^2$$

$$\frac{du}{dx} = u^2 + 9.$$

$$\int \frac{1}{u^2+9}\, du = \int dx.$$

$$\int \frac{1}{(3)^2+(u)^2}\, du = \int dx. \Rightarrow \frac{1}{3}\tan^{-1}\left(\frac{u}{3}\right) = x + c_1$$

$$\tan^{-1}\left(\frac{u}{3}\right) = 3x + 3c_1$$

$$\tan^{-1}\left(\frac{u}{3}\right) = 3x + c$$

Q1 (b).

$$\frac{u}{3} = \tan(3x + c).$$

$$y + 9x = 3\tan(3x + c).$$

$$\boxed{y = -9x + 3\tan(x + c).}$$

Q2 (a) Estimate the general solution

$$x^3 dx + y^3 dy = 0$$

Sol:-

$$M dx + N dy = 0$$

$$M = x^3 , \quad N = y^3$$

$$\frac{\partial M}{\partial y} = 0 \qquad \frac{\partial N}{\partial x} = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \qquad \text{so exact}$$

$$u = \int M dx + k(y).$$

$$u = \int x^3 dx + k(y).$$

$$u = \frac{x^4}{4} + k(y) \rightarrow (i)$$

$$\frac{\partial u}{\partial y} = 0 + \frac{d}{dy} k(y).$$

$$\frac{\partial u}{\partial y} = \frac{d}{dy} k(y)$$

since we know that

$$\frac{\partial u}{\partial y} = N = y^3$$

$$y^3 = \frac{d}{dy} k(y) \Rightarrow \int y^3 = \int d\, k(y)$$

$$k(y) = y^4/4 + c_1$$ Put in 1

$$u = \frac{x^4}{4} + \frac{y^4}{4} + c_1$$

$$c_2 = \frac{x^4}{4} + \frac{y^4}{4} + c_1$$

$$c_2 - c_1 = \frac{x^4}{4} + \frac{y^4}{4} \Rightarrow \boxed{c = \frac{x^4}{4} + \frac{y^4}{4} = 4}$$

Ans

Q3
(a) Find the general solution

$$4y'' - 20y' + 25y = 0.$$

Sol:

$$4y'' - 20y' + 25y = 0$$

$a = -20 \quad b = 25$

$N^2 + aN + b = 0$

$4N^2 - 20N + 25 = 0$

$4N^2 - 10N - 10N + 25 = 0.$

$2N(2N-5) - 5(2N-5) = 0$

$(2N-5)(2N-5) = 0.$

$2N - 5 = 0 \quad , \quad 2N - 5 = 0.$

$N = \frac{5}{2} \qquad N = \frac{5}{2}.$

So roots are real and equal.

i.e Case II

So $y = (c_1 + c_2 x)e^{Nx}.$

$y = (c_1 + c_2 x)e^{5/2 x}.$

Q3 (b): Estimate general solution of $4y'' - 6y' - 7y = 0$

Sol: $4y'' - 6y' - 7y = 0$

$4\lambda^2 - 6\lambda - 7 = 0$

So by quadratic formula

Here

$a = 4, \quad b = -6 \quad c = -7.$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$\lambda = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(4)(-7)}}{2(4)}.$$

$$\lambda = \frac{6 \pm \sqrt{36 + 112}}{8}$$

$$\lambda = \frac{6 \pm \sqrt{36 + 112}}{8} \Rightarrow \frac{6 \pm \sqrt{148}}{8}$$

Taking 2 as common

$$\lambda = \frac{3 \pm \sqrt{37}}{4}.$$

So $\lambda_1 = \dfrac{3 + \sqrt{37}}{4}.$

$\lambda_2 = \dfrac{3 - \sqrt{37}}{4}.$

(7)

Roots are Real, so

$$y = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$$

$$\boxed{y = c_1 e^{\frac{3+\sqrt{37}}{4}t} + c_2 e^{\frac{3-\sqrt{37}}{4}t}}$$

Ans