

Q No 1

A = {sum is ~~even~~ even}

B = {sum is even}

let = C = {sum is greater than 8}

D = {2 dice same outcome}

A = {(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}

B = {(2,2), (2,6), (4,2), (6,2) ...}

C = {(6,6), (6,4)}

D = {(1,1), (2,2), (4,4), (6,6)}

$$P(A) = \frac{6}{36}$$

$$P(B) = \frac{4}{36}$$

$$P(C) = \frac{2}{36}$$

$$P(D) = \frac{4}{36}$$

$$P(A \cap B) = \frac{\phi}{36} = \phi = 0$$

$$P(A \cap C) = \frac{\phi}{36} = \phi = 0$$

$$P(A \cap D) = \frac{\phi}{36} = \phi = 0$$

Hence

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{4/36} = 0$$

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{0}{2/36} = 0$$

$$P(A|D) = \frac{P(A \cap D)}{P(D)} = \frac{0}{4/36} = 0$$

The results show that probability is zero in all the cases given.

Q2

Throwing off one single dice can give 6 outcomes i.e. 1, 2, 3, 4, 5, 6.

So, throw of 2 dice can give us  $6 \times 6 = 36$  outcomes

Sequence leading to get atleast 1.

(1,6), (2,6), (2,5), (3,4), (3,5), (3,6),  
(4,3), (4,4), (4,5), (4,6), (5,2), (5,3),  
(5,4), (5,5), (5,6), (6,1), (6,2), (6,3),  
(6,4), (6,5), (6,6).

Therefore, probable outcomes for getting at least 7 =  $1+2+3+4+5+6 = 21$

So, Probability is =  $\frac{21}{36} = \frac{7}{12} = 0.58$



Q No. 3

Solution

$$\text{Given } p = \frac{2}{3} \quad n = 8$$

$$q = 1 - p \\ = 1 - \frac{2}{3}$$

$$q = \frac{1}{3}$$

Let  $x$  denotes number of games  
won by A.

$$\begin{aligned} \textcircled{1} P(X=4) &= \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4 \\ &= \frac{1120}{6561} \\ &= 0.1707 \end{aligned}$$

$$\begin{aligned} \textcircled{2} P(X \geq 4) &= 1 - P(X < 4) \\ &= 1 - \sum_{x=0}^3 \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x} \\ &= 1 - \left[ \left(\frac{1}{3}\right)^8 + 8 \left(\frac{2}{3}\right) \left(\frac{1}{3}\right)^7 + 28 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^6 + 56 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^5 \right] \\ &= 1 - \frac{1}{6561} [1 + 16 + 112 + 448] \\ &= 1 - \frac{577}{6561} \\ &= \frac{6561 - 577}{6561} \\ &= \frac{5984}{6561} = 0.9121 \end{aligned}$$

$$\textcircled{3} \quad P(3 \leq X \leq 6)$$

$$\sum_{x=3}^6 \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x}$$

$$= \binom{8}{3} \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^5 + \binom{8}{4} \left(\frac{2}{3}\right)^4$$

$$\left(\frac{1}{3}\right)^4 + \binom{8}{5} \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^3 +$$

$$\binom{8}{6} \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^2$$

$$= \frac{8}{(3)^8} (56 + 140 + 224 + 224)$$

$$= \frac{8 \times 644}{6561} = \frac{5152}{6561}$$

$$= 0.7852$$

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 $\alpha$ 
 $\alpha$

Q5

The binomial distribution

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

for  $x=0, 1, 2, \dots$ 

$$\mu = np, \quad \sigma^2 = np(1-p)$$

$$E(X) = np, \quad \text{var}(X)$$

Let  $U_1, \dots, U_n$  be the independent Binomial random variables.

$$E(U_i) = p, \quad \text{var}(U_i) = p(1-p)$$

$$X = U_1 + \dots + U_n$$

$$E(X) = E(U_1 + \dots + U_n)$$

$$\begin{aligned} E(X) &= E(U_1) + \dots + E(U_n) \\ &= p + \dots + p = np \end{aligned}$$

Let  $U_1, \dots, U_n$  be the independent Binomial variables

$$E(U_i) = p, \quad \text{var}(U_i) = p(1-p)$$

$$X = U_1 + \dots + U_n$$

$$\text{var}(X) = \text{var}(U_1 + \dots + U_n)$$

$$\text{var}(X) = \text{var}(U_1) + \dots + \text{var}(U_n)$$

$$= p(1-p) + \dots + p(1-p)$$

$$= np(1-p)$$

The binomial theorem

$$(a+b)^n = \sum_{y=0}^n \binom{n}{y} a^y b^{n-y}$$



$$\begin{aligned}
 E(X) &= \sum_{x=0}^n x P(x) \\
 &= \sum_{x=0}^n x \binom{n}{x} p^x (1-p)^{n-x} \\
 &= \sum_{x=0}^n x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}
 \end{aligned}$$

here

$$\begin{aligned}
 &= n p \sum_{x=1}^n \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} (1-p)^{n-x} \\
 &= n p \sum_{x=1}^n \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} (1-p)^{n-x} \\
 &= n p^{x-1} (1-p)^{(n-1)-(x-1)}
 \end{aligned}$$

$$\begin{aligned}
 m &= n-1 \\
 y &= x-1
 \end{aligned}$$

Now

$$\begin{aligned}
 n p \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} \\
 n p \sum_{y=0}^m \binom{m}{y} p^y (1-p)^{m-y}
 \end{aligned}$$

$$\begin{aligned}
 V_2(X) &= E[(X-M)^2] \\
 &= \sum (x-M)^2 P(x)
 \end{aligned}$$

$$E[(X-M)^2] = E(X^2) - [E(X)]^2$$

$$E(X^2) = \sum_{x=0}^n x^2 \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$E[x(x-1)] = \sum_{x=2}^n \frac{n!}{(n-2)!(n-x)!} p^2 (1-p)^{n-x}$$

$$n(n-1)p^2 \sum_{x=2}^n \frac{(n-2)!}{(n-2)!(n-x)!}$$

$$p^{n-2} (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^n \frac{(n-2)!}{(n-2)!(n-x)!}$$

$$(1-p)$$

here  $m = n-2$   
 $y = x-2$

$$E[x(x-1)] = n(n-1)p^2 \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} [(p + (1-p))^m - 1]$$

$$E[x(x-1)] = n(n-1)p^2$$

$$E[x^2 - x] = n(n-1)p^2$$

$$\text{Var}(x) = E[x^2] - [E(x)]^2$$

$$= n(n-1)p^2 + np - (np)^2$$

$$= np[(n-1)p + 1 - np]$$

$$= np(1-p)$$



Q6

Binomial Distribution.

→ If  $X$  is a discrete random variable with probability mass function

Binomial Distribution formula

$$P(X) = {}^n C_x p^x (1-p)^{n-x}$$

$X$  = random variable  $x$

$x$  = No of Success

$C$  = Combination of  $x$  Successes from  $n$  trials

$p$  = Probability of success

$1-p$  = Probability of failure

$(n-x)$  = number of failures

Important points

→ where  $f.c.v$  should be greater than 1

→  $0 < p < 1$  because ~~probability~~ probability cannot be more

-  $X$  is a discrete random variable which may take on only countable number of distinct values such

e.g. 1, 2, 3, 4, ...



Q 7. SolData A

$$\text{Coefficient variation} = \frac{\text{Standard deviation}}{\text{Mean}} \times 100$$

Here.

$$SD = 3$$

$$\text{Mean} = 45$$

$$CV = \frac{3}{45} \times 100$$

$$= 6.666$$

$$= 7 \text{ (rounding off to nearest 10)}$$

Data B

$$SD = 11, \text{ mean} = 60$$

$$CV = \frac{SD}{\text{mean}} \times 100$$

$$= \frac{11}{60} \times 100$$

$$= 18.3 = 18$$

Data C

$$SD = 5, \text{ mean} = 50$$

$$= \frac{5}{50} \times 100$$

$$= 10 \text{ (rounding off to nearest 10)}$$

Conclusion

Hence the results of all 3 datasets are mentioned above as 7, 18 and 10, being the highest value of 18 and lowest of 7.

Conditions for binomial distribution

- ① The random experiment is ~~performed~~ performed repeatedly a finite and fixed number of times
- ② The outcome of the random experiment (trials) results in the dichotomous classification of events
- ③ All the trials are independent

Formulas

$$\text{Mean } \mu = n \cdot p$$

$$\text{Variance } \sigma^2 = n \cdot p \cdot q$$