**Final Term Assignment (Spring 2020)**

**Program: MBA-90**

**Semester: Summer Semester-2020**

**Course: Business Mathematics & Statistics**

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## *Question No.1*

i. When 3 balanced coins are tossed and $X$ be a random variable that denotes the number of heads then $f(0) =$

**Answer:** **(a)** $1/8$

ii. Let $X$ be a continuous random variable and $f(x) = x,\ 0 \le x \le 1,\quad f(x) = 0$ otherwise, then $E(X) =$

**Answer:** **(b)** 1/3

iii. If $E(X) = 2$ and $E(X^2) = 5$, then $V(X) =$

**Answer:** **(a)** 1

iv. The sum of squares of residuals for a straight line is

**Answer:** **(a)** $S = \sum Y^2 - a\sum Y - b\sum XY$

v. The value of $E(aX + b) =$

**Answer:** **(b)** $aE(X) + b$

vi. The variation of the Y values around the regression line is measured by

**Answer:** **(d)** None of above

vii. If, $P(A) = 2$ and $P(A \cap B) = 8$ then $P(B/A) =$

**Answer:** **(c)** 4 (Probability cannot be greater than 4)

viii. A random variable $X$ is normally distributed with $\mu = 50$, $\sigma^2 = 25$ and . $x = 0$ The value of

Standardized $z =$

**Answer:** **(a)** -10

ix. If a fair coin is tossed two times then the probability of at least one head appears is

**Answer:** **(c)** 3/4

x. If A and B are mutually exclusive events then $P(A \cup B) =$

**Answer:** **(a)** $P(A) + P(B)$

Q 2:-

Answer:-

Solution:- $f(x) = 3/4\,(3-x)(x-5) \quad 3 \le x \le 5$

$$= 3/4\,(3x - 15 - x^2 + 5x)$$

$$= 3/4\,(-x^2 + 8x - 15)$$

Variance $= E(x^2) - \{E(x)\}^2$

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= 3/4 \int_3^5 x(-x^2 + 8x - 15)\,dx$$

$$= -3/4 \int_3^5 x^3\,dx + 3/4 \int_3^5 8x^2 - 3/4 \times 15 \int_3^5 x\,dx$$

$$= -3/4 \cdot x^4/4 \Big/_3^5 + 3/4 \times 8 \; x^3/3 \Big/_3^5 - 3/4 \times 15 \; x^2/2 \Big/_3^5$$

$$= -3/16 \cdot x^4 \Big/_3^5 + 2 \cdot x^3 \Big/_3^5 - 45/8 \; x^2 \Big/_3^5$$

$$= -3/16 \left\{(5)^4 - (3)^4\right\} + 2\left\{(5)^3 - (3)^3\right\} - 45/8\left\{(5)^2 - (3)^2\right\}$$

$$= -3/16\,(625 - 81) + 2\,(125 - 27) - 45/8\,(25 - 9)$$

$$= -3/16\,(544) + 2\,(98) - 45/8\,(16)$$

$$= -3\,(34) + 2\,(98) - 45 \times 2$$

$$= -102 + 196 - 90$$

$E(x) = 4$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$

$= 3/4 \int_3^5 (x^2(-x^2+8x-15)\, dx$

$= 3/4 \int_3^5 (-x^4+8x^3-15x^2)\, dx$

$= -3/4 \int_3^5 x^4 dx + 3/4 \times 8 \int_3^5 x^3 dx - 3/4 \times 15 \int_3^5 x^2\, dx$

$= -3/4 \left. x^5/5 \right|_3^5 + 6 \left. x^4/4 \right|_3^5 - 45/4 \left. x^3/3 \right|_3^5$

$= -3/20 \{(5)^5-(3)^5\} + 6/4 \{(5)^4-(3)^4\} - 45/12 \{(5)^3-(3)^3\}$

$= -3/20 (3125-243) + 6/4 (625-81) - 45/12 (125-27)$

$= -3/20 (2882) + 6/4 (544) - 45/12 (98)$

$= -4323/10 + 6 \times (136) - 45/6 \times (49)$

$= -4323/10 + 816/1 - 2205/6$

$= \frac{-12969 + 24480 - 11025}{30}$

$= \frac{486}{30}$

Pg-4/10

$Ex^2 = 81/5$

$Var(x) = Ex^2 - \{E(x)\}^2$

$= 81/5 - (4)^2$

$= 81/5 - 16$

$= \frac{81-80}{5}$

$= 1/5$

$= 0.2$

$S.D = \sqrt{Var(x)}$

$= \sqrt{1/5}$

$= \sqrt{0.2}$

$= 0.44$

x———x———x———x———x———x

Pg-5/10

Q3: Answer

Solution: (a) $(A \cup B)^c = A^c \cap B^c$

Proof: we have to prove that

if $U = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23\}$

$A = \{3, 6, 9, 12, 15, 18\}$

$B = \{10\}$

$C = \{3, 6, 9, 12, 15, 18\}$

then

$(A \cup B)^c = A^c \cap B^c$

L.H.S =>

$(A \cup B)^c = U/(A \cup B)$

$= \{1, 3, 5, 7 ---- 23\} / \{1, 3, 6, 9, 12, 15, 18\} \cup \{10\}$

$= \{1, 3, 5, 7 ---- 23\} / \{3, 6, 9, 10, 12, 15, 18\}$

$(A \cup B)^c = \{1, 5, 11, 13, 17, 19, 21, 23\}$ — i

R.H.S =>

$A^c = U/A$

$= \{1, 3, 5, ----- 23\} / \{3, 6, 9, 12, 13, 18\}$

$= \{1, 5, 7, 11, 13, 17, 19, 21, 23\}$

$B^c = U/B$

$= \{1, 3, 5 ----- 23\} / \{10\}$

$= \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23\}$

$A^c \cap B^c = \{1, 5, 11, 13, 19, 21, 23\}$ — (ii)

So from eq (i) and eq (ii)

L.H.S = R.H.S

$(A \cup B)^c = A^c \cap B^c$

Hence proved.

Q3: Part (b) Answer

ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof:- we have to prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S =>

$A \cap (B \cup C) = \{3,6,9,12,15,18\} \cap [\{10\} \cup \{3,6,9,12,15,18\}]$

$= \{3,6,9,12,15,18\} \cap \{3,6,9,10,12,15,18\}$

$= \{3,6,9,12,15,18\}$ — (i)

R.H.S =>

$(A \cap B) \cup (A \cap C) = [\{3,6,9,12,15,18\} \cap \{10\}] \cup [\{3,6,9,12,15,18\} \cap \{3,6,9 \ldots 18\}]$

$= \{\ \} \cup \{3,6,9,12,15,18\}$

$= \{3,6,9,12,15,18\}$ — ii

From eq (i) and eq (ii)

L.H.S = R.H.S

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Hence Proved.

Q NO 4:- Answer



Solution:-

| $X$ | $Y$ | $XY$ | $X^2$ |
|---|---|---|---|
| 5 | 16 | 80 | 25 |
| 6 | 19 | 114 | 36 |
| 8 | 23 | 184 | 64 |
| 10 | 28 | 280 | 100 |
| 12 | 36 | 432 | 144 |
| 13 | 41 | 533 | 169 |
| 15 | 44 | 660 | 225 |
| 16 | 45 | 720 | 256 |
| 17 | 50 | 850 | 289 |
| 102 | 302 | 3853 | 1308 |

$$\hat{Y} = a + bx$$

$$a = \bar{Y} - b\bar{x}$$

$$b = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}$$

$$= \bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{3853 - \frac{(102)(302)}{9}}{1308 - \frac{(102)^2}{9}}$$

$$= \frac{102}{9} \Rightarrow 11.33$$

$$= \frac{3853 - 3422.6}{1308 - 1156} \Rightarrow \frac{430.4}{152}$$

$$= 2.83$$

Pg-8/10

$$\bar{Y} = \frac{\Sigma Y}{n}$$

$$= \frac{302}{9}$$

$$= 33.5$$

$\bar{Y} = 33.5$

$\bar{X} = 11.33$

$b = 2.83$

$$a = 33.5 - 11.33(2.83)$$

$$= 33.5 - 32.064$$

$$= 1.4361$$

$$\hat{Y} = 1.4361 + 2.83X$$

The estimated regression cofficient is $b = 2.83$, which indicates that the value of Y increase by 2.83 unit for a unit increase in X.

X ———— X ———— X ———— X ———— X ———— X

Pg-9/10

Question NO 5 :- Answer

Solution :-

| Sr # | $x$ | $\log x$ |
|---|---|---|
| 1 | 9 | 0.9542 |
| 2 | 12 | 1.0792 |
| 3 | 15 | 1.1761 |
| 4 | 15 | 1.1761 |
| 5 | 16 | 1.2041 |
| 6 | 18 | 1.2553 |
| 7 | 20 | 1.3010 |
| 8 | 20 | 1.3010 |
| 9 | 25 | 1.3979 |
| 10 | 30 | 1.4771 |
| | 179 | 12.331 |

$$G.M = \text{Antilog}\left(\frac{\sum \log x}{n}\right)$$

$$= \text{Antilog}\left(\frac{12.331}{10}\right)$$

$$= \text{Antilog}\,(1.2322)$$

$$= 17.06$$

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ item}$$

$$= \left(\frac{11}{2}\right)^{th} \text{ item}$$

$$= 5.5^{th} \text{ item}$$

$$= \frac{5^{th} \text{ item} + 6^{th} \text{ item}}{2}$$

$$= \frac{16 + 18}{2}$$

$$= \frac{34}{2}$$

$$\text{Median} = 17$$

Mode = 20 and 15 (most repeated value)

x——x——x——x——x——x