

Day: MTWTFs

Date: ___/___/___

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Subject : Numerical analysis

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Checked By:.....Parents:.....Excellent Good 

Q1 Apply both Euler's method and the improved Euler method of solution of

$$\frac{dy}{dx} = 2x, \quad y(0) = 1$$

For $0 \leq x \leq 0.5$ using $h = 0.1$
Compare your answer with the analysis sol. work throughout to three decimal places.

Ans By Euler's method :

(a) Given data
 $y(0) = 1, \quad h = 0.1, \quad x_0 = 0$

By formula

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(2x_n)$$

1st Iteration:

$$x = 0$$

$$y_1 = y_0 + h(2x_0)$$

$$y_1 = 1 + 0.1(2(0))$$

$$y_1 = 1 + 0.1$$

$$\boxed{y_1 = 1.1}$$

$$\rightarrow x_{n+1} = x_n + h$$

$$x_1 = x_0 + h$$

$$x_1 = 0 + 0.1$$

$$\boxed{x_1 = 0.1}$$

2nd Iteration.

$$n = 1$$

$$y_2 = y_1 + h (2x_1)$$

$$y_2 = 1.1 + 0.1 (2(0.1))$$

$$\boxed{y_2 = 1.12}$$

$$x_{n+1} = x_n + h$$

$$x_2 = x_1 + h$$

$$x_2 = 0.1 + 0.1$$

$$\boxed{x_2 = 0.2}$$

3rd Iteration:

$$n = 2$$

$$y_3 = y_2 + h (2x_2)$$

$$y_3 = 1.12 + 0.1 (2(0.2))$$

$$\boxed{y_3 = 1.16}$$

$$x_{n+1} = x_n + h$$

$$x_3 = x_2 + 0.1$$

$$x_3 = 0.2 + 0.1$$

$$\boxed{x_3 = 0.3}$$

(b) By Modified Euler method

$$\frac{dy}{dx} = 2$$

Given data:

$$y_0 = 1, x_0 = 0, h = 0.1$$

Formula

$$y_{n+1}^* = y_n + h [f(x_n)]$$

$$y_{n+1}^* = y_n + h (2x_n) \quad \text{--- (1)}$$

$$y_{n+1} = y_n + \frac{h}{2} [f(x_n, y_n) + f(x_{n+1}, y_{n+1}^*)]$$

$$= y_n + \frac{h}{2} [2x_n + 2x_n]$$

$$= y_n + \frac{h}{2} [4x_n]$$

1st Iteration:

$$n = 0$$

$$x_{n+1} = x_n + h$$

$$x_1 = x_0 + h$$

$$x_1 = 0 + 0.1$$

$$x_1 = 0.1$$

$$y_1 = y_0 + \frac{h}{2} (4x_0)$$

$$y_1 = 1 + \frac{0.1}{2} (4(0))$$

$$y_1 = 1$$

2nd Iteration:

$$n = 1$$

$$x_2 = x_1 + h$$

$$x_2 = 0.1 + 0.1$$

$$x_2 = 0.2$$

$$y_2 = y_1 + \frac{h}{2} (4x_1)$$

$$y_2 = 1 + \frac{0.1}{2} (4(0.1))$$

$$y_2 = 1.02$$

3rd Iteration:

$$n = 2$$

$$x_3 = x_2 + h$$

$$x_3 = 0.2 + 0.1$$

$$x_3 = 0.3$$

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$$y_3 = y_2 + \frac{h}{2} (4u_2)$$

$$= 1.02 + \frac{0.1}{2} (4(0.2))$$

$$y_3 = 1.06$$

Ans

Question No 2

Use the fourth order Runge Kutta method to obtain as of $\frac{dy}{dx} = x^2 + x - y$

Sol

Given data:

$$y = 0, x = 0, h = 0.2 \quad 0 \leq x \leq 0.6$$

$$y_{n+1} = y_n + k$$

 \Rightarrow 1st Iteration:

$$n = 0$$

$$y_1 = y_0 + k, \quad k = \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = hf(x_n, y_n)$$

$$k_1 = h(x_0^2 - x_0 - y_0)$$

$$k_1 = 0.2(0^2 - 0 - 0)$$

$$\boxed{k_1 = 0}$$

$$\Rightarrow k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}\right)$$

$$= 0.2f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}\right)$$

$$= 0.2f\left(0 + \frac{0.2}{2}, 0 + \frac{0.2}{2}\right)$$

$$= 0.2f(0.1, 0.1)$$

$$= 0.2(0.1^2 + 0.1 - 0.1)$$

$$\boxed{k_2 = 0.0020}$$

$$\begin{aligned}
 k_3 &= hf \left(x_{n+h} , y_n + \frac{k_2}{2} \right) \\
 &= 0.2f \left(0 + \frac{0.2}{2} , 0 + \frac{0.002}{2} \right) \\
 &= 0.2f (0.1, 0.001) \\
 &= 0.2 (0.1^2 + 0.1 - 0.001)
 \end{aligned}$$

$$k_3 = 0.0218$$

$$\begin{aligned}
 k_4 &= hf (x_{n+h} , y_n + k_3) \\
 &= 0.2f (0 + 0.2, 0 + 0.0218) \\
 &= 0.2f (0.2, 0.0218) \\
 &= 0.2 (0.2^2 + 0.2 - 0.0218)
 \end{aligned}$$

$$k_4 = 0.0436$$

$$k = \frac{1}{6} (0 + 2(0.002) + 2(0.0218) + 0.0436)$$

$$k = 0.0152$$

$$y_1 = 0 + 0.0152$$

$$y_1 = 0.0152$$

Q_{no} 3

Given data:

$$a = 0 \quad b = 10 \quad n = 10$$

$$h = \frac{b-a}{n} = \frac{10-0}{10} = 1$$

Sol: ∴

x	0	1	2	3	4	5	6	7	8	9	10
F(x)	10.1	17.2	24.4	29.2	34.6	41.2	50.9	57.8	60.3	61.2	62.1

Using formula.

$$f(x) \quad \Delta x = \frac{h}{2} \left[f(x_0) + 2(f(x_1) + f(x_2) + f(x_3) + \dots + f(x_9) + f(x_{10})) \right]$$

$$= \frac{1}{2} \left[10.1 + 2(17.2 + 24.4 + 29.2 + 34.6 + 41.2 + 50.9 + 57.8 + 60.3 + 61.2) + 62.1 \right]$$

$$= \boxed{412.9} \quad \text{Ans}$$

Q.No 4 $\int_2^3 \ln(x^3 + 1) dx$

use 10 strips

Sol: $n = 10$

$$h = \frac{3-2}{10} = 0.1$$

x	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
$f(x)$	0.693	0.846	1.003	1.162	1.320	1.476	1.628	1.777	1.922	2.062

Now using formula

$$\int_a^b f(x) dx = \frac{h}{3} \left[f(x_0) + 4(f(x_1) + f(x_3) + \dots) \right. \\ \left. + 2[f(x_2) + \dots] + f(x_n) \right]$$

$$= \frac{0.1}{3} \left[0.693 + 4(0.846 + 1.162 + 1.476 + 1.777) \right. \\ \left. + 2(1.003 + 1.320 + 1.628 + 1.922) \right. \\ \left. + 2.062 \right]$$

$$= \boxed{1.184} \quad \text{Ans}$$