

Quiz # 1

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Question :-

A yarn merchant sells brands A, B, C of yarn each of which is a blend of Pakistani, Egyptian and American cotton in the ratio 1:2:1, 2:1:1, 2:0:2 respectively. If one kilogram of A, B, C costs, 40, 50 and 60 rupees respectively. Find the cost of a kilogram of cotton of each country.

Solution :-

1:2:1, 2:1:1, 2:0:2

40	50	60
P E	P P	P P
A E	A E	A A
B ₁	B ₂	B ₃

Let x , y and z be the cost/kg of Pakistani, Egyptian and American cotton respectively. Then according to the given conditions.

$$\left. \begin{aligned} \frac{1}{4}x + \frac{2}{4}y + \frac{1}{4}z &= 40 \\ \frac{2}{4}x + \frac{1}{4}y + \frac{1}{4}z &= 50 \\ \frac{2}{4}x + \frac{2}{4}z &= 60 \end{aligned} \right\} \dots (S')$$

$$\left. \begin{aligned} 1x + 2y + 1z &= 160 \\ 2x + 1y + 1z &= 200 \\ 1x + 1z &= 120 \end{aligned} \right\} \dots (S)$$

In matrix form, we can write it as.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \underline{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \underline{b} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

$$AX = \underline{b}$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 180 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 180 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 180 \end{bmatrix}$$

$$|A| = -2 \quad |A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1(1 \times 1 - 0 \times 1) - 2(2 \times 1 - 1 \times 1) + 1(2 \times 1 - 1 \times 1)$$

$$|A_1| = -120 \quad |A_1| = \begin{vmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 180 & 0 & 1 \end{vmatrix} = 160(1 \times 1 - 0 \times 1) - 2(200 \times 1 - 180 \times 1) + 1(200 \times 1 - 180 \times 1)$$

$$|A_2| = -40 \quad |A_2| = \begin{vmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{vmatrix} = 1(200 \times 1 - 120 \times 1) \\ - 160(2 \times 1 - 1 \times 1) \\ + 1(2 \times 1 - 1 \times 200).$$

$$|A_3| = 120 \quad |A_3| = \begin{vmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 1 & 120 \end{vmatrix} = 1(1 \times 120 - 0 \times 200) \\ - 2(2 \times 120 - 1 \times 200) \\ + 160(2 \times 120 - 1 \times 1)$$

$$|A_1| = -2,$$

$$|A_1| = -120, |A_2| = -40,$$

$$|A_3| = -120$$

According to Cramer's rule.

$$x = \frac{|A_1|}{|A|} = \frac{-120}{-2} = 60$$

$$y = \frac{|A_2|}{|A|} = \frac{-40}{-2} = 20.$$

$$z = \frac{|A_3|}{|A|} = \frac{-120}{-2} = 60.$$

$$(x, y, z) = (60, 20, 60).$$