Department of Electrical Engineering
Assignment
Date: 14-04-2020

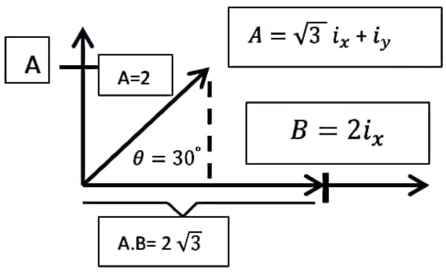
Course Details

Course Title: Electro Magnetic Field Theory **Module:** ____________

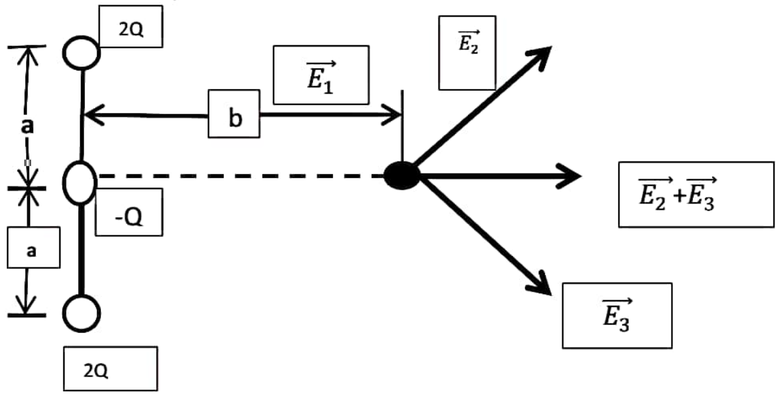
Instructor: ____________ **Total Marks:** 30

Student Details

Name: ____________ **Student ID:** ____________

Q1: Solve the following short Question	(a)	Transform the vector $B = yi\ (x+z)j$ located at point (-2,6,3) into cylindrical coordinates	Marks 2 CLO 1
	(b)	Convert the point (3,4,5) from Cartesian to spherical coordinates	Marks 2 CLO 1
	(c)	Find the spherical coordinates of A(2,3,-1)	Marks 2 CLO 1
	(d)	Find the Cartesian coordinates of B(4.25,120)	Marks 2 CLO 1
	(e)	Find the force between two charges when they are brought in contact and separated by 4cm apart, charges are 2nC and -1nC, in µN.	Marks 2 CLO 2
	(f)	Find the electric field intensity of two Charges -2C and -1C separated by a distance 1m in air	Marks 2 CLO 2
	(g)	Determine the charge that produce an electric field strength of 40 v/cm at a distance of 30cm in vacuum (in 10^{-8}c)	Marks 2 CLO 2
	(h)	A charge of $2 * 10^{-7}$ C is acted upon by a force of 0.1N. determine the distance to the other charge of $4.5 * 10^{-7}$C, both the charges are in vacuum	Marks 2 CLO 2
Q2:	**(a)**	**Find the angle between the vectors shown in figure.** 	**Marks 4** **CLO 1**

	(b)	**Find the gradient of each of the following functions where a and b are constant** **(i)** $f = ax^2 + by^3z$ **(ii)** $f = ar^2 \sin \emptyset + brz \cos 2 \emptyset$	**Marks 4** **CLO 1**
Q3:		**Three pointer charges are placed on the y-axis as shown. Find the electric field at point P on the x-axis.**	**Marks 6** **CLO 2**



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Q NO 1

(a) :→

Solution:→

$$B = y a_x + (x+z) a_y$$

point = $P(-2, 6, 3)$.

At point P :

$x = -2, \quad y = 6, \quad z = 3.$

$\rho = \sqrt{x^2 + y^2} \Rightarrow \rho = \sqrt{-2^2 + 6^2}$

$\rho = \sqrt{4 + 36} \Rightarrow \rho = 2\sqrt{10}$.

$\phi = \tan^{-1} y/x$.

$\phi = \tan^{-1} 6/-2$

$\phi = 108.43°$

$z = 3$.

for vector B

$B_x = y, \quad B_y = x + z, \quad B_z = 0.$

$$\begin{bmatrix} B_\rho \\ B_\phi \\ B_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ x+z \\ 0 \end{bmatrix}.$$

P.T.O

$\Rightarrow$

$$B_\rho = y\cos\phi + (x+z)\sin\phi$$

$$B_\phi = -y\sin\phi + (x+z)\cos\phi$$

$$B_z = 0$$

Now

$$x = \rho\cos\phi \;,\; y = \rho\sin\phi$$

$\Rightarrow$

$$B = (B_\rho, B_\phi, B_z) = [\rho\cos\phi\,\rho\sin\phi + (\rho\cos\phi + z)\sin\phi]\,a_\rho$$
$$+ [-\rho\sin^2\phi + (\rho\cos\phi + z)\cos\phi]\,a_\phi + 0$$

Now At P

$$\rho = 2\sqrt{10} \quad \& \quad \phi = 108.43^\circ$$

$$\Rightarrow \cos\phi = \frac{-2}{\sqrt{40}} \;,\; \sin\phi = \frac{6}{\sqrt{40}}$$

$\Rightarrow$

$$B = \left[2\sqrt{10}\cdot\frac{-2}{2\sqrt{10}}\cdot\frac{6}{2\sqrt{10}} + \left(\sqrt{40}\cdot\frac{2}{\sqrt{40}} + 3\right)\cdot\frac{6}{\sqrt{40}}\right)a_\rho$$
$$+ \left[-2\sqrt{10}\cdot\frac{36}{40} + \left(2\sqrt{10}\cdot\frac{-2}{2\sqrt{10}} + 3\right)\frac{2}{2\sqrt{10}}\right]a_\phi$$

$$B = \frac{-6}{2\sqrt{10}}\,a_\rho - \frac{38}{2\sqrt{10}}\,a_\phi$$

$$B = -0.949\,a_\rho - 6.01\,a_\phi$$

Ans

Q NO 1

(b)

Solution:

point = P(3,4,5)

in Cartesian plane $x = 3$, $y = 4$, $z = 5$

So we have

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r = \sqrt{3^2 + 4^2 + 5^2}$$

$$r = \sqrt{9 + 16 + 25}$$

$$r = 5\sqrt{2} \Rightarrow 7.07$$

$$\theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z}$$

$$\theta = \tan^{-1} \frac{\sqrt{3^2 + 4^2}}{5}$$

$$\theta = \tan^{-1} \frac{\sqrt{25}}{5}$$

$$\theta = \tan^{-1} 1$$

$$\theta = 45^\circ$$

$$\phi = \tan^{-1} y/x \Rightarrow \phi = \tan^{-1} 4/3$$

$$\phi = 53.13$$

so

$$P(3,4,5) = P(7.07, 45^\circ, 53.13^\circ)$$

Ans

Q No 1:

(c)

Point = A(2, 3, -1)

Solution:

In Cartesian

$x = 2, \; y = 3, \; z = -1.$

$\Rightarrow$

$$\rho = \sqrt{x^2 + y^2}$$

$$\rho = \sqrt{2^2 + 3^2}$$

$$\rho = \sqrt{13} \Rightarrow \rho = 3.61.$$

$$\phi = \tan^{-1} y/x.$$

$$\phi = \tan^{-1} 3/2.$$

$$\phi = 56.3^\circ.$$

$$z = -1.$$

$$A(2, 3, -1) = A(3.61, 56.3^\circ, -1)$$

Q No (1)

(e)

$q_1 = 2nC$

$q_2 = -1nC$

$r = 4cm$

$\varepsilon_0 = 8.85 \times 10^{-12}\ F/m$

Solution:

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$F = \frac{1}{4\pi \times 8.85 \times 10^{-12}} \times \frac{2 \times 10^{-9} \times (-1 \times 10^{-9})}{(4 \times 10^{-2})^2}$$

$$F = \frac{2 \times 10^{-18}}{4 \times \pi \times 8.85 \times 16 \times 10^{-16}}$$

$$F = \frac{-2 \times 10^{-2}}{1779.4}$$

$$F = -1.124 \times 10^{-5}$$

$$\boxed{F = -11.2\ \mu N}$$ Ans.

Q No (1)

(f) :→ $q_1 = -2C$

$q_2 = -1C$

$r = 1\,m.$

$\varepsilon_0 = 8.8 \times 10^{-12}\, F/m.$

Sol:

$E = F/Q$

$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}.$

$F = \frac{1}{4 \times \pi \times 8.85 \times 10^{-12}} \times \frac{(-2)(-1)}{1^2}.$

$F = \frac{2}{111.2 \times 10^{-12}}$

$F = 0.01798 \times 10^{12}.$

$F = 17.9 \times 10^9\, N.$

$E_1 = F/q_1 \Rightarrow E_1 = \frac{17.9 \times 10^9}{-2}.$

$E_1 = -8.95 \times 10^9\, N/C.$

$E_2 = F/q_2 \Rightarrow E_2 = \frac{17.9 \times 10^9}{-1}$

$E_2 = -17.9 \times 10^9.$

$E = E_1 + E_2 = (-8.95 - 17.9)\, 10^9$

$\boxed{E = -26.85\, M\, N/C.}$ Answer

Q NO (1)

(Q):1

$E = 40\ V/cm$

$r = 30cm.$

$q_1 = 10^{-8}C.$

$q_2 = ?.$

$\varepsilon_0 = 8.85\times10^{-12}\ F/m.$

Solution

$E = F/Q.$

$F = E\times Q.$

$F = 4.0\times10^{-2}\times10^{-8}.$

~~F = 400nN.~~

$F = 4.00nN.$

As $F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}.$

$\Rightarrow q_2 = \frac{F\times4\times\pi\times\varepsilon_0\times r^2}{q_1}.$

$q_2 = \frac{4.00\times10^{-9}\times4\pi\times8.85\times10^{-12}\times(30\times10^{-2})^2}{10^{-8}}.$

~~$q_2 = 444.85\times900\times10^{-25}$~~ / 10^{8}.

$q_2 = 4.\times10^{5}\times10^{-17}$

$\boxed{q_2 = 4\times10^{-12}\ C}$ Ans.

Q No (1)

(h)

$q_1 = 2 \times 10^{-7} C$.

$F = 0.1 N$.

$q_2 = 4.5 \times 10^{-7} C$.

$r = ?$.

$\epsilon_0 = 8.85 \times 10^{-12}$.

Solution.

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}.$$

$$r^2 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{F}.$$

$$r^2 = \frac{1}{4 \times \pi \times 8.85 \times 10^{-12}} \times \frac{2 \times 10^{-7} \times 4.5 \times 10^{-7}}{0.1}.$$

$$r^2 = \frac{9 \times 10^{-14}}{11.12 \times 10^{-12}}.$$

$$r^2 = 0.81 \times 10^{-2}.$$

$$r = \sqrt{0.0081}$$

$$r = 0.09$$

$\Rightarrow$ $\boxed{r = 9 cm}$ Ans.

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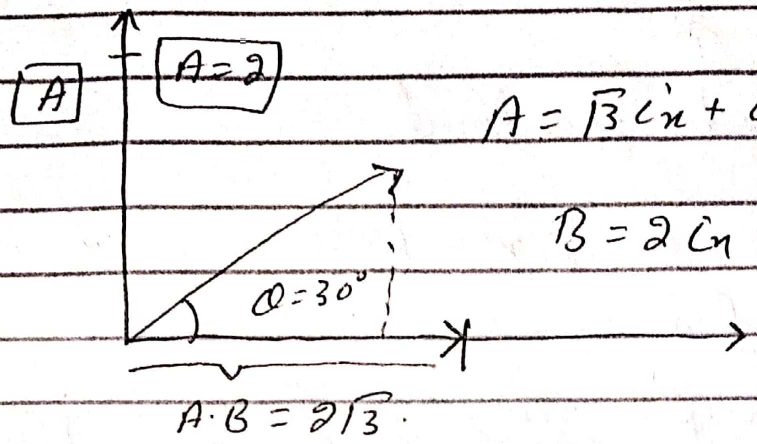
Q NO (2)

(a).



$A = \sqrt{3}\,i_x + i_y$.

$B = 2\,i_x$.

Sol:

$$A \cdot B = |A| \cdot |B| \cos\theta .$$

$$\cos\theta = \frac{A \cdot B}{|A| \cdot |B|} \quad \text{—— (1)}.$$

$$|A| = \sqrt{(\sqrt{3})^2 + 1^2}$$

$$|A| = 2.$$

$$|B| = \sqrt{2^2}$$

$$|B| = 2.$$

(1) ⇒ $$\cos\theta = \frac{2\sqrt{3}}{2 \times 2}.$$

$$\cos\theta = \sqrt{3}/2$$

$$\theta = \cos^{-1} \sqrt{3}/2.$$

$$\boxed{\theta = 30^\circ}$$

~~Q NO 1~~
Q NO 2
(b)
Find gradient:

(i) $f = ax^2 + by^3 z$.

Sol:

$$\nabla f = \frac{\partial f}{\partial x} a_x + \frac{\partial f}{\partial y} a_y + \frac{\partial f}{\partial z} a_z$$

$$\nabla f = \frac{\partial}{\partial x}(ax^2+by^3z) + \frac{\partial}{\partial y}(ax^2+by^3z) + \frac{\partial}{\partial z}(ax^2+by^3z) a_z$$

$$\nabla f = 2ax\, a_x + 3by^2z\, a_y + by^3 a_z$$

$$\boxed{\nabla f = 2ax\, a_x + 3by^2z\, a_y + by^3 a_z}$$ Ans

(ii) $f = ar^2 \sin\phi + brz\cos 2\phi$.

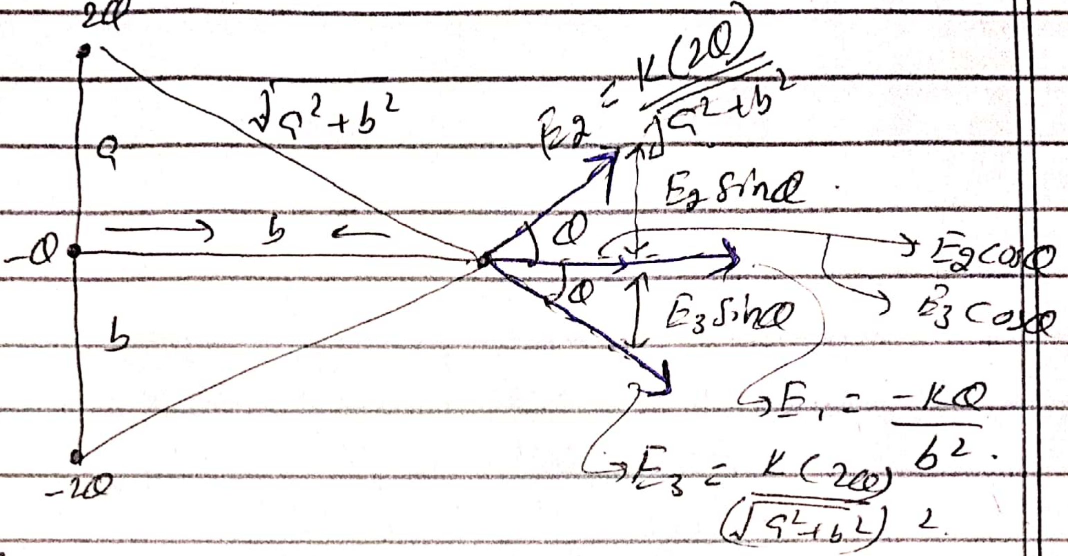
Sol:

$$\nabla f = \frac{\partial f}{\partial r} a_r + \frac{\partial f}{\partial \phi} a_\phi + \frac{\partial f}{\partial z} a_z$$

$$\Rightarrow \boxed{\nabla f = (2ar\sin\phi + bz\cos 2\phi)a_r + (ar^2\cos\phi - 2brz\sin 2\phi)a_\phi + br\cos 2\phi}$$

Ans

Q NO3 :→



Sol:

E at point P is due to +2Q, -Q & +2Q and will be along x-axis. So we will use superposition principle from figure

$$E_P = E_1 + E_2\cos\theta + E_3\cos\theta$$

$$\Rightarrow E_P = \frac{-KQ}{b^2} + \frac{K(2Q)}{a^2+b^2}\left(\frac{b}{\sqrt{a^2+b^2}}\right) + \frac{K(2Q)}{a^2+b^2}\cdot\frac{b}{\sqrt{a^2+b^2}}$$

$$E_P = \frac{-KQ}{b^2} + \frac{4KQb}{(a^2+b^2)^{3/2}}$$

$$E_P = \frac{KQ}{b^2}\left[\frac{4b}{(1+a^2/b^2)^{3/2}} - 1\right]$$

$$E_P = \frac{KQ}{b^2}\left[\frac{4b}{(1+a^2/b^2)^{3/2}} - 1\right]$$

$$E_P = \frac{KQ}{b^2}\left[4b - 1\right]$$

$$\Rightarrow \boxed{E_P = \frac{4KQ}{b} - \frac{KQ}{b^2}}$$