Name Zia Ur Rehman
ID 11473. Digital Signal processing.
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Q. Consider the following analog signal.

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t.$$

i) Determine the minimum sampling rate required to avoid aliasing?

Ans: According to sampling theorem.

$F_1 = 100\ Hz$ $F_2 = 200\ Hz$

$F_s \geq f_{max}$ $f = \omega/2\pi$

$f_s > 2 \times 100$ $f_s = 200\ Hz$

(ii) Suppose the signal is sampled at the rate $f_s = 100\ Hz$ what is the discrete-time signal.

Sol:

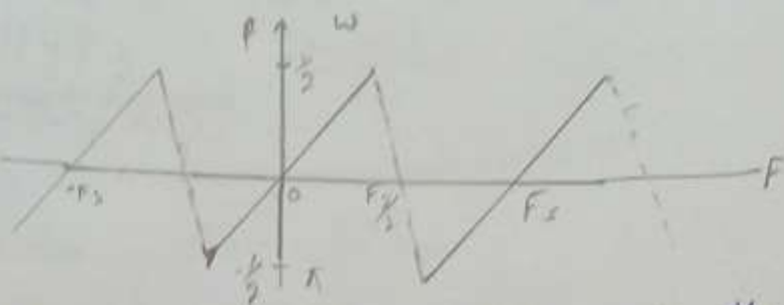
$f_s = 100\ Hz$

$f = 100/2 = 50\ Hz$

This is the maximum freq that can be represented uniquely by the sampled signal

$$x_a[n] = 3\cos\pi(50/100)n + 4\sin 2\pi\left(\frac{100}{100}\right)n$$

$$= 3\cos\pi(5/10)n + 4\sin 2\pi n$$



The effect of sampling rate on the newly generated discrete time signal is that there will be no dislay phenomenon mean there will not present unwanted component in Re construt of the signal. The reconstrut orignal signals.

(iii) what is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation.

Ans: folding freq $= f_s/2 = 100/2 = 50Hz$.

$f_1 = 50H$ $f_2 = 100 Hz$

Both frequency are either equal or greater the folding frequency.

Hence for ideal interpolation we can construct the orignal signal..

$$x_a(t) = 3\cos 100\pi + 4\sin 200\pi t$$

since only the freq components at 100Hz are present on the sampled signals the.

The analog signal we can remove or reconstruct is.

$$\boxed{y_a(t) = 3\cos 100\pi t}$$ Ans:

(b) Consider a discrete time signal which is given by.

$$x(n) = \begin{cases} 0.5^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

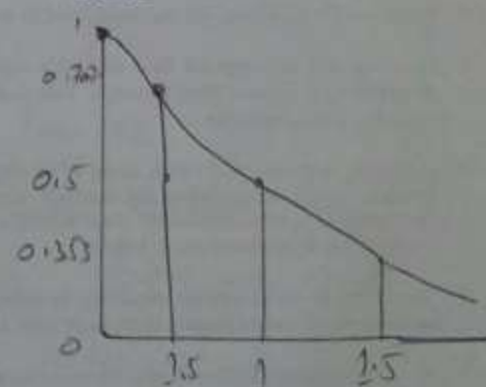
This signal is sampled at rate $F_s = 2$ Hz

i) Draw the sampled signal.

$$f_s = 1/T \quad = \quad T = 1/f_s$$

$$= 1/2 \quad = 0.5 \text{ sec}$$

x_n	0.5^n
0	1
0.5	0.7071
.7	0.5
1.5	0.353



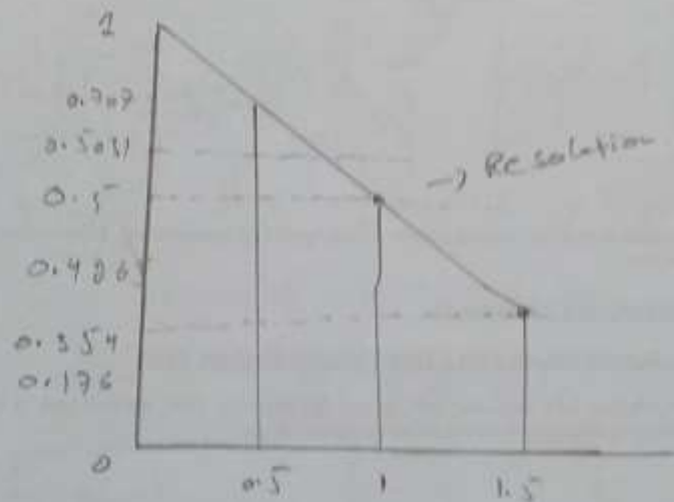
ii) The samples of the signals (1) are Intended to carry ....

Ans: $L = 2^n$

$n = \text{bits} = 3$

$L = 2^3 = 8$ levels.

$$\text{Resolution} = \frac{X_{max} - X_{min}}{L} = \frac{1-0}{8} = 0.125$$



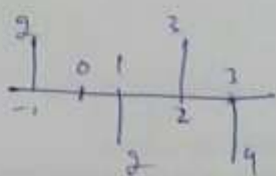
	Dist signal	Correction	Reading	Error
0	1	1.0	1.0	0.0
1	0.9535	0.8	0.9	0.1
2	0.707	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4065	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.1	0.1	-0.2

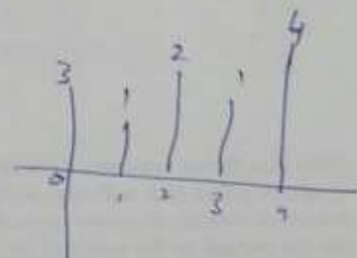
Q2 Determine the response of the signal the following input signal with given Impulse response.

$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$ $\quad h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$

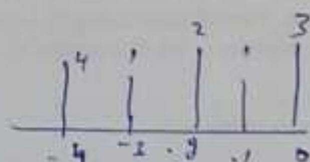
Sol.

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k).$$





$h(-k)$ = folded signal.



$h(-k)$.

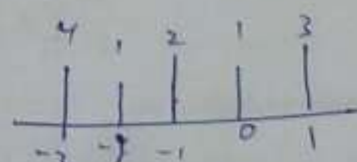
$$y(0) = \sum_{k=-1}^{0} x(-1]\, h(-1) + 1(0)\,(h(0)).$$

$$= 2 \times 1 + 1\,(3)$$

$$= 5$$

For $n = 1$

$h(1-k)$



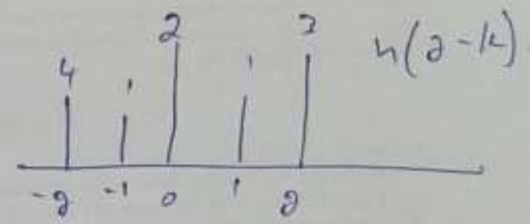
$$y[1] = \sum_{k=-3}^{3} x(n)\,[h(1-k)].$$

$= x(-1)h(-1) + x(0)h(0) + x(1)h(1)$

$= 2(2) + 1(1) + 3(-2)$

$= 4+1-6 \quad = -1$

$n=2$. $\quad h(2-k)$



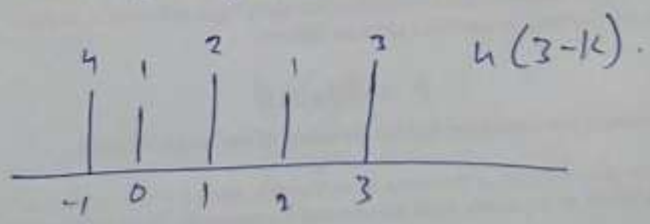
$y(2) = \sum_{k=-1}^{1} x(k)\, h(2-k)$

$x(-1)h(-3) + 2(0) + h(0) + x(1)h(1)$

$x(-1)h(-1) + x(0) + h(0) + x(1)(1)$

$= 2(1) + 1(2) + (-2)(1) + 3(3)$

$= 2+2-2+9 \quad = 11$



$y(3) = \sum_{k=2}^{3} x(k)\, h(3-k)$

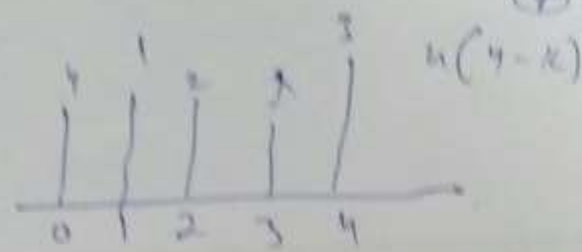
$= x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2)$

$= 2\times4 + 1\times2 + (-2)(2) + 3(1) + (-4)(3)$

$= 4+1-4+3-12$

$= -8$

$n=4$

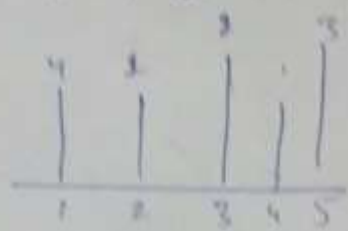


$$y(4) = \sum_{k=0}^{3} x(k)\,h(4-k)$$

$$= x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$= 4 - 2 + 6 - 4$$

$n=5$



$$y(5) = \sum_{k=1}^{5} x(k)\,h(5-k).$$

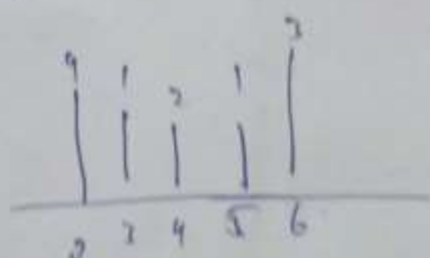
$$x(1)h(1) + x(2)h(2) + x(3)h(3).$$

$$= (-2)(4) + 3(1) + (-4)(2).$$

$$= -8 + 3 - 8$$

$$= 13$$

$n=6$



$$y(6) = \sum_{k=1}^{3} x(2)\,h(2) + x(3)\,h(3)$$

$$= 3(4) + 1(-4) = 8$$

Compute the convolution $y(n)$ ⑧ of the following signal.

$$x(n) = \begin{cases} a^{n+1} & -3 \le n \le 5 \\ 0, & \text{else where.} \end{cases}$$

$$h(n) = \begin{cases} 2^{n} & 0 \le n \le 4 \\ 0 & \text{else where.} \end{cases}$$

Sol: we have

$x(n) = x(k) = \{\alpha^{-2}, \alpha^{-1}, 1, \alpha, \alpha^{2}, \alpha^{3}, \alpha^{4}, \alpha^{5}, \alpha^{6}, 0, 0 \ldots\}$

$h(n) = h(k) = \{\ldots 0, 1, 2, 4, 8, 16, 0 \ldots\}$

To find $y(n)$;

$$y(n) = \sum_{k=-\infty}^{\infty} x[k]\, h(n-k).$$

for $n=0$ first to find $h(n-k) = h(0-k)$

So by inverting $h(k)$ we get $h(-k)$

$\Rightarrow h(-k) = \{16, 8, 4, 2, \underset{\uparrow}{1}\} \rightarrow$ ②

So $y(0) = \sum_{k=-\infty}^{\infty} x(k) \times h(-k)$

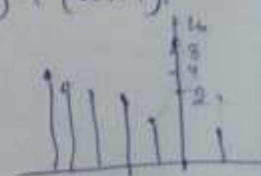
$y(0) = (\alpha^{-2} \times 8) + (\alpha^{-1} \times 4) + (1 \times 2) + (\alpha \times 1)$

~~for~~ for $n=1$ $h(1-k) = \{16, 8, 4, 2, 1\}$

So $y(1)\ (\alpha^{-2} \times 16) + (\alpha^{-2} \times 8) + (1 \times 4) + (\alpha + 2) + (\alpha^{2} \times 1)$.

$y(1) = 16\alpha^{-2} + 8\alpha^{-1} + 4 + 2\alpha + \alpha^{2}$

$= \alpha^{2} + 2\alpha + 4 + 8\alpha^{-1} + 16\alpha^{-2}$

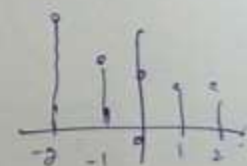


Now for $n=2$.

$$x(2-k) = \{16, 8, 4, 2, 1\}$$

$$y(2) = \{(\alpha^{-1}\times 16) + (1\times 8) + (\alpha + 4) + (\alpha^{2}\times 2) + (\alpha^{3}\times 1)$$

$$= 16\alpha^{-1} + 8 + 4\alpha + 2\alpha^{2} + \alpha^{3}$$

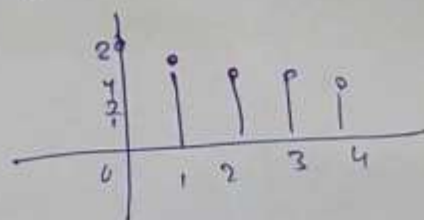


Similarly for $n=3$.

$$x(3-k) = \{16, 8, 4, 2, 1\}$$

$$y(3) = (1\times 16)(\alpha\times 8) + (\alpha^{2} + 4) + (\alpha^{3}\times 2) + (\alpha^{4}\times 1)$$

$$= 16 + 8\alpha + 4\alpha^{2} + 2\alpha^{3} + \alpha^{4}$$



Now for $n=2$.

$$x(2-k) = \{16, 8, 4, 2, 1\}$$

$$y(2) = \{(\alpha^{-1}\times 16) + (1\times 8) + (\alpha\times 4) + (\alpha^{2}\times 2) + (\alpha^{3}\times 1)$$

$$= 16\alpha^{-2} + 8 + 4\alpha + 2\alpha^{2} + \alpha^{3}$$

Similarly for $n=3$

$$x(3-k) = \{16, 8, 4, 2, 1\}$$

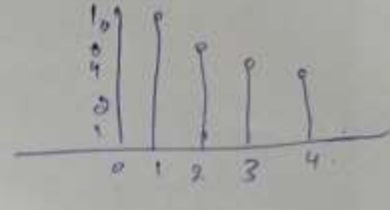
$$y(3) = (1\times 16)(\alpha\times 8) + (\alpha^{2} + 4) + (\alpha^{3}\times 2) + (\alpha^{4}\times 1)$$

$$= 16 + 8\alpha + 4\alpha^{2} + 2\alpha^{3} + \alpha^{4}$$

Now $h(4-k) = \{16, 8, 4, 2, 1\}$.

$$y(4) = (\alpha \times 16) + (\alpha^2 \times 8) + (\alpha^3 \times 4) + (\alpha^4 \times 2) + (\alpha^5 \times 1).$$

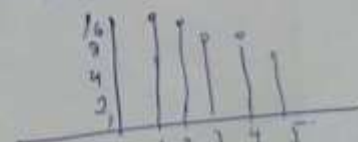
$$= 16\alpha + 8\alpha^2 + 4\alpha^3 + 2\alpha^4 + \alpha^5$$



$\Rightarrow h(5-k) = \{0, 16, 8, 4, 2, 1\}$.

$$y(5) = (\alpha \times 0) + (\alpha^2 \times 16) + (\alpha^3 \times 2) + (\alpha^4 \times 4) + (\alpha^5 \times 2) + (\alpha^6 \times 1)$$

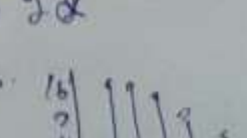
$$= 16\alpha^2 + 8\alpha^3 + 4\alpha^4 + 2\alpha^5 + \alpha^6.$$



Similarly if we calculate for rest of the values of n. upto till there are any common values we get.

$$y(6) = 0 + 0 + 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$$

$$= 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6.$$



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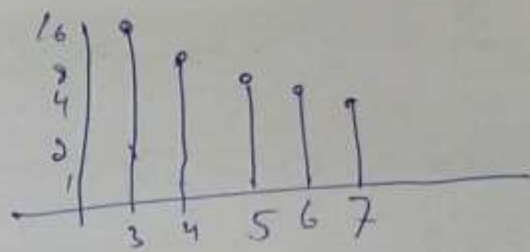
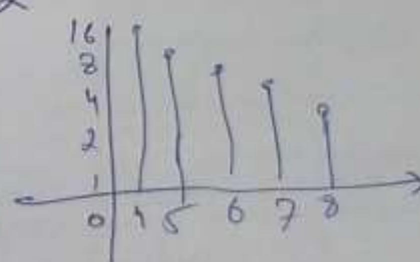
$y(7) = 0+0+0+16\alpha^4+8\alpha^5+4\alpha^6$

$= 6\alpha^4+8\alpha^5+4\alpha^6$



$y(8) = 0+0+0+0+16\alpha^5+8\alpha^6$

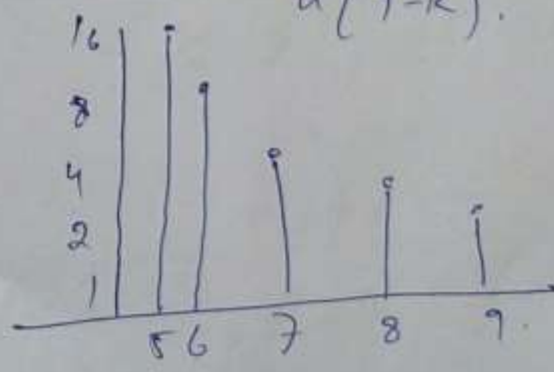
$= 16\alpha^5+8\alpha^6$



$y(9) = 0+0+0+0+16\alpha^6$

$= 16\alpha^6$

$u(9-k)$.



Q3) Determine the Z- transform

Sol: As we know that

Z- transform.

$$X(z) = \sum_{n=0}^{\infty} (1/4)^n e + \sum_{n=\infty}^{0} (1/3)^n z^n - 1$$

using geometric series.

$$= \frac{1}{1-1/4 z^{-1}} + \sum_{n=0}^{\infty} (1/3)^n z^n - 1$$

$$= \frac{1}{1-1/4 z^{-3}} + \frac{1}{1-1/3} - 1$$

$$= \frac{1 - \frac{1}{1/4} z^{-1} + 1 - 1/1 - 1}{(1 - 1/4 z^{-2})(1-1/3 z^{-2})^{-1}}$$

$$= \frac{1 - 1/3 z + 1 - 1/4 z^{3} - (1 - 1/4 z^{-2})(1 - 1/3 z)}{(1 - 1/4 z^{-3})(1 - 1/3 z)}$$

$$= \frac{1 - 1/3 z + 1 - 1/4 z' - 1 + 1/3 + 1/4 z + 1/12}{(1 - 1/4 z')\cdot(1 - 1/3 z)}$$

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$$X_2(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} - \sum_{n=0}^{\infty} 2^n z^{-n}$$

$$= \frac{1}{1-\frac{1}{2}z^{-3}} - \frac{1}{1-2z^{3}}$$

$$= \frac{-\frac{5}{2} z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-2z^{-1}\right)}$$

As seen the ROC use $|z|>2$

The sketch are.