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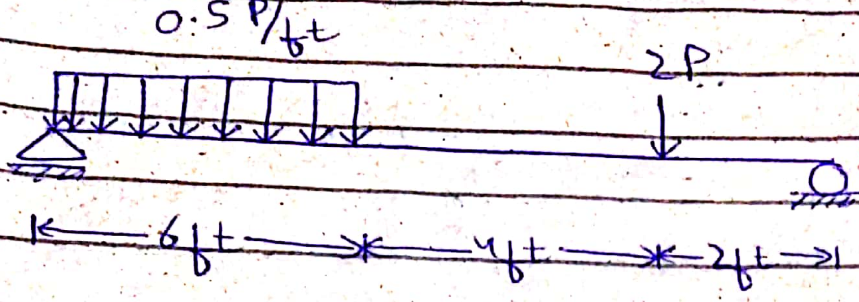
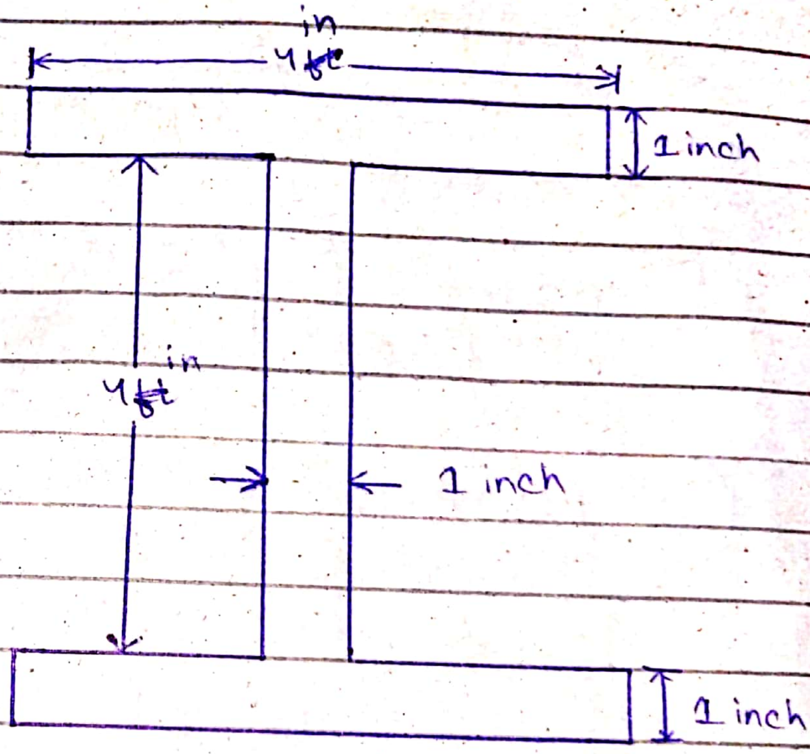
Section: "A"

Paper: MOS(II)

Examination: Mid-Term

Department: Civil Engineering

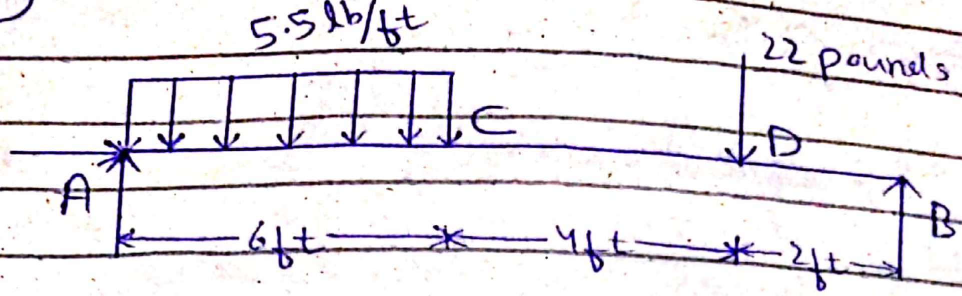




Solution:

In the given diagram the value of P = 11, So the 2P load becomes 22 pounds and 0.5 P/ft becomes 5.5 lb/ft

The free body diagram for the given beam is



Now we will find the support reactions at point A and B

Taking Moment at A

$$\Sigma M_A = 0$$

$$(R_B \times 12) - (22 \times 10) - (5.5 \times 6 \times 3) = 0$$

$$\Rightarrow 12 R_B - 220 - 99 = 0$$

$$\Rightarrow 12 R_B = 220 + 99$$

$$\Rightarrow R_B = \frac{319}{12}$$

$$\Rightarrow R_B = 26.58 \text{ lb}$$

Now Taking $\Sigma F_y = 0$

$\Rightarrow R_A + R_B - 5.5 \times 6 - 22 = 0$

$\Rightarrow R_A + 26.58 - 33 - 22 = 0$

$\Rightarrow R_A = -26.58 + 33 + 22$

$\Rightarrow \boxed{R_A = 28.42 \text{ lb.}}$

Now we will find Shesr force at different points

Shear force at A $= +28.42$ lb

Shesr force at C $= +28.42 - (5.5 \times 6)$
$= +28.42 - (33)$
$= -4.58$ lb

Shear force at D $= -4.58 - 22$
$= -26.58$ lb

Shesr force at B $= -\cancel{26.58} + \cancel{26.58}$
$= 0$

Now we will find the bending moment at different points.

Bending mement at A = 0

The bending moment is maximum at a point where the Shear force is zero and this point lies b/w A and C. The maximum bending moment can be find after drawing the shear force diagram.

Bending moment at C = ~~(5.5×6)~~
(28.42×6) − (33×3)

= 71.52 lb.ft

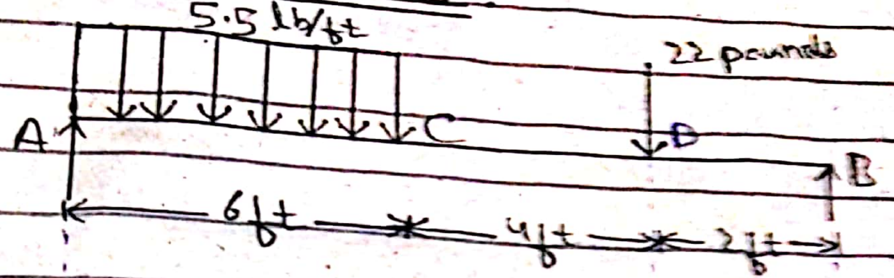
Bending moment at D = (28.42×10) − (33×7)

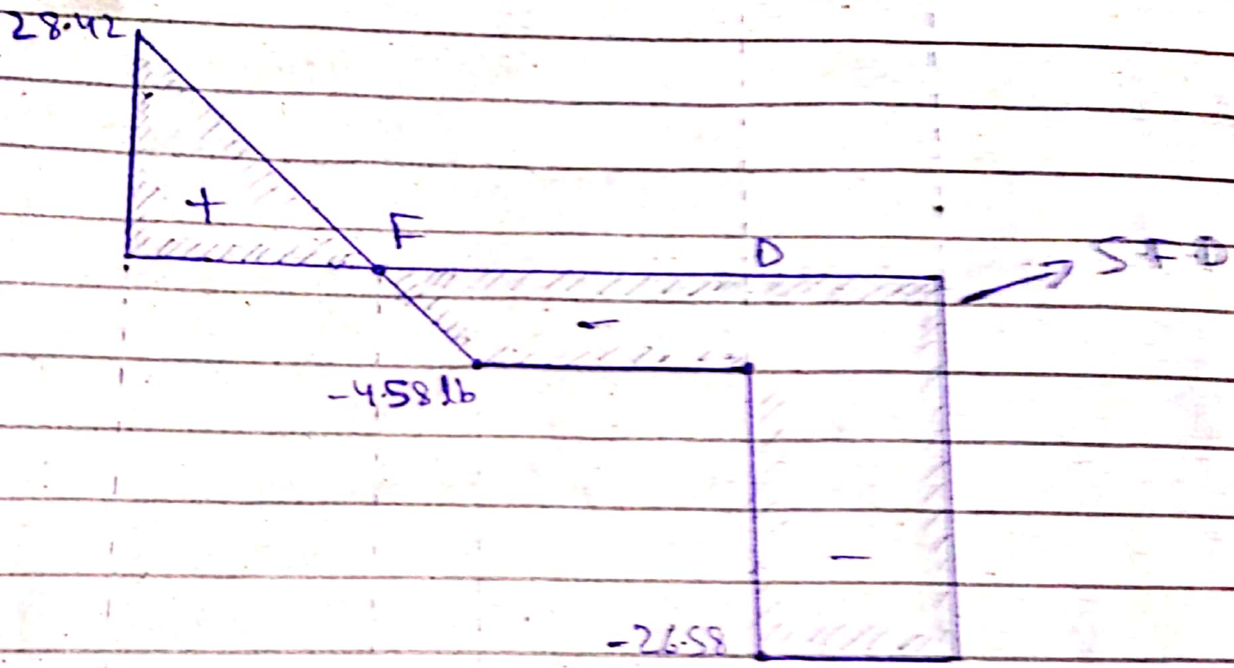
= 53.2 lb.ft

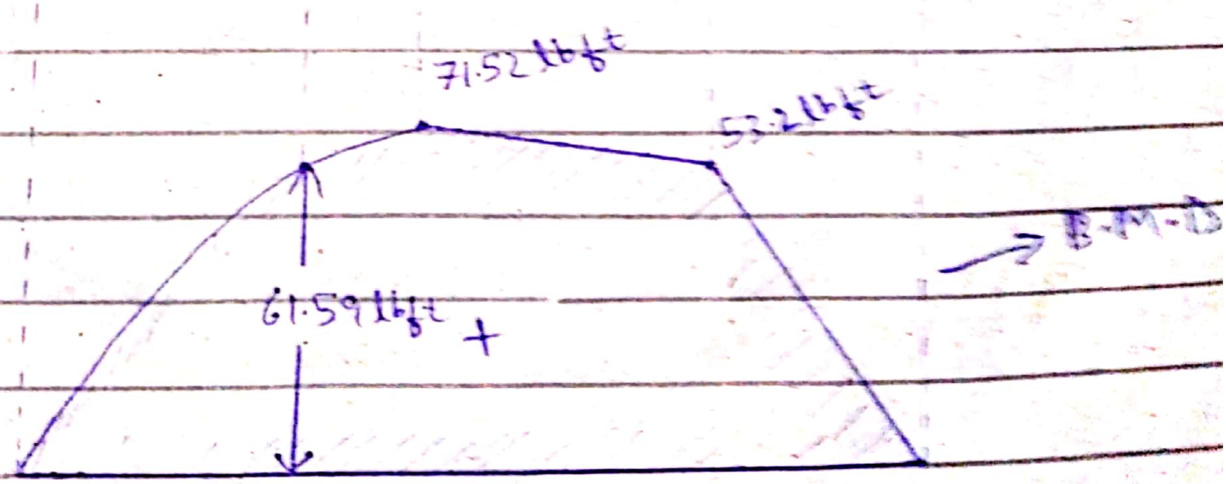
Bending moment at B = 0

(5)

S.F.D and B.M.D:

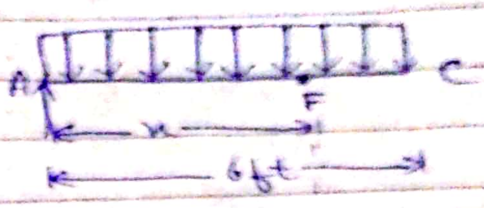

5.5 lb/ft
22 pounds
A
C
D
B
6ft
4ft
2ft


28.42
+
F
D
-
-4.58 lb
-
-26.58
SFD

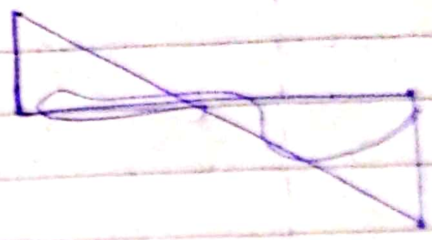

71.52 lbft
53.2 lbft
61.59 lbft
+
B.M.D

Now we will find the maximum bending moment from Shear force diagram.

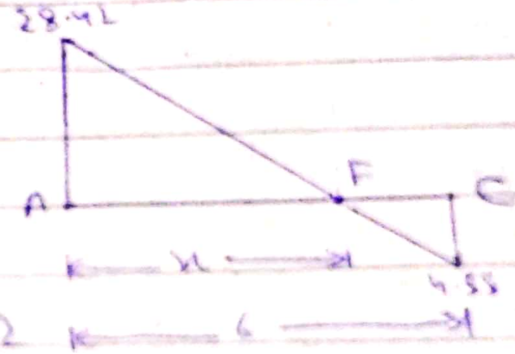
From the law of similar triangles we have



$$\frac{28.42}{x} = \frac{4.58}{(6-x)}$$



$$28.42(6-x) = 4.58x$$



$$170.52 - 28.42x = 4.58x$$

$$\Rightarrow 28.42x + 4.58x = 170.52$$

$$\Rightarrow 33x = 170.52$$

$$\Rightarrow x = \frac{170.52}{33}$$

$$\Rightarrow \boxed{x = 5.167}$$

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Now the maximum bending moment at point F is

$M_{max} = (28.42 \times 5.167) - (5.5 \times 6)\left(\frac{5.167}{2}\right)$

$M_{max} = 146.84 - 85.25$

$\boxed{M_{max} = 61.59 \text{ lb ft}}$

Now we will find the Shear Stress $\tau = \frac{VQ}{Ib}$. For Shear

Stress it is necessary to first find the moment of inertia "I"

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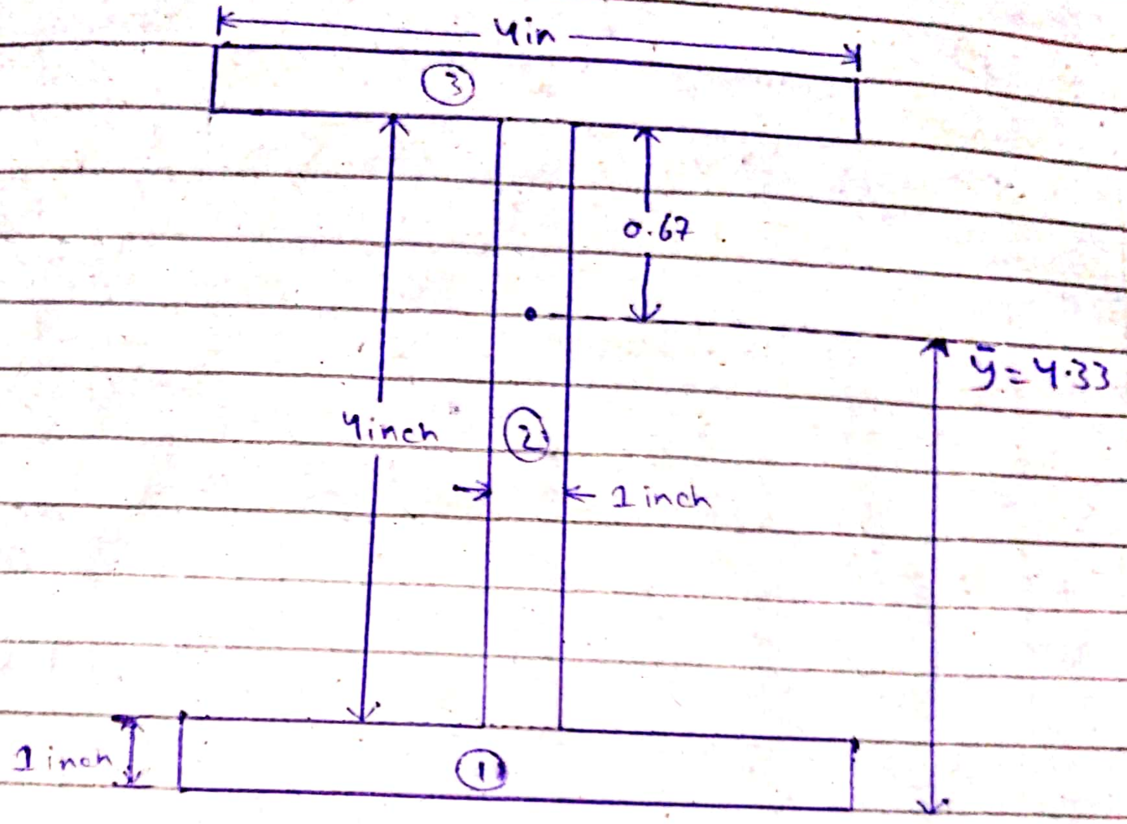
Moment of Inertia of "I" Section:



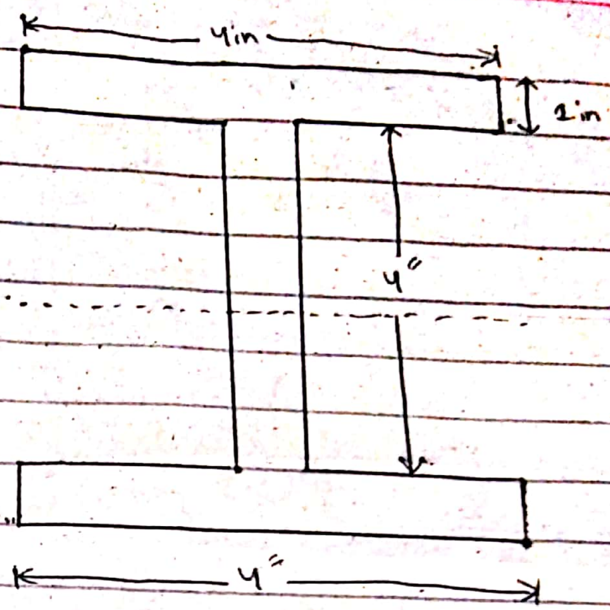
The centroid of the section about y-axis

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$\bar{y} = \frac{(4)(0.5) + (4)(3) + (4)(9.5)}{4 + 4 + 4}$$

$$\boxed{\bar{y} = 4.33}$$

1



Moment of inertia of I-Section

$$I_{xx} = \left[\frac{(4)(1)^3}{12} + 4(1)(2.5)^2\right] \times 2 + \frac{(1)(4)^3}{12}$$

$$I_{xx} = 56 \text{ in}^4$$

Now we will find the Shear Stress at different points

For Shear Stress $\tau = \frac{VQ}{Ib}$

where V = maximum Shear force

$Q = \bar{y}A$

~~Here~~

$\cancel{\bar{y} = \frac{0.67 + 0.5 + 0.5}{2}}$

Case 1.

Shear stress at top fibre

$$\tau_{Top} = \frac{VQ}{Ib} = \frac{(28.42)[0]}{56 \times 4}$$

$$\tau_{Top} = 0$$

Case 2:

τ at 0.5in below top fibre

$$\tau_{0.5} = \frac{(28.42)(1.42)(0.5 \times 4)}{56 \times 4}$$

Here $\bar{y} = 0.67 + 0.5 + \frac{0.5}{2}$

$$\bar{y} = 1.42 \text{ in}$$

$$\boxed{\tau_{0.5} = 0.36 \text{ psi}}$$

Case 3:

τ at 1 in below the top fibre

$$\tau_1 = \frac{(28.42)(1.17 \times 4)}{(56)(4)}$$

Here $\bar{y}' = 0.67 + \frac{1}{2} = 1.17$

$A = 1 \times 4 = 4$

$$\boxed{\tau_1 = 0.59 \text{ psi}}$$

Case 4:

Shear Stress at centroidal axis:

In this case taking area above the centroidal axis and then find Q

$$Q = Q_1 + Q_2$$

$$\Rightarrow Q_1 = \bar{y}_1' A_1$$

$$\Rightarrow Q_1 = \frac{0.67}{2}(0.67 \times 1)$$

$$\Rightarrow \boxed{Q_1 = 0.224}$$

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$$Q_2 = \left(0.67 + \frac{1}{2}\right) \times (4 \times 1)$$

$$\boxed{Q_2 = 4.68}$$

$$\Rightarrow Q = Q_1 + Q_2$$

$$\Rightarrow Q = 0.224 + 4.68$$

$$\Rightarrow \boxed{Q = 4.904}$$

$$\tau_{max} = \frac{(28.42)(4.904)}{56 \times 1}$$

$$\boxed{\tau_{max} = 2.488 \text{ psi}}$$

<u>Case 5:</u> Shear stress below the centroidal axis and above the bottom fibre

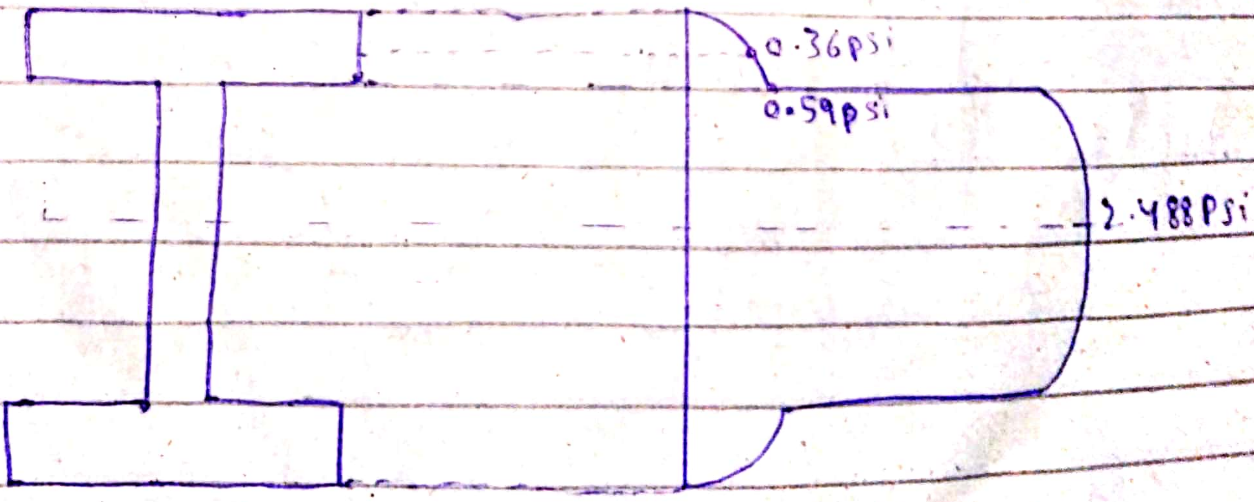
$$\tau = \frac{VQ}{Ib} = \frac{28.42(2.165 \times 3)}{56 \times 4}$$

Here $\bar{y}' = \frac{4.33}{2} = 2.165$

$A = 3 \times 1 = 3 \, in^2$

$$\boxed{\tau = 0.824 \, psi}$$

Shear Stress variation diagram for "I" section:



Now we will find the flexural stress at different points along the leng of Beam.

Flexural Stress $= \dfrac{My}{I}$

Case 1:

Flexural Stress at top fibre.

$$\sigma_{Top} = \frac{71.752 \times 1.67}{56}$$

$$\boxed{\sigma_{Top} = 2.132 \text{ psi}}$$

Case 2:

Flexural Stress at 0.5 in below the top fibre

$$\sigma_{0.5} = \frac{My}{I} = \frac{71.52 \times 1.17}{56}$$

$$\boxed{\sigma_{0.5} = 1.494 \text{ psi}}$$

ural

along

Case 3.

Flexural stress at 1 in below the top fibre

$$\sigma_1 = \frac{My}{I} = \frac{71.52 \times 0.67}{56}$$

$$\boxed{\sigma_1 = 0.855 \text{ psi}}$$

op

Case 4:

Flexural stress at centroid

$$\sigma_{cen} = \frac{My}{I} = 0 \quad , \quad y = 0$$

n

Case 5:

Flexural stress at 3 in above the bottom fibre

$$\sigma = \frac{My}{I} = \frac{71.52 \times 1.33}{56}$$

Here $y = 4.33 - 3$
$y = 1.33$

$$\sigma_3 = 1.33 \text{ psi}$$

Case 6:

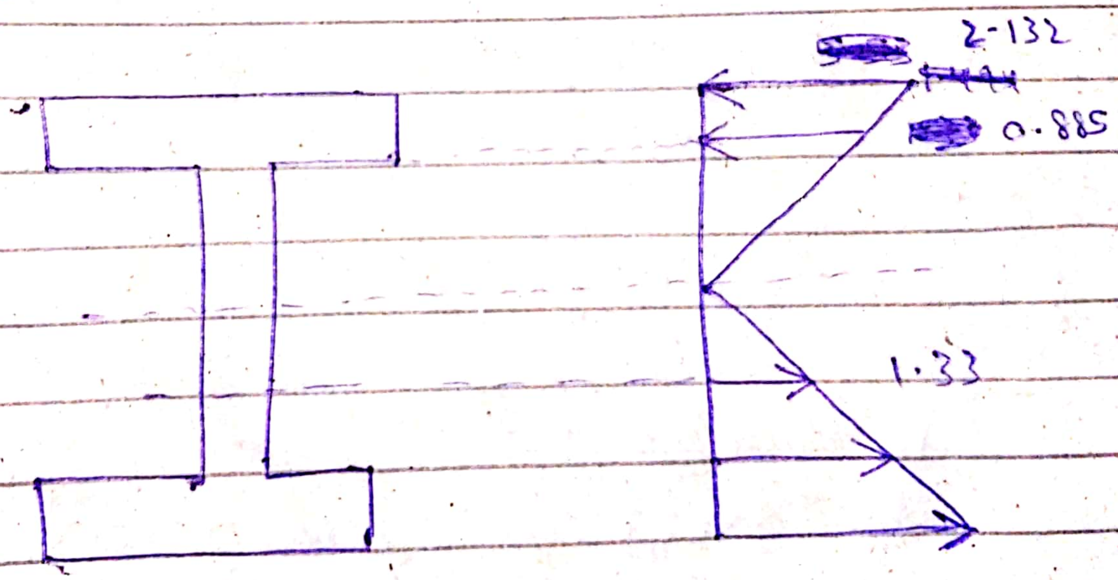
Flexural stress at bottom fibre

$$\sigma_{max} = \frac{My}{I}$$

$$\sigma_{max} = \frac{71.52 \times 4.33}{56}$$

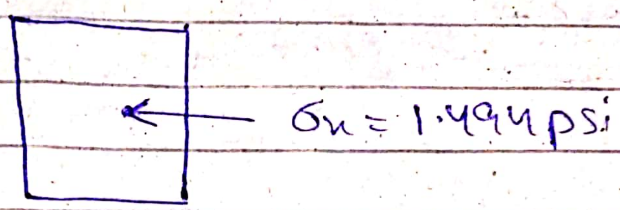
$$\boxed{\sigma_{max} = 5.530\,psi}$$

Flexural stress distribution diagram:



Now we will find the stresses on a plane of point which is in compression above the centroid of a section.

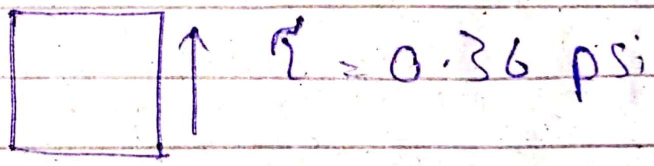
Consider point "C" on planer element



The Sheer Stress on a plane will be

$$\boxed{\tau = 0.36 \text{ psi}}$$

From Case - 2



Here $\sigma_x = -1.494$ psi

$\sigma_y = 0$

$\tau_{xy} = -0.36$ psi

Equation for finding the normal stress is

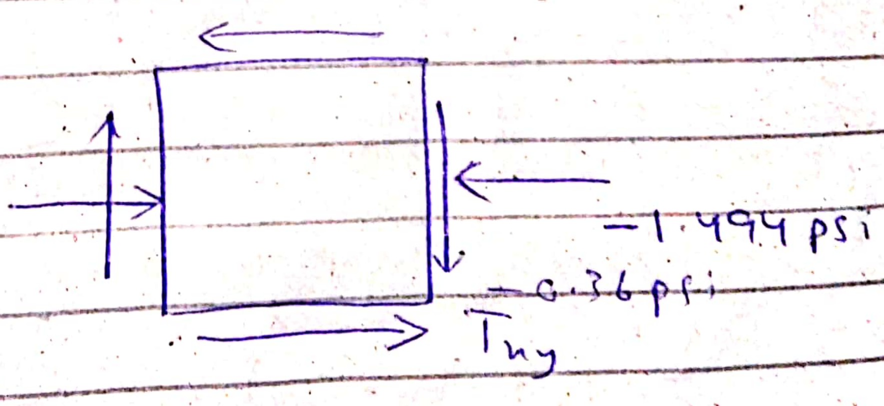
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 (\tau_{xy})^2}$$

$$\sigma_{1,2} = \frac{-1.494 + 0}{2} \pm \sqrt{\left(\frac{-1.494 - 0}{2}\right)^2 + (-0.36)^2}$$

$$\sigma_{1,2} = -0.747 \pm 0.829$$

$$\sigma_{1,2} = 0.082, -1.576$$

The above values are the principle stresses acting on a point



Construction of Moh's circle:

Radius of Moh's circle

$$r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$r = \sqrt{\left(\frac{-1.494 - 0}{2}\right)^2 + (0.36)^2}$$

$$r = 0.829 \text{ psi}$$