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Q1 :-

a)

Ans:- The Lagrange form is

$$P(x) = 2\frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)} + 1\frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)}$$

$$+0\frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)} - 1\frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)}$$

$$= -\frac{1}{3}(x^3-6x^2+11x-6) + \frac{1}{2}(x^3-5x^2+6x)$$

$$-\frac{1}{6}(x^3-3x^2+2x)$$

$$= (-x+2) \text{ Ans.}$$

Q1:-

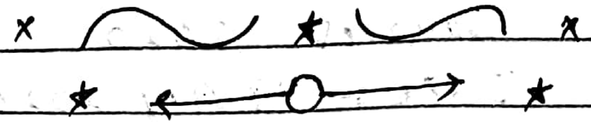
b)-

Ans:- This parabola is called the degree 2 interpolating polynomial passes through the three points.

So:- The points (0,1) (2,2) & (3,4) are interpolated by the ftn:-

$$P(x) = \frac{1}{2}x^2 - \frac{1}{2}x + 1.$$

The ftn $y = P(x)$, interpolates the data points (x_1, y_1) ..... (x_n, y_n) if $P_x = y_i$ for each $1 \leq i \leq n$.



Q2:-

a)- The two-point diff... formula

$$f'(x) \simeq \frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{2.1} - \frac{1}{2}}{0.1}$$

$$= -0.2381$$

difference is $f'(x) = -x^{-2}$ at $x = 2$.

$$-0.2381 - (0.2500) = 0.0119$$

Since $f''(x) = 2x^{-3}$ so:-

$$(0.1) 2^{-3} \simeq 0.0125 \quad \& \quad (0.1)(2.1)^{-3}$$

$$= 0.0108.$$

Applying taylor theorem:-

if "f" is 3- times continuesly differentiated then:-

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{6} f'''(c_1).$$

&

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2} f''(x) - \frac{h^3}{6} f'''(c_2).$$

So:-

where $x-h < c_2 < x < c_1 < x+h$

So:-

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

$$- \frac{h^2}{12} f'''(c_1) - \frac{h^2}{12} f'''(c_2).$$

Q 2:-

b)

Ans: Applying the definition leads to the table:-

0	1		
		$\frac{1}{2}$	
2	2		$\frac{1}{2}$
3	4	2	

Computed as:-

$$\frac{2-1}{2-0} = \frac{1}{2}$$

$$\frac{4-2}{3-2} = 2 \qquad \frac{2-\frac{1}{2}}{3-0} = \frac{1}{2}$$

So the interpoling polynomial can be written as:-

$$P(x) = 1 + \frac{1}{2}(x-0) + \frac{1}{2}(x-0)(x-2)$$

or in nested form:-

$$P(x) = 1 + (x-0)\left(\frac{1}{2} + (x-2)\cdot\frac{1}{2}\right)$$

As: if $x_1 = 0$, $x_2 = 2$.

So:- we can write

$$P(x) = 1 + \frac{1}{2}x + \frac{1}{2}x(x-2)$$

$$\left(P(x) = \frac{1}{2}x^2 - \frac{1}{2}x + 1\right).$$

Q3:-

a)-

Ans:-

The normal equation $A^T Ax = A^T b$ are:-

$$\begin{bmatrix} 9 & 6 \\ 6 & 29 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 45 \\ 75 \end{bmatrix}.$$

The solution of the normal equation are $\bar{x}_1 = 3.8$ & $\bar{x}_2 = 1.8$ The residal vector is

$$r = b = \overline{Ax} = \begin{bmatrix} -3 \\ 15 \\ 9 \end{bmatrix} - \begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 3.8 \\ 1.8 \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ 15 \\ 9 \end{bmatrix} - \begin{bmatrix} 3.4 \\ 13 \\ 11.2 \end{bmatrix} = \begin{bmatrix} 0.4 \\ 2 \\ -2.2 \end{bmatrix}.$$

i.e:-

$$\|e\|_2 = \sqrt{(0.4)^2 + 2^2 + (-2.2)^2}$$

$$\left(\|e\|_2 = 3\right).$$

Q3:-
b)-
Ans:-

The model is $Y = C_1 + C_2 f$ and the goal is to find the best C_1 & C_2 substituting the data points:-

$$C_1 + C_2(1) = 2.$$

$$C_1 + C_2(-1) = 1.$$

$$C_1 + C_2(1) = 3.$$

or in matrix form:-

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$