

NASRULLAH

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B

PROBABILITY AND STATISTICS

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QNO1

Compute and analyze the result of the least square regression equation and coefficient of correlation of Y on X for the following data. Compare your manual result with the output of SPSS?

Temperature	33	62	57	71	78	70	86	87	96	91	94	94
Chirps per minute	20	32	45	60	80	100	120	140	160	180	200	210

QNO1

Solution

X	Y	XY	X <sup>2</sup>	Y <sup>2</sup>
53	20	1060	2809	400
62	32	1984	3844	1024
57	45	2565	<del>3241</del> 3249	2025
71	60	4260	<del>6081</del> 5041	3600
78	80	6240	6084	6400
70	90	7000	4900	10000
86	120	10320	7396	14400
87	140	12180	7596	19600
96	160	15360	9216	25600
91	180	16380	8281	32400
94	200	18800	8836	40000
94	210	11280	8836	44100
Σ939	Σ1347	Σ107429	Σ76061	Σ199549

$$y = a + bx \quad \text{--- (i)}$$

$$a = \bar{y} - b\bar{x} \quad \text{--- (ii)}$$

$$\text{So } \bar{y} = \frac{\Sigma y}{n} = \frac{1347}{12} = 112.25 \quad \text{--- (iii)}$$

$$\bar{x} = \frac{\sum x}{n} = \frac{939}{12} = 78.25 \text{ --- (iv)}$$

where

$$b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}$$

$$b = \frac{12 [107429] - [1264833]}{12 [76061] - [881721]}$$

$$\boxed{b = 0.7840} \text{ --- (v)}$$

Putting eq (iii), (iv) & (v) in eq (ii)

$$\begin{aligned} a &= \bar{y} - b\bar{x} \\ &= 112.25 - 0.7840 (78.25) \end{aligned}$$

$$\boxed{a = 50.90}$$

Hence the desired estimated regression line  
y on x is

$$\hat{y} = 50.90 + 0.7840x$$

The estimated regression coefficient  $b = 0.7840$ , which indicates that the values of y increase by 0.7840 units for a unit increase in x.

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Now coefficient of correlation

As we know

$$r = \frac{\sum xy - (\sum x)(\sum y)/n}{\sqrt{[\sum x^2 - (\sum x)^2/n][\sum y^2 - (\sum y)^2/n]}}$$

$$\sqrt{[\sum x^2 - (\sum x)^2/n][\sum y^2 - (\sum y)^2/n]}$$

$$r = \frac{(107429) - \frac{(939)(1347)}{12}}{\sqrt{[\sum x^2 - (\sum x)^2/n][\sum y^2 - (\sum y)^2/n]}}$$

$$\sqrt{(76061) - \left[\frac{(939)^2}{12}\right] \left[199549 - \frac{(1347)^2}{12}\right]}$$

$$r = 2626.25$$

$$r = \frac{2626.25}{\sqrt{(76061) - \left[\frac{(939)^2}{12}\right] \left[199549 - \frac{(1347)^2}{12}\right]}}$$

$$r = 0.600016$$

Q No (2)

(A)

A box contains 4 red, 4 white and 5 green balls. Three balls are drawn from the box together. Find the probability that they may be (i) all of different colours (ii) all of the same colours.

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Q2 (A)

Solution

$$(i) \quad n(S) = \binom{13}{3} = 286$$

let A = Event all balls are different colours.

$$n(A) = \binom{4}{1} \binom{4}{1} \binom{5}{1} = 4 \times 4 \times 5 = 80$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{80}{286} = \boxed{0.28}$$

(ii)

let B = Event all balls of same colours.

$$n(B) = \binom{4}{3} \text{ (Red)} \text{ or } \binom{4}{3} \text{ (White)} \text{ or } \binom{5}{3} \text{ (Green)}$$

$$= \binom{4}{3} + \binom{4}{3} + \binom{5}{3} = 4 + 4 + 10 = 18$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{18}{286} = \boxed{0.063}$$

Q No (2)

(B) of 12 eggs in a refrigerator, 2 are bad. From these, 4 are chosen at random to make a cake. What are the probability that (i) exactly one is bad (ii) at least one is bad?

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Q No (2)

(B)

Solution

$$n(S) = \binom{12}{4} = 495$$

Let A = denote the event the exactly one egg is bad.

$$n(A) = \binom{2}{1} \binom{10}{3} = 2 \times 120 = 240$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{240}{495} = \boxed{0.48}$$

(ii) let B = be the event at least one egg is bad is selected.

$$n(B) = \binom{2}{1} \binom{10}{3} + \binom{2}{2} \binom{10}{2}$$

$$= 2 \times 120 + 1 \times 45 = 240 + 45 = 285$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{285}{495} = \boxed{0.58}$$

Q No (03)

The following are the score made by three batsmen A, B & C in a series of innings.

A	12	15	6	73	7	70	199	36	84	29
B	47	12	76	48	4	70	37	48	13	3
C	15	23	52	4	24	70	74	52	13	4

- (A) Find the range of batsmen A, B & C?
- (B) Who is more consistent player
- (C) Compare A with B, B with C & A with C?



Solution

A	B	C
12	47	15
15	12	23
6	76	52
73	48	4
27	4	24
70	70	70
199	37	74
36	48	52
84	13	13
29	3	4

$$\text{Range} = x_m - x_0$$

$$\begin{aligned} \text{Range of A} &= x_m - x_0 \\ &= 199 - 6 \\ &= 193 \end{aligned}$$

$$\begin{aligned} \text{Range of B} &= x_m - x_0 \\ &= 88 - 3 \\ &= 85 \end{aligned}$$

$$\begin{aligned} \text{Range of } C &= X_m - X_0 \\ &= 88 - 4 \\ &= 84 \end{aligned}$$

BATSMAN A		BATSMAN B		BATSMAN C	
X	X <sup>2</sup>	Y	Y <sup>2</sup>	Z	Z <sup>2</sup>
12	144	47	2209	15	225
15	225	12	144	23	529
6	36	76	5776	52	2704
73	5329	48	2304	4	16
<del>70</del>	49	4	16	24	576
<del>70</del>	4900	70	4900	70	4900
<del>199</del>	36601	37	1369	74	5476
<del>36</del>	1296	48	2304	52	2704
<del>84</del>	7056	13	169	13	169
29	841	3	9	4	16
$\Sigma X = 531$	$\Sigma X^2 = 59477$	$\Sigma Y = 358$	$\Sigma Y^2 = 19200$	$\Sigma Z = 331$	$\Sigma Z^2 = 17315$

BATSMAN A

$$\bar{X} = \frac{\sum X}{n}$$

$$\therefore n = 10$$

$$= \frac{531}{10} = 53.1$$

$$S_x = \sqrt{\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2}$$

$$= \sqrt{\frac{59477}{10} - \left(\frac{531}{10}\right)^2}$$

$$S_x = 55.92$$

$$C.V = \frac{55.92 \times 100}{\bar{X}} = \frac{55.92}{53.1} \times 100$$

$$C.V = 105.31\%$$

BATSMAN B

$$y = \frac{\sum y}{n} = \frac{358}{10} = 35.8$$

$$s_y = \sqrt{\frac{\sum y^2}{n} - \left(\frac{\sum y}{n}\right)^2}$$

$$= \sqrt{\frac{19200}{10} - \left(\frac{358}{10}\right)^2}$$

$$= 25.26$$

$$C.V = \frac{25.26}{35.8} \times 100$$

$$= 70.76\%$$

BAFSTMANC

$$\bar{z} = \frac{\sum z}{n} = \frac{331}{10} = 33.1$$

$$s_z = \sqrt{\frac{\sum z^2}{n} - \left(\frac{\sum z}{n}\right)^2}$$

$$= \sqrt{\frac{17315}{10} - \left(\frac{331}{10}\right)^2}$$

$$= 25.21$$

$$C.V = \frac{25.21}{33.1} \times 100$$

$$\approx 76.25\%$$

$$76.25\%$$

$\Rightarrow$  A is value of B is less so B is more consistent.

$\Rightarrow$  A with B, B is more consistent

$\Rightarrow$  B with C, B is more consistent

$\Rightarrow$  A with  $C, C$  is more consistent.