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Section "C"

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Subject Differential Equation.

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$$\frac{dy}{dx} = e^{y-t} \sec(y)(1+t^2) \qquad y(0)=0$$

Solution :-

$$\frac{dy}{dx} = \frac{e^y}{e^t} \times \frac{1}{\cos y}(1+t^2)$$

or $\quad e^{-y}\cos y\, dy = e^{-t}(1+t^2)\,dt$

Integrating both side :-

$$\Rightarrow \int e^{-y}\cos y\, dy = \int e^{-t}(1+t^2)\,dt \rightarrow (i)$$

By Parts.

$$\Rightarrow \cos y \cdot \frac{e^{-y}}{-1} - \int \mp \sin y \cdot \frac{e^{-y}}{+1}\,dy = \int e^{-t}(1+t^2)\,dt$$

$$\Rightarrow -e^{y}.\cos y - [-\sin y \cdot e^{-y} + \int \cos y \cdot e^{-y}\,dy] =$$

$$\int e^{-t}(1+t^2)\,dt$$

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$\Rightarrow -e^{-y}\cdot \cos y + e^{-y} \sin y - \int \cos y \cdot e^{-y}\, dy =$

$\int e^{-t} (1+t^2)\, dt$

$\Rightarrow e^{-y} (\sin y - \cos y) - \int \cos y \cdot e^{-y}\, dy =$

$\int e^{-t} (1+t^2)\, dt$

From equation (i)

$\Rightarrow \int e^{-t}(1+t^2)\, dt = \int e^{-y} \cos dy.$

So $\Rightarrow e^{-y}(\sin y - \cos y) - \int e^{-t}(1+t^2)\, dt =$

$\int e^{t} (1+t^2)\, dt.$

$\Rightarrow e^{-y} (\sin y - \cos y) = \int e^{-t} (1+t^2)\, dt$

$\int e^{-t}(1+t^2)\, dt$

$\Rightarrow e^{-y} (\sin y - \cos y) = 2\int e^{-t} (1+t^2)\, dt$

$\Rightarrow e^{-y}(\sin y - \cos y) = 2\,(1+t^2)\frac{e^{-t}}{-1} + \int 2t \cdot e^{-t} dt$

$$= e^{-y}(\sin y - \cos y) = 2\left[e^{-t}(1+t^2) + 2\left(t\frac{e^{-t}}{-1} + \int e^{-t}\,dt\right.\right.$$

$$= e^{-y}(\sin y - \cos y) = 2\left[e^{-t}(1+t^2) + 2[te^{-t} - e^{-t}\right] + C$$

$$= e^{-y}(\sin y - \cos y) = 2\left[-e^{-t} - e^{-t}.t^2 - 2t.e^{-t} - 2e^{-t}\right] + C$$

$$= e^{-y}(\sin y - \cos y) = 2\left[-3e^{-t} - t^2.e^{-t} - 2t.e^{-t}\right] + C$$

$$\boxed{= e^{-y}(\sin y - \cos y) = -2e^{-t}\left[t^2 + 2t + 3\right] + c}$$

Q NO # 02

Solution :-

$$\left(\sqrt{x+y} + \sqrt{x-y}\right)dx - \left(\sqrt{x+y} - \sqrt{x-y}\right)dy = 0$$

$$\frac{dy}{dx} = \frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} \rightarrow (i)$$

This is Homogenous differential equation in x and y to solve this put

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Thus the equation (i) become.

$$v + x\frac{dv}{dx} = \frac{\sqrt{x+vx} + \sqrt{x-vx}}{\sqrt{x+vx} - \sqrt{x-vx}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}} \times \frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{1+\cancel{v}+1-\cancel{v}+2\sqrt{1-v^2}}{2v}$$

$$v + x\frac{dv}{dx} = \frac{\cancel{2}(1+\sqrt{1-v^2})}{\cancel{2}v}$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v} - v$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}-v^2}{v}$$

$$x\frac{dv}{dx} = \frac{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})}{v}$$

$$\frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \frac{dx}{x}$$

taking Integral on both side.

$$\int \frac{v\,dv}{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)} = \int \frac{dx}{x}$$

Put $1+\sqrt{1-v^2} = t$

$$\Rightarrow \frac{1}{2}\left(1-v^2\right)^{-1/2}(-2v)\,dv = dt$$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\int -\frac{dt}{t} = \int \frac{dx}{x}$$

$$-\ln t = \ln x + \ln c$$

$$-\ln\left(1+\sqrt{1-v^2}\right) = \ln cx$$

$$\cancel{\ln}\left(1+\sqrt{1-v^2}\right) = \left(\frac{1}{cx}\right)\cancel{\ln}$$

$$1+\sqrt{1-v^2} = \frac{1}{cx}$$

$$1+\sqrt{\frac{x^2-y^2}{x^2}} = \frac{1}{cx}$$

$$x + \sqrt{x^2 - y^2} = \frac{1}{c}$$

$$x + \sqrt{x^2 - y^2} = c_1 \quad \because \frac{1}{c} = c_1$$

That is Required Solution.

Q NO - 03

$$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x.$$

Solution::

$$= (D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$$

$$\Rightarrow f(D)y = f(x)$$

As it is non homogenous linear equation so solution will be

$$y = y_c + y_p \rightarrow (i)$$

Complementry Solution y_c

$$D^4 + D^2 = 0 \Rightarrow D^2(D^2+1) = 0$$

Either $D^2 = 0 \Rightarrow \boxed{D = 0}$

$$D^2 + 1 = 0 \Rightarrow D^2 = -1$$

$$D = \sqrt{-1} \Rightarrow \boxed{D = i} \text{ or } D = \boxed{0 + i}$$

Root are Real and Complex

$$y_c = C_1 e^{0x} + e^{0x}(C_2 \cos x + C_3 \sin x)$$

$$y_c = C_1 + C_2 \cos x + C_3 \sin x$$

$$y_p = \frac{1}{f(D)} Fx$$

$$y_p = \frac{1}{D^4 + D^2}(3x^2 + 4\sin x - 2\cos x)$$

$$= \frac{3x^2}{D^4 + D^2} + \frac{4\sin x}{D^4 + D^2} - \frac{2\cos x}{D^4 + D^2}$$

$$f(D) = D^4 + D^2$$

at $D = 0 \Rightarrow f(D) = 0$

So $f'(D) = 4D^3 + 2D$

Now also for $D = 0 \Rightarrow f'(D) = 0$

again differentiating.

$f''(D) = 12D + 2$

So for $D = 0$

$f''(0) = 12(0) + 2 = 2.$

So replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

$$\Rightarrow y_p = \frac{x^2 3x^2}{12D+2} + \frac{x^2}{12D+2} \cdot 4\sin x -$$

$$\frac{x^2}{12D+2} \cdot 2\cos x.$$

Putting $D = 0$ in all

So

$$y_p = \frac{x^2 \cdot 3x^2}{12(0)+2} + \frac{x^2 \cdot 4\sin x}{12(0)+2} - \frac{2x^2\cos x}{12(0)+2}$$

$$y_p = \frac{3x^4}{2} + \frac{4x^2\sin x}{2} - \frac{2x^2\cos x}{2}$$

$$y_p = \frac{3}{2}x^4 + 2x^2\sin x - x^2\cos x$$

So

putting in equation (i)

$$y = C_1 + C_2 \cos x + C_3 \cos x + \tfrac{3}{2} x^4 + 2x^2 \sin x - x^2 \cos x$$

$$y = C_1 + (C_2 - x^2)\cos x + (C_3 + 2x^2)\sin x + \tfrac{3}{2} x^4 .$$

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