⇒ Solve the following Objective Type Questions:

(i) The order of matrix "A" is m×p and the order of B is p×n. Then the order of matrix AB is ?

Ans:- (The order of Matrix is equal to the No of its rows multiply by no of the Columns, so the order of Matrix AB is) m×n.

(ii) The number of non-zero rows in an Echelon form?

Ans:- One.

(iii) if $B=\begin{bmatrix}1 & 4\\ 2 & a\end{bmatrix}$ is a Singular matrix then a = ?.

Ans:- a = 8

$\begin{bmatrix}1 & 4\\ 2 & a\end{bmatrix}$

$|M| = a\times1 - 4\times2$
$= a-8$
$a = 8$.

(iv) If $A=\begin{bmatrix}2i & i\\ i & -i\end{bmatrix}$ then $|A| = ?$

Ans:- $|A| = 3$

$|A| = 2i(-i) - i(i)$
$= -2i^2 - i^2$
$= -2(-1) - (-1)$
$= 2+1$
$= 3$.

(v) The matrix $= \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ (2) is ?

Ans:- Scalar Matrix.

(vi) Solution of $dy/dx + 2xy = y$ = ?

$\Rightarrow dy/dx + 2xy = y$

Separting variable.

$\Rightarrow \frac{dy}{dx} = y - 2xy.$

$\Rightarrow \frac{dy}{dx} = y(1-2x).$

$\Rightarrow \frac{dy}{y} = (1-2x)dx.$

$\Rightarrow \int \frac{1}{y} dy = \int 1 dx - \int 2x dx.$

$\Rightarrow \ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + c.$

$\Rightarrow \ln y = x - x^2 + c.$

vii) The order and degree of differential equation $\left(\frac{dy}{dx}\right)^2 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is ?

order = 1

degree = 3.

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viii) The order and degree of differential equation $\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$ is?

Order $= 2$

Degree $= 1$

xi) The differential equation $2\frac{dy}{dx} + x^2y = 2x+3$, $y(0) = 5$ is?

Sol:

$$2\frac{dy}{dx} + x^2y = 2x+3.$$

$y(0) = 5$

Taking Integration.

$$\Rightarrow \int 2dy = \int (2x+3-x^2y)\,dx.$$

$$\Rightarrow 2y = \frac{2x^2}{2} + 3x - \frac{x^3y}{3} + c.$$

$$\Rightarrow y = \frac{\cancel{2}x^2}{\cancel{2}\times 2} + \frac{3x}{2} - \frac{x^3y}{3\times 2} + c.$$

$$\Rightarrow y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3y}{6} + c.$$

put $x = 0$ And $y = 5$

$$5 = 0 + 0 - 0 + c$$

$$c = 5.$$

Then

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3y}{6} + 5.$$

x) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ? (4)

Sol:- Expanding from C1

$1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - 1\begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1\begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix}$

$\Rightarrow 1(bc^2 - cb^2) - 1(a^2c - ac^2) + 1(ab^2 - a^2b)$

$\Rightarrow bc^2 - cb^2 - a^2c + ac^2 + ab^2 - a^2b$

$\Rightarrow a^2c - a^2b + ab^2 - cb^2 + bc^2 - ac^2$

$\Rightarrow a^2(c-b) + b^2(a-c) + c^2(b-a)$

Question No 2

Part A.

Express the Determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

As the product of factors which are linear in a, b, c.

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Solution:-

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expanding from R_1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}.$$

$$\Rightarrow a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$\Rightarrow ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2b^3c - a^3b^2c$$

Taking abc as Common.

$$\Rightarrow abc(\cancel{b}c^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b).$$

$$\Rightarrow abc[bc(c-b) - ac(c-a) + ab(b-a)]$$

Question 2:-

Part B:-

$$\begin{vmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{vmatrix}$$

Solution:-

$$\begin{vmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{vmatrix}$$

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Characteristic equation $\rightarrow [A-\lambda I]=0 \rightarrow$ A

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now Taking DeterminenT.

$$[A-\lambda I]=0.$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix}$$

Expanding from R_1.

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \longrightarrow B$$

Again.

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ Expand by R_1

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$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$\Rightarrow 3-\lambda \left[(3-\lambda)(2-\lambda) - (-1)(-1)\right] + 1 \left[(-1)(2-\lambda) - (-1)(-1)\right]$$
$$-1\left[-1(-1) - (3-\lambda)(-1)\right]$$

$$\Rightarrow 3-\lambda\left[(6-3\lambda-2\lambda+\lambda^2)-1\right] + 1(-2+\lambda-1) - 1(1+3-\lambda)$$

$$\Rightarrow (3-\lambda)(\lambda^2-5\lambda+5) + (-3+\lambda) - (4-\lambda)$$

$$\Rightarrow 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda$$

$$\Rightarrow \boxed{-\lambda^3 + 8\lambda^2 - 18\lambda + 8} \longrightarrow (a)$$

$$\Rightarrow \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$ Expanding from c_1

$$\Rightarrow -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1) + 1(-2+\lambda-1)$$

$$\Rightarrow -\lambda^2 + 5\lambda - 5 - 3 + \lambda$$

$$\Rightarrow \boxed{-\lambda^2 + 6\lambda - 8} \longrightarrow (b)$$

$$\Rightarrow -1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$ Expanding from C_1

$$-\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0.\right]$$

$$\Rightarrow -\left[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right.$$

$$\Rightarrow -(3-\lambda+\lambda^2-5\lambda+5).$$

$$\boxed{-\lambda^2+6\lambda-8} \rightarrow (c)$$

put (a), (b) and (c) in equation (B)

$$(2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right)-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$$

$$\Rightarrow -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2+6\lambda$$
$$-8-\lambda^2+16\lambda-8$$

$$\Rightarrow \lambda^4-2\lambda^3-8\lambda^2+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2-36\lambda-8\lambda$$
$$+6\lambda+6\lambda\cancel{+16}\cancel{-16}.$$

$$\Rightarrow \lambda^4-10\lambda^3+30\lambda^2-32\lambda=0.$$

By Synthatic division we get:

$$\lambda(\lambda-2)(\lambda^2-8\lambda+18)=0.$$

$\Rightarrow \boxed{\lambda = 0} \rightarrow (1).$

$\Rightarrow \lambda - 2 = 0 \Rightarrow \boxed{\lambda = 2} \rightarrow (2).$

$\Rightarrow \lambda^2 - 8\lambda + 16 = 0.$

By factorization Method.

$\lambda^2 - 8\lambda + 16 = 0. \Rightarrow \lambda^2 - 4\lambda - 4\lambda + 16 = 0.$

$\lambda(\lambda - 4) - 4(\lambda - 4) = 0.$

$(\lambda - 4)(\lambda - 4) = 0.$

$(3) \leftarrow \boxed{\lambda = 4} \quad \boxed{\lambda = 4} \rightarrow (4)$

Question No 3

$(x^2 + 3y^2)\,dx - 2xy\,dy = 0.$

$x = 2,\ y = 6.$

Solution:- $(x^2 + 3y^2)\,dx - 2xy\,dy = 0.$

$\Rightarrow (x^2 + 3y^2)\,dx = 2xy\,dy.$

Dividing Both Sides by $(2xy)(dx)$ we get..

$dy/dx = x^2+3y^2/2xy.$

$$\frac{dy}{dx} = \frac{x^{\cancel{2}}}{2\cancel{x}y} + \frac{3y^{\cancel{2}}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{x}{2y} + \frac{3y}{2x}.$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \rightarrow (*)$$

let $y = Vx.$

Diff: $dy = Vdx + xdv.$

Dividing by dx.

$$\boxed{\frac{dy}{dx} = V + x\frac{dv}{dx}} \rightarrow (a)$$

put (a) in eq (*)

$$V + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{V\cancel{x}} + 3\frac{V\cancel{x}}{\cancel{x}}\right]$$

$$V + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{V} + 3V\right]$$

Multiplying Both Sides by "2"

$\Rightarrow \quad 2v + 2x \frac{dv}{dx} = \frac{1}{v} + 3v$.

$\Rightarrow \quad 2x \frac{dv}{dx} = \frac{1}{v} + 3v - 2v$.

$\Rightarrow \quad 2x \frac{dv}{dx} = \frac{1}{v} + v$.

$\Rightarrow \quad 2x \frac{dv}{dx} = \frac{1+v^2}{v}$

Multiplying Both Sides by $\frac{dx}{dv}$.
we get.

$2x \, dv = \frac{1+v^2}{v} \, dx$.

Multiplying Both Sides by $\frac{v}{x(1+v^2)}$.
we get.

$\frac{v}{1+v^2} \, dv = \frac{1}{x} \, dx$

Take "$\int$" on Both Sides.

$\int \frac{2v}{1+v^2} \, dv = \int \frac{1}{x} \, dx + c$.

$\ln|1+v^2| = \ln x + \ln c$

Take "e" on both Sides.

$$e^{\ln|1+v^2|} = e^{\ln x \ln c}$$

$$1+v^2 = xc$$

put $v = y/x$

$$\Rightarrow 1+\left(\frac{y}{x}\right)^2 = xc$$

$$\Rightarrow \frac{x^2+y^2}{x^2} = xc.$$

$$\Rightarrow x^2+y^2 = x^3 c \longrightarrow (*)(*).$$

put $x=2$, $y=6$ in above equation.

$$4+36 = 8c. \Rightarrow \boxed{c = 40/8 = 5}$$

put in eq (**)

(So $x^2+y^2 = 5x^3$

$$y^2 = 5x^3 - x^2. \Rightarrow y^2 = x^2(5x-1)$$

Taking Square Root on Both Sides.

$$y = +x\sqrt{5x-1}, \quad y = -x\sqrt{5x-1}.$$

OR

$$y = \pm x\sqrt{5x-1} \quad \text{Ans.}$$