Department of Computer Science

Date: 13th April 2020

Midterm Assignment –Spring 2020

Course Title: Differential Equations

Instructor: Engr. Latif Jan

Program: BS (CS-SE-EE) **Total Marks: 30** **Time Allowed:** **6 days Note:**

Attempt all Questions:

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Q 1: a) Define differential equation along with 2 examples? **(1+1 Marks)** **b)** Define a S eparable Differential Equation (DE)? **(1+4+3 Marks)**

i. Solve the following **Initial Value Problem (IVP) using separable DE** and find the interval of validity of the solution.

$$\text{(a)}\quad y' = \frac{xy^3}{\sqrt{1+x^2}} \qquad y(0) = -1$$

$$\text{(b)}\quad y' = e^{-y}(2x-4) \qquad (5) = 0$$

Q 2: a) Solve the following IVP using Linear Differential method **(2+5+3 Marks)**

(i) **Explain the steps for solving Linear Differential Equation.** (ii) $co(x)y' + sin(x)y = 2cos^3(x)sin(x) - 1 \quad y\left[\frac{\pi}{4}\right] = 3\sqrt{2}, \quad 0 \le x \le \frac{\pi}{2}$

(iii) $x' + 2x = sint$

Q 3: Solve the following IVP for the exact equation and find the interval of validity for the solution. **(5+5 Marks)**

(i) $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0, \quad (0) = -3$

(ii) $\frac{2ty}{t^2+1} - 2t - (2 - l(t^2 + 1))y' = 0 \quad y(5) = 0$

Q1 a) Define differential equation along with 2 examples.

Ans **Differential Equation:-**

Differential equation is an equation that relates one are more functions & their derivatives In applications, the function generally represent physical quantities, the derivatives represents their rates of change & the differential equation defines a relationship between the two.

Example:

① $\frac{dy}{dx} + xy = e^{2x}$

② $U_{xx} + U_{yy} + 0$

As both of the above example contains the derivative. So these are differential equation.

(b) Define a separable Differential Equation?

Ans **Separable:** A separable differential equation is one that can be broken into set of separate equations of lower dimensionality by a method of separation of variable.

(1) Solve The following initial value problem (IVP) (2)
using separable DE & find the interval of validity of
The solution.

a) $y' = \dfrac{xy^3}{\sqrt{1+x^2}}$ $y(0) = -1$

Sol:

$$\int \frac{dy}{y^3} = \int \frac{x}{\sqrt{1+x^2}}\, dx.$$

$$\Rightarrow \int y^{-3}\, dy = \int \frac{x}{\sqrt{1+x^2}}\, dx.$$

$$\Rightarrow 1+x^2 = u$$

$$2x\, dx = du$$

$$\Rightarrow x\,du = \frac{du}{2}$$

$$\Rightarrow \int y^{-3}\, dy = \int \frac{1}{\sqrt{u}} \frac{du}{2}.$$

$$\Rightarrow \frac{y^{-3+1}}{-3+1} = \frac{1}{2} \frac{u^{-1/2+1}}{-1/2+1} + C$$

$$\Rightarrow \frac{y^{-2}}{-2} = \frac{2}{2} \sqrt{u} + C$$

$$\Rightarrow \frac{1}{-2y^2} = \sqrt{u} + C$$

$$\Rightarrow \frac{1}{-2y^2} = \sqrt{1+x} + C$$

$$\Rightarrow y(0) = -1$$

$\frac{1}{-2(-1)^2} = \sqrt{1} + c$ (2)

$\Rightarrow \frac{1}{-2} = 1 + c$

$\Rightarrow c = -1 - \frac{1}{2} = c$

$\Rightarrow c = \frac{-2-1}{2}$

$\Rightarrow \frac{y^{-2}}{2} = \sqrt{1-1x^2} - 3/2$ Ans

(b) $y' = e^{-y}(2x-4) \quad y(5)=0.$

Sol $\frac{dy}{dx} = e^{-y}(2x-4)$

$\Rightarrow \int \frac{dy}{e^{-y}} = \int (2x-4)\,dx.$

$\Rightarrow \int e^{y}\,dy = \frac{2x^2}{2} - 4x + c$

$\Rightarrow e^{y} = x^2 - 4x + c$

$\Rightarrow y = \ln(x^2-4x) + c$

$\Rightarrow y = \ln((5)^2 - 4(5)) + c.$

$\Rightarrow 0 = \ln(25-20) + c.$

$\Rightarrow 0 = \ln 5 + c.$

$\Rightarrow c = -\ln 5$

$\Rightarrow y = \ln(x^2-4x) - \ln 5$

③ ④

Q2a) Solve the following IVP using Linear Differential method

i) Explain the ~~steps~~ steps for solving Linear Differential Equation.

Following are the steps for solving Linear Differential Equation

1 Substitute = $y = uv$, and

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} + P(x)y = Q(x)$$

② Factor the parts involving v.

③ Involving v term equal to zero in D.E.U.

④ Solve using separation of variable to find u.

⑤ Substitute u back into the equation we got at step 2.

⑥ Solve that to find v.

⑦ Finally substitute u & v into $y = uv$ to get solution.

ii) $\cos(x)\, y' + \sin(x) y = 2\cos^3(x)\sin(x) - 1 \quad y\left[\frac{\pi}{4}\right] = 3\sqrt{2}$

$0 \le x \le \pi/2.$

Sol:. Dividing $\cos(x)$

$$\Rightarrow y' + \frac{\sin(x)\, y}{\cos(x)} = \frac{2\cos^3(x)\sin(x) - 1}{\cos(x)}$$

$$\Rightarrow y' + \tan(x) y = \frac{2\cos^3(x)\cdot\sin(x) - 1}{\cos(x)} \quad -(1)$$

It has the form $y' + P(x)y = Q(x)$

where $P(x) = \tan x$ & $Q(x) = \dfrac{2\cos^3(x)\cdot\sin(x) - 1}{\cos(x)}$

(5)

where $P(t) = 2$ and $Q(t) = \sin t$.

Integrating Factor $= e^{\int P(t)\,dt}$

$I.F = e^{\int 2\,dt} = e^{2t}$

multiplying (1) by e^{2t} we get

$$e^{2t}x' + 2xe^{2t} = 2e^{2t}\sin t$$

$$\Rightarrow \frac{d}{dt}\left[xe^{2t}\right] = e^{2t}\sin t$$

$$\Rightarrow \int d\left[xe^{2t}\right] = \int e^{2t}\sin t\,dt$$

$$\Rightarrow xe^{2t} = \int e^{2t}\sin t\,dt$$

Solving by Integration by Parts method.

$$\Rightarrow xe^{2t} = \frac{(2\sin t - \cos t)e^{2t}}{5} + c$$

$$\Rightarrow x = e^{-2t}\left[c + \frac{2e^{2t}\sin t}{5} - \frac{e^{2t}\cos t}{5}\right].$$

Q3 Solve the following IVP for the exact equation & find the interval of validity of the solution.

(1) $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0$ $y(0) - 3$.

Sol: $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0$ —(i)

$$\Rightarrow (2y + x^2 + 1)\frac{dy}{dx} = -2x + 9x^2.$$

$\Rightarrow (2y + x^2 + 1)\, dy = (9x^2 - 2xy)\, dx$

$\Rightarrow (9x^2 - 2xy)\, dx - (2y + x^2 + 1)\, dy = 0$

$(9x^2 - 2xy)\, dx + (-2y - x^2 - 1)\, dy = 0$

$M(x,y)\, dx + N(x,y)\, dy = 0$

$\frac{\partial M}{\partial y} = -2x$

$\frac{\partial N}{\partial x} = -2x$

So, Exact Equation. Solution exist.

$\int M dx + \int$ (term of N, (free of x) dx.

y-axis.

$\int (9x^2 - 2xy)\, dx + \int (-2y - 1)\, dy = c$

$\Rightarrow \frac{9x^3}{3} - \frac{\cancel{2}xy}{\cancel{2}} - \frac{\cancel{2}y^2}{\cancel{2}} - y = c$

$\frac{9x^3}{3} - x^2y - y^2 - y = c$

$y(0) = -3$

$-(-3)^2 - (-3) = c$

$-9 + 3 = c$

$\boxed{c = -6}$

⑦ ⑧

ii) $\frac{2ty}{t^2+1} - 2t - (2 - \ln(t^2+1))y' = 0 \quad y(5) = 0$

Sol: $\frac{2ty}{t^2+1} - 2t - (2 - \ln(t^2+1)] \frac{dy}{dx} = 0.$

$(\frac{2ty}{t^2+1} - 2t)\,dx - (2 - \ln(t^2+1)\,dy = 0.$

$M(t,y) = \frac{2ty}{t^2+1} - 2t.$

$N(t,y) = \ln(t^2+1) - 2.$

$\frac{\partial M}{\partial y} = \frac{2t}{t^2+1}$

$\frac{\partial N}{\partial t} = \frac{2t}{t^2+1}$

So Exact Equation, solution exist.

$\int M\,dx + \int(\text{term of } N \text{ free of } x)\,dy.$

y-axis.

$\int(\frac{2ty}{t^2+1} - 2t)\,dt + \int -2\,dy = c.$

$y\ln(t^2+1) - t^2 - 2y\ c.$

$y(5) = 0.$

$-(5)^2 = c.$

$\boxed{c = -25}$