FINAL TERM

(summer)

Name: Syed Najeeb

ID: 7855

Section: "B"

Semester: "6th"

Subject: Fluid Mechanics II

Instructor: Abdul waheed

01 7855 Question # 01

Q#01 Part A

Answer:

Forces on immersed body:

A body which is widly immersed in a homogeneous fluid may be subjected to two kind of forces using relative motion between body and fluid.

These forces are termed as drag and lift depending on weather force is parallel or at right angles to motion.

Drag forces on submerged body can have two components.

1: Pressure Drag FP

2: Friction Drag:

1: Pressure Drag FP:

It is equal to the integration of components in the direction of motion at all pressure forces exerted on surface of the body.

$$FP = CP \rho \frac{V^2}{2} A$$ CP — depends on shape

02 7855 Question #01 A

2: Friction Drag:

It is equal to integration of components of shear stress along the surface of the body in direction of motion.

$$F_D = C_D \rho \frac{U^2}{2} BC$$ C_D – depends on viscosity

Friction Drag of Boundary layer.

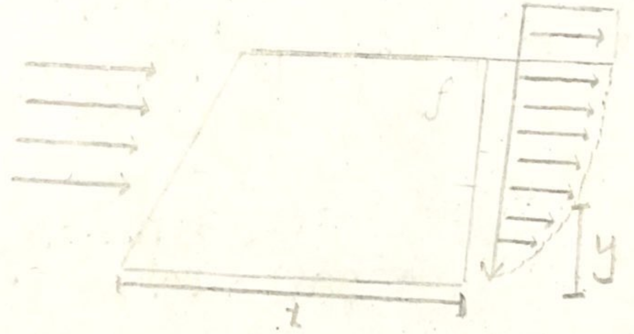
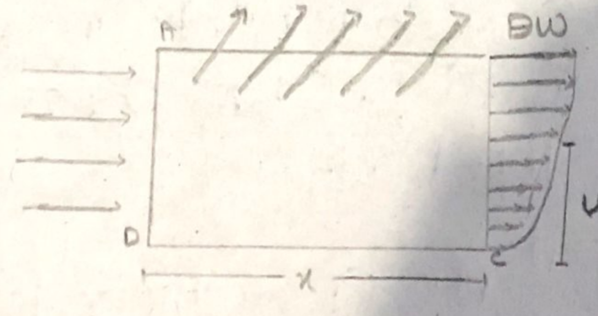


Figure shows growth of boundary layer along one side of smooth plate in steady flow of incompressible fluid concider a control volume.



03 7855

where δ is distance from boundary layer to plate - U is undistributed velocity.

Now - Fx = - drag = rate of momentum in x-direction leaving through BC + rate of momentum in x-direction leaving through AB - rate of momentum in x-direction entering through DA.

According to impulse momentum principle.

$$\Sigma F = \frac{d(mv)}{dt} = \frac{(\rho \times vol) \times v}{dt} = \rho Q v$$

$$\Sigma F_x = \rho Q_2 v_2 - \rho Q_1 v_1$$

A: $\rho U (UB\delta)$

B: $\rho(UB\delta - B\int_0^\delta u\,dy)U$

C: $\rho B \int_0^\delta u^2 dy$

Putting values

$$F_D = \rho B \int_0^\delta u(U-u)\,dy$$

Solving this,

$F\alpha = \rho B U^2 \delta_2$ where α is a function of boundary layer velocity distribution only.

"How to find local shear stress

04/ 7855

$$\tau = \frac{F_x}{Area} \longrightarrow \tau_0 = \frac{dF_x}{B \cdot dx}$$

$$F_x - \rho B U^2 \delta_\alpha$$

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$$

Laminar Boundary Layer:

In case of laminar flow

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0} = \frac{\mu}{\delta}\left(\frac{du}{dy}\right) = \frac{\mu u}{\delta}\left(\frac{d f}{d n}\right)_{n=0}$$

By solving

$$\tau_0 = \frac{\mu U B}{\delta} \quad \text{—(ii)}$$

Equating $\Rightarrow \tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$

$$\delta\, d\delta = \frac{\mu B}{\rho U \alpha}\, dx$$

Solving it

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho U \alpha} x + C$$

At $x = 0$, $\delta = 0$, $C = 0$

$$\delta = \sqrt{\frac{2\mu B x}{\rho U \alpha}} = \sqrt{\frac{2B}{\alpha}} \cdot \frac{x}{\sqrt{R_x}} \qquad R_x = \frac{x U \rho}{\mu}$$

05 7855

Experimentaly $B = 1.63$, $\alpha = 0.133$

Putting values in eq ①

$$\frac{\delta}{x} = \sqrt{\frac{2 \times 1.63}{0.133}} \times \frac{x}{\sqrt{R_x}}$$

$$= \frac{4.91}{\sqrt{R_x}}$$

$$\tau_o = 0.332 \frac{\mu U}{x} \sqrt{R_x}$$

Where R_x may be called the local reynold number. It should be noted that R_x increases linearly in down stream direction.

now

$$F_D = B \int_0^L \tau_o \, dx \longrightarrow$$

$$\tau_o = 0.332 \frac{\mu U}{x} \sqrt{R_x}$$

$$R_x = \frac{x U \rho}{\mu}$$

Thus $F_D = 0.664 B \sqrt{\rho \mu L U^3}$

Where $F_D = C_D \rho \frac{U^2}{2} BL$

Equ bo both

$$C_D = 1.328 \sqrt{\frac{\mu}{\rho L U}} = \frac{1.328}{\sqrt{R}}$$

Ob 7855

where R is based on characteristics length of whole plate. The laminar boundary layer will remain laminar if R_x is of about 50900.

Turbulant Boundary layer:

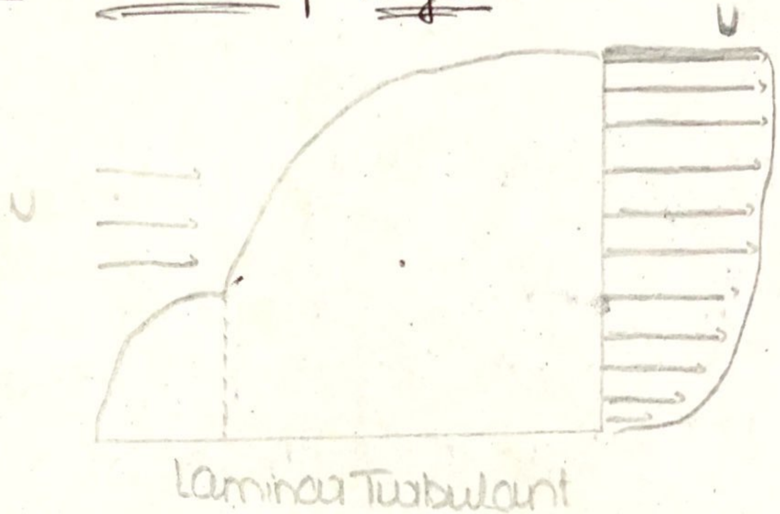


Figure shows the velocity distribution of boundary layer which is steeper near wall and flatly through out remainder of layer.
The shear stress is greater in turbulant layer than laminar.

Thus $\tau_0 = f \frac{\rho v^2}{8}$ where "v" is always velocity.

To obtain relation between average and max velocity we have $\frac{v}{U_{max}} = \frac{1}{1+1.33\sqrt{f}}$

where f = 0.028 from moody's chart.

Thus

$U = 1.233V$

07 7835

Now $\tau_0 = f \frac{\rho u^2}{8}$

where $u = \frac{U}{1.235}$

and $f = \frac{0.316}{R^{0.25}} = \frac{0.316}{\left(\frac{Du}{\nu}\right)^{1/4}}$

Putting values

$\tau_0 = \frac{0.316}{\left(Du/\nu\right)^{1/4}} \times \rho \times \frac{u^2}{8}$ where $u = \frac{U}{1.235}$, $D = 2\delta$

$\tau_0 = 0.316$

$\left[\frac{2\delta}{\nu} \times \left(\frac{U}{1.235}\right)\right]^{1/4} \times \frac{\rho}{8} \times \left(\frac{U}{1.235}\right)^2$

$\tau_0 = \frac{0.023 \rho U^2}{\left(\frac{\delta U}{\nu}\right)^{1/4}} \longrightarrow (1)$

As we have general equation

$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx} \longrightarrow (2)$

Equation (1) & (ii) are integrating for boundary layer condition.

i.e. $x = 0$, $\delta = 0$

$\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5} \left(\frac{\nu}{Ux}\right)^{1/5} \times x$

For, $\alpha = 0.0972$

$\delta = \frac{0.377}{(R_x)^{1/5}} \times x \longrightarrow (3)$

Putting eq (3) in (ii)

08 7855

$$\tau_0 = 0.0587 \rho \frac{U^2}{2} \left(\frac{\nu}{Ux}\right)^{1/5}$$

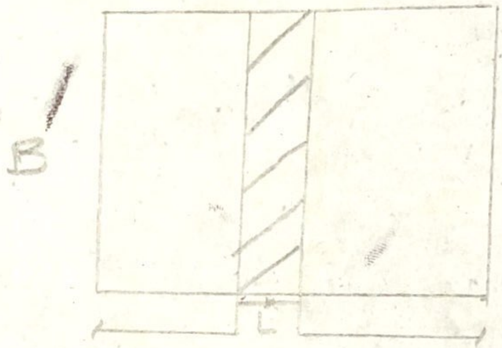
now

$$F_D = B \int_0^L \tau_0 \, dx$$

$$F_D = 0.0735 \rho \frac{U^2}{2} \left(\frac{\nu}{UL}\right)^{1/5} \times BL$$

As we have

$$F_D = C_D \rho \frac{U^2}{2} \times BL$$



Equating B/S

$$C_D = \frac{0.0735}{R^{1/5}}$$

$\therefore (500 < R < 10^7)$

For $R > 10^7$

$$C_D = \frac{0.455}{\log R^{2.58}}$$

09 7855 Question # 01 / B

Q#01 Part / B

Derivation: $E = y + \frac{v^2}{2g}$

Assuming 'q' is discharge per unit width "b"

Thus $q = \frac{Q}{b}$

Average velocity will be

$v = Q/A = \frac{qb}{bxy} = \frac{q}{y}$ or $q = vxy$

Now

$E = y + \frac{v^2}{2g} = y + \frac{q^2}{y^2 \times 2g}$

→ Lets consider how E will vary with y if "q" remains constant.

Thus,

$(E - y) = \frac{q^2}{2g \times y^2}$

$\Rightarrow (E - y)y^2 = \frac{q^2}{2g}$ (constant)

For particular "q" there will be three kind of values of "y" with two roots positive and one negative thus two values represent two "y" different situation as slow and deep and a fast and shallow. The point dividing the flow is critical point where energy is minimum and depth is critical depth.

→ The relation of critical depth is

$E = y + \frac{1}{2g} \times \frac{q^2}{y^2}$

For minimum specific energy. $\frac{dE}{dy} = 0$

Thus, $\frac{dE}{dy} = 1 - \frac{2q^2}{2g(y)^3} = 0$

Thus

$$\frac{q^2}{g(y)^3} = 1 \Rightarrow q^2 = gy^3 \text{ or } \left(\frac{q^2}{g}\right)^{1/3} = y_{cr}$$

As

$q = v_{cr} y$ thus, $q^2 = gy^3$

$\rightarrow (v_{cr} y)^2 = gy^3 \Rightarrow v_{cr}^2 = g y_{cr}$

$$v_{cr} = \sqrt{gy}$$

or $$y_{cr} = \frac{v_{cr}^2}{g}$$

$\rightarrow$ As E is minimum at critical point

Thus

$$\frac{y_{cr}}{2} = \frac{v_{cr}^2}{2g}$$

$$E_{min} = y_{cr} + \frac{v_{cr}^2}{2g} \longrightarrow \frac{y_{cr}}{2} = \frac{3}{2} y_{cr}$$

$$E_{min} = \frac{3}{2} y_{cr} \text{ or } y_{cr} = \frac{2}{3} E_{min}$$

subcritical	critical	supercritical
$y > y_{cr}$	$y = y_{cr}$	$y < y_{cr}$
$v < v_{cr}$	$v = v_{cr}$	$v > v_{cr}$

11   7855   Question # 02

Q#02 ⑥

Sol:

Given data

$Q = 3.5\,m^3/s$

$S_o = 0.0008$

$n = 0.0219$

$D = 7855\,mm = 7.855\,m$

Required:

$y = ?$

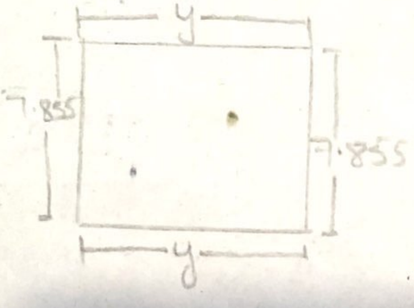
$y_{cr} = ?$

$N_{cr} = ?$

$$Q = \frac{1}{n} A R_h^{2/3} S_o^{1/2} \longrightarrow ①$$

$$A = y \times b = y \times 7.855 \Rightarrow A = y \times 7.855$$

$$R_h = \frac{A}{P} = \frac{7.855y}{2y + 15.71}$$



Putting values in eq ①

$$3.5 = \frac{1}{0.0219} \times 7.855y \left(\frac{7.855y}{2y+15.71}\right)^{2/3} \times (0.0008)^{1/2}$$

$y = 0.7217\,m$

$$\boxed{y = 721.7\,mm}$$

12 7855

Now

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

$$q = \frac{Q}{b} = \frac{3.5}{7.855} = 0.446$$

$$y_{cr} = \left(\frac{0.446^2}{9.81}\right)^{1/3}$$

$$y_{cr} = 0.273$$

$$\boxed{y_{cr} = 273mm}$$

Now

$$v_{cr} = ?$$

$$v_{cr} = \sqrt{gy_{cr}}$$

$$v_{cr} = \sqrt{9.81 \times 0.273}$$

$$\boxed{v_{cr} = 1.64 m/s}$$

Since

$$y > y_{cr}$$

Thus the flow is subcritical.

13 7855 Question # 03

Q#03
Sol:

Given data:

wide $.B = 200mm$
length. $L = 800mm$
specific gravity $= 0.89$
undisturbed velocity, $U = 5$
Kinetic viscosity $.V = 0.93 \times 10^{-4} m^2/s$

Solution:

As we know that

$$R = \frac{LU}{V}$$

$$N = 0.93 \times 10^{-4} m^2/s$$

$$L = 0.80$$

$$U = 5$$

Putting these values

$$R = \frac{0.80 \times 5}{0.93 \times 10^{-4}} = 43010 < 500,000$$

Thus

$$C_b = \frac{1.328}{\sqrt{R}}$$

$$= \frac{1.328}{\sqrt{43010}} = 0.0064$$

Now

$$F_b = C_b \rho \frac{U^2}{2} \times BL$$

Scanned with CamScanner

14 7855

$$F_b = 0.006 \times 0.89 \times 1000 \times \frac{(5)^2}{2} \times 0.20 \times 0.80$$

$$= 53.4$$

$$\frac{S}{x} = \frac{4.91}{\sqrt{Rx}} \quad \text{at} \quad x = L$$

$$S = \frac{4.91}{\sqrt{43010}} \times 80\,cm$$

$$S = 189\,cm$$

$$F_b = 0.664 \times B \sqrt{SU.LV^3}$$

$$= 0.664 \times 0.20 \sqrt{0.89 \times 1000 \times 1000 \times 0.89 \times 0.93 \times 10^{-4} \times 0.80 \times (5)^3}$$

$$\boxed{F_b = 11.39\,N}$$