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SECTION: "C"

SUBJECT: Earth quake & dyn engineering

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FINAL PAPER.

IQRA NATIONAL UNIVERSITY.

(1)

QUESTION NO 1

ANSWER:-

GIVEN DATA:

- → length of Beam $= L = 10'$
- → Beam pulled in downward direction $= 1/2''$
- → $E = 29000$ KSI
- → $I = 150 \text{ in}^4$
- → $S_{st} = 7730$ lb
- → Ignore self wt & damping effect.

REQUIRED:

1. Natural time period = ?
2. develop & solve equation of motion for vibrations at free end = ?
3. Develop eq showing variation in the Equivalent static force with time = ?
4. amplitude of equivalent static force = ?

(5) Graph = ?

Solution:-

The general E.O.M for SDOF system is

$$ku + c\dot{u} + m\ddot{u} = P(t)$$

undamped system (c=0) undergoing free vibration $P(t) = 0$

So general EOM become

$$ku + m\ddot{u} = 0 \quad \text{—}(1)$$

$$k = \frac{3EI}{L^3}$$

$$k = \frac{3 \times 29000 \times 150}{(10 \times 12)^3}$$

$$\boxed{k = 7.55 \ k/in}$$

In order to eliminate the chances of mistake during calculation, it is more appropriate to use fundamental unit like lb, ft or kg, m, sec.

$K = 7.55 \text{ k/in} = 90625 \text{ lb/ft}$

$m = \dfrac{7730 \text{ lbs sec}^2}{32.2 \text{ ft}} = 240.06 \text{ slug}$

$\omega_n = \sqrt{k/m} = \sqrt{\dfrac{96025}{240.06}}$

$\omega_n = 20 \text{ rad/sec}$

$T_n = 2\pi/\omega_n = 2\pi/20 = \boxed{0.314 \text{ Sec}}$

putting values in eq ①

$$\boxed{90625u + 240.06\ddot{u} = 0}$$

where k is in lb/ft & m is in lb sec/ft²

General solution to the EOM for undamped free vibration is,

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)$$

$u(0) = \frac{1}{2}'' = \frac{1}{24}$ ft & $\dot{u}(0) = 0$.

$$u(t) = \left(\frac{1}{24}\right) \times \cos(20t) + 0$$

$$\boxed{u(t) = \left(\frac{1}{24}\right) \times \cos 20t}$$

Equivalent static force at any time "t" is

$$f_s(t) = k \cdot u(t) = \frac{90625 \times \cos(20t)}{24}$$

$$\boxed{f_s(t) = 3776 \cos(20t)}$$

Amplitude of dynamic displacement, u_o for undamped free vibration is

$$u_o = \sqrt{\left[(u(0))^2 + \left(\frac{\dot{u}(0)}{\omega_n}\right)^2\right]}$$

(2)

$$= \sqrt{\left(\left(\tfrac{1}{24}\right)^2 + 0\right)}$$

$$\boxed{u_0 = \tfrac{1}{24} \text{ ft}}$$

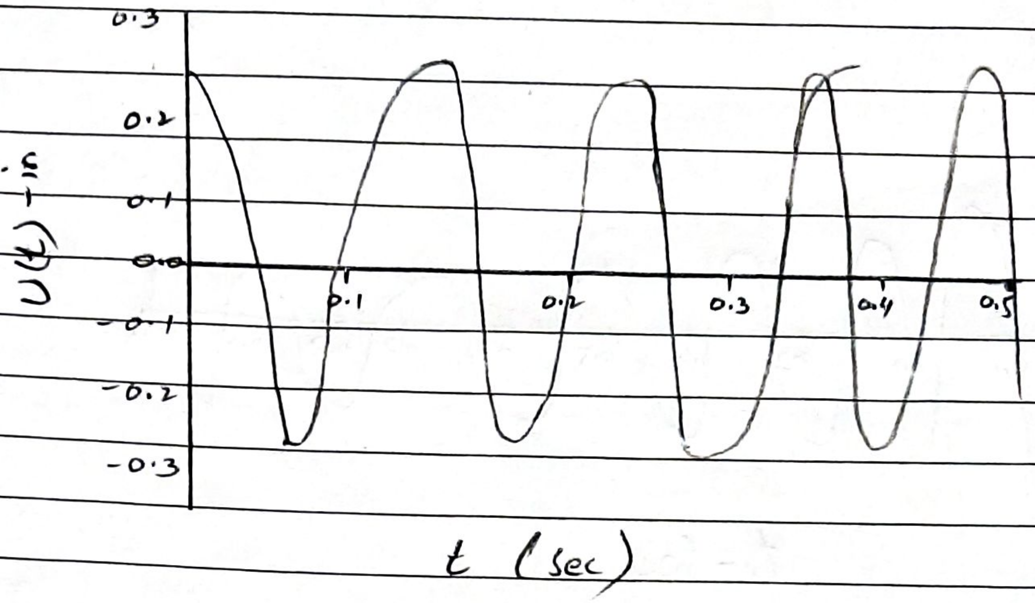
Amplitude of equivalent static force, f_{s0}

$$\boxed{ku_0 = 90625 \times \tfrac{1}{24} = 3776 \text{ lb}}$$

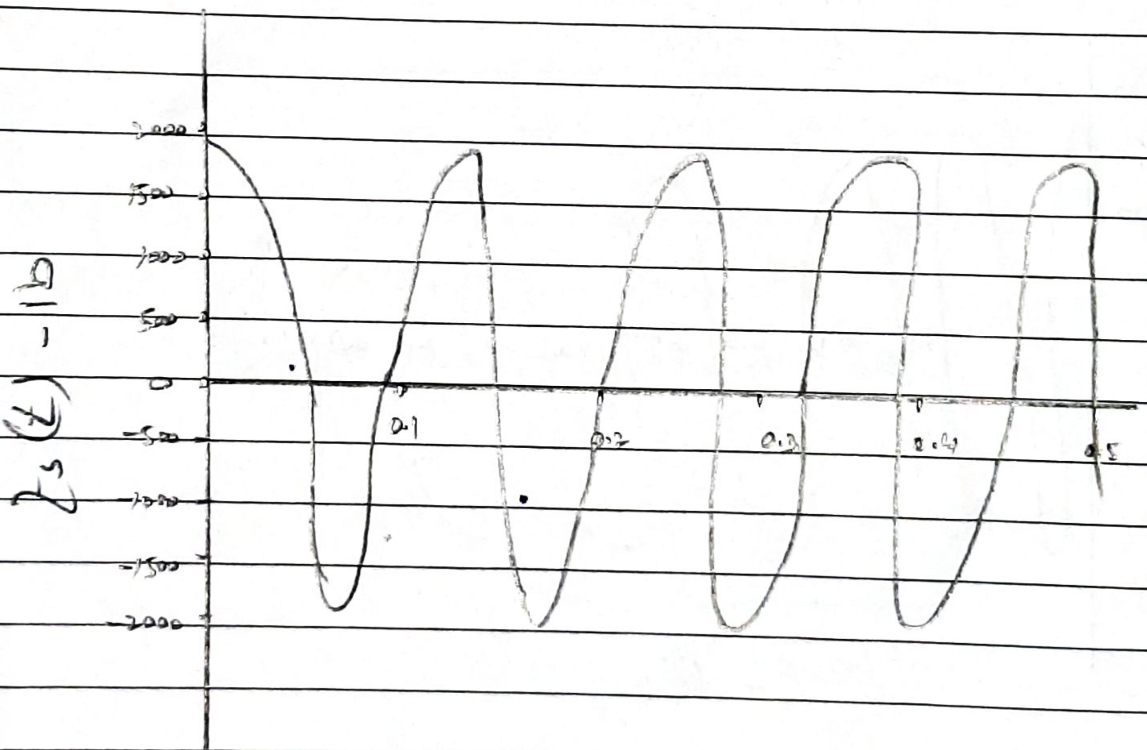
⑤

Date:

UNDAMPED FREE VIBRATION :-



Variation of displacement with time



QUESTION NO. 02.

ANSWER:-

GIVEN DATA:

use required data from Question "01"

REQUIRED DATA:

① develop & solve the eq of motion for vibrations resulting at free end.

② develop eqs showing variation in equivalent static forces with time.

③ Draw Graph to show the variation of displacement with time & variation of equivalent static forces with time.

SOLUTION:-

ξ (Damping ratio) for RCC

Min = 0.80% , Avg = 1.3%

So we consider 1.3%

E.O.M for damped free vibration

$$ku + c\dot{u} + m\ddot{u} = 0 \quad \text{——— (1)}$$

from prob (1)

$k = 90625$ lb/ft.

$m = 240.06$ lb. sec^2/ft.

$c = \xi \times 2m\omega_n$

$c = 2 \times 240.06 \times 20 \times 0.013$

$$c = 124.83 \text{ lb.sec/ft}$$

Putting values of k, m, & c in eq (1)

$$(1) \Rightarrow \boxed{90625u + 124.83\dot{u} + 240.06\ddot{u} = 0}$$

(3)

Solution to the EOM for damped free vibration.

$$u(t) = e^{-\zeta \omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\dot{u}(0) + u(0)\zeta\omega_n\right]\cdot \sin(\omega_D t)\right].$$

$$\omega_D = 20 \text{ rad/sec}.$$

$$u(t) = e^{-0.013 \times 20 t}\left[1/24 \times \cos(20t) + \frac{1}{20} \times \left[0 + 1/24 \times 0.013 \times 20 \times \sin(20t)\right]\right]$$

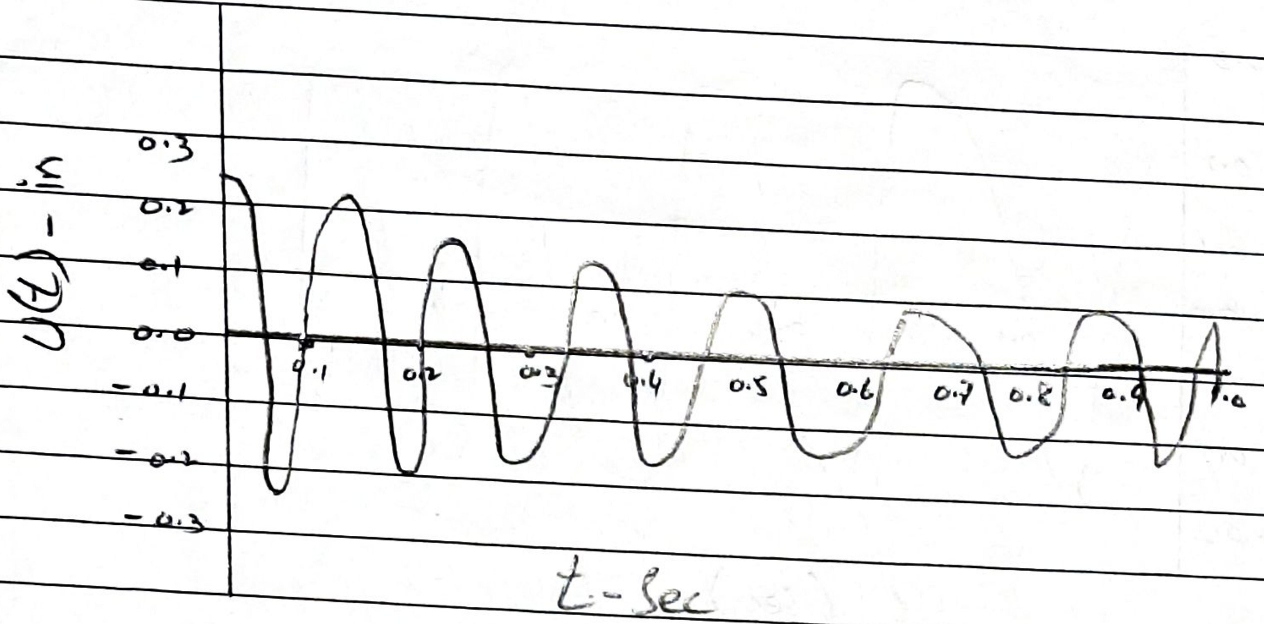
$$u(t) = e^{-0.26t}\left[0.041 \times \cos(20t) + 0.0005416 \sin(20t)\right].$$

$$f_s(t) = k \cdot u(t) = 90625 \times u(t).$$

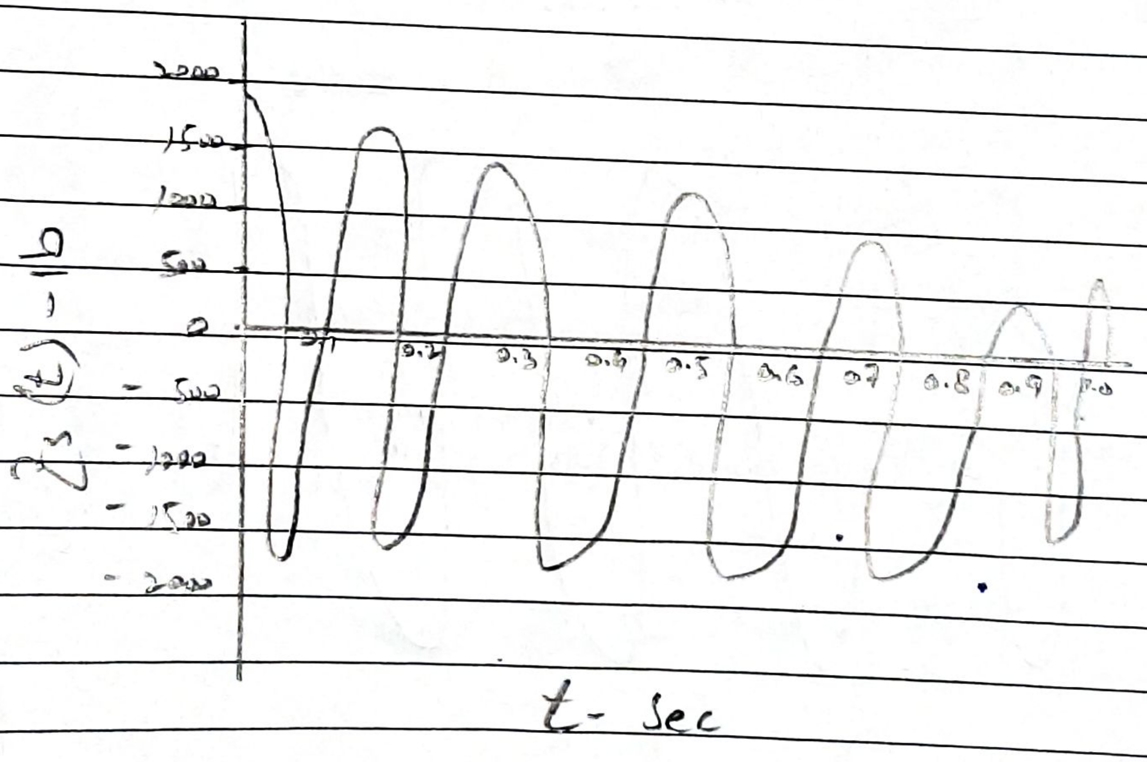
$$f_s(t) = e^{-0.26t}\left[3715 \cos(20t) + 49.08 \sin(20t)\right].$$

Date:

⇒ DAMPED FREE VIBRATION



⇒ DAMPED FREE VIBRATION



QUESTION NO 3.

ANSWER:-

GIVEN:-

- Force = F = 60 kips.
- displacement = 7730/1000 = 7.73"
- no of cycle 7 are complete in 3.57 sec.
- Amplitude = 2.286 cm = 0.9"
- Ignore vertical vibration.

REQUIRED:-

a. Damping ratios
b. Natural period of un-damped vibration
c. Stiffness of Structures
d. weight of tank
e. Damping Coefficient.
f. No of cycles to reduce displacement to 0.5"

SOLUTION:-

$u_1 = 7.73''$

After $j = 7$, $u_{j+1} = u_8 = 0.9''$

(a) ζ = Damping ratio = ?

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$7 = \frac{1}{2\pi\zeta} \ln(7.73/0.9)$$

$$\zeta = \frac{2.15}{2\pi(7)}$$

$$\boxed{\zeta = 0.0488 = 4.88\%}$$

(b) T_n = ?

7 cycles of vibration are completed in 3.57 secs.

Now

Time required for 1 cycle $= T_D = 3.57/7$.

(d)

$T_D = 0.51$ sec

Now

$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$

$2\pi / \omega_D = 2\pi / \omega_n \sqrt{1-\zeta^2}$

$T_D = \dfrac{T_n}{\sqrt{1-\zeta^2}}$

$T_n = T_D \times \sqrt{1-\zeta^2}$

$T_n = 0.51 \times \sqrt{1-(0.0488)^2}$

$T_n = 0.5093 = 0.51$ sec

(e) $k = ?$

$k = \dfrac{60 \times \cos 60^\circ}{2} = 15$ k/in

$k = 18000$ lb/ft.

(d) weight of tank, $W = ?$

$$\omega_n = \sqrt{K/m} = \sqrt{\frac{K}{W/g}} = \sqrt{\frac{K \times g}{W}}$$

$$\omega_n^2 = \frac{K \times g}{W}$$

$$W = \frac{K \times g}{\omega_n^2}$$

Also $\omega_n = 2\pi/T_n$.

$$W = \frac{K \times g}{4\pi^2/T_n^2}$$

$$W = K \times g \times \frac{T_n^2}{4\pi^2}$$

~~Wt = 18000 ... / Ff~~

$$W = 18000 \times 32.2 \times \frac{(0.51)^2}{4\pi^2}$$

$$\boxed{W = 3818.64 \text{ lb} = 3.81 K}$$

(e) $c = ?$

it is known that $\zeta = \frac{c}{2m\omega_n}$

$$c = \zeta \times 2m\omega_n$$

$$c = \zeta \times 2m\left(2\pi/T_n\right)$$

$$c = 0.0488 \times 2 \times \left(\frac{3818.64}{32.2}\right)\left(\frac{2\pi}{0.51}\right)$$

$$\boxed{c = 142.59 \text{ lb}\cdot\text{sec/ft}}$$

(f) No of cycle to reduce displacement amplitude from 7.73" to 0.5", $j = ?$

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$j = \frac{1}{2\pi \times 0.0488} \ln\left[\frac{7.73}{0.5}\right]$$

$$\boxed{j = 8.93 \text{ or } 9 \text{ cycles}}$$